The D-criterion for the Perseid stream

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An analysis of the upper limit value of the D-criterion of family association of meteoroid bodies in the Perseid stream is presented. On the basis of modeling of ejections of meteoroids in the stream out of the parent comet at different points of the comet it is shown that D value for the Perseids is not higher than 0.1 over the whole most probable range of ejection speeds.

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1 Introduction

Criteria of family association of the orbits of celestial bodies are widely used in astronomy. For example, they are used for identifying small bodies of the solar system with possible parent bodies, discovering family connections with asteroid groups, meteor associations, and designation of the elements of mean orbits of meteoroid streams.

Similarity of the Keplerian elements of their orbits is considered to be a criterion for family association of two bodies. This question, which is based on the solution of a limited three-body problem, was considered for the first time by F. Tisserand in 1889. It was concluded that a necessary condition for the family association of two comets is the proximity of their invariants:

$$C = a^{-1} + 0.16860 p^{1/2} \cos(i) \tag{1}$$

where a, p, and i are the orbit parameters of comet. The value C was called the Tisserand constant or Tisserand criterion. As for the small meteoroid streams the distribution according to the value of the Tisserand constant mostly corresponds to the short-period comet distribution. The maximum of both distributions coincide when $C = +0.55$, besides about 40% of the small streams have Tisserand constants in the interval from +0.45 to +0.60, which corresponds to the Jupiter group of short-period comets. The result of studying the value of the Tisserand criterion statistically can show connections between some asteroids, comets and small meteoroid streams.

Later the D-criterion, in which the proximity of orbital parameters between bodies in the 5-dimensional phase space (Southworth & Hawkins, 1963; Drummond, 1981; Klačka, 1999) and others is taken as a measure of the family association, was suggested to research the evolution of meteoroid streams. The most widespread criterion was suggested by R.B. Southworth & G.S. Hawkins (1963). For the two bodies being considered the D-criterion is represented by the formula:

$$\begin{aligned} D^2 &= (e_2 - e_1)^2 (q_2 - q_1)^2 \\ &+ (2\sin(I/2))^2 + ((e_2 - e_1)/2)^2 (2\sin(W/2))^2 \end{aligned} \tag{2}$$

where

$$\begin{aligned} \left[2\sin\left(\frac{I}{2}\right)\right]^2 &= \left[2\sin\left(\frac{i_2 - i_1}{2}\right)\right]^2 \\ &+ \sin(i_1)\sin(i_2)\left[2\sin\left(\frac{\Omega_2 - \Omega_1}{2}\right)\right]^2 \end{aligned}$$

and

$$\begin{aligned} W &= \omega_2 - \omega_1 \\ &\pm 2\arcsin\left(\cos\left(\frac{i_2 + i_1}{2}\right)\right. \\ &\left.\sin\left(\frac{\Omega_2 - \Omega_1}{2}\right)\sec\left(\frac{I}{2}\right)\right) \end{aligned}$$

and I is the reciprocal angle of orbits, W is the angle between directions towards perihelion, and e, a, q, i, w, ω and Ω are orbital elements. The $\pm$ is negative when $|\Omega_2 - \Omega_1| > 180°$. It is noted that the two bodies will have the same origin if the distance between their orbits in the given area will appear to be less than some given value of D. The method assumes the measurement errors in the orbital elements to be considerably smaller than the real dispersion of the orbits in the stream.

The basic problem of using the D-criterion is in the choice of the value of D as the measure of the common origin of two bodies. The upper limits of D are from 0.115 to 0.30 in different sources. While researching the meteor association or finding mean stream orbit for the Perseid, Geminid, Orionid and other meteor showers, D is taken equal to 0.2. But the studies show that for example showers slightly inclined to the ecliptic (such as the Taurids) cannot be clearly distinguished from the background by using this value. In conclusion, we can assume that using the same upper limit for all showers can be used just as a first approach. For a better identification of meteor bodies the upper limit of D has to be defined for every meteor complex individually. Most probably the definition of the D-criterion for meteor complexes is different and serves to be as some evolutionary characteristic of a given meteoroid stream.

2 Results

This particular work considers various D-criterion values for the Perseid meteoroid stream. Following the disintegration of the comet nucleus, the orbits of ejected fragments are connected with the parent body, so the D-criterion value will depend on the particle ejection speed and the point of its ejection round the orbit. That is why the limit of the D value can be found from the

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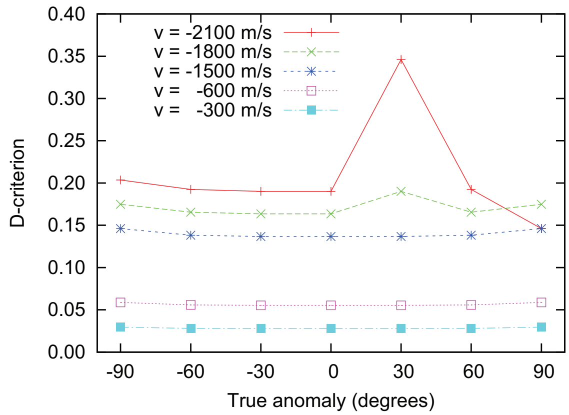

Figure 1 – The D-criterion at the moment of ejection in 1348 depending on the ejection speed and the location of the ejection point in the comet orbit.

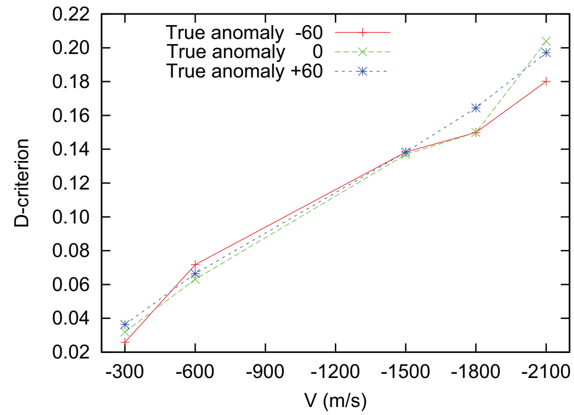

Figure 2 – The D-criterion at the moment of ejection in 1348 depending on the ejection speed.

physically possible speeds of particles from the nucleus of a comet.

The process of modeling the formation of a Perseid meteoroid stream made by the authors earlier is described in detail in their works (Ishmukhametova & Kondrat'eva, 2004; Ishmukhametova & Kondrat'eva, 2006). The elements of a parent comet 109P/Swift-Tuttle are taken from the catalogue of B. Marsden (1995). Using the results of modeling the ejection of Perseids from their parent comet in 1348, we analyze the D-criterion value for different ejection speeds of model particles in different points of the comet orbit.

As an example, we will consider only the ejection perpendicularly to the radius vector and in the opposite direction to the comet movement (type III, vector T, $V < 0$). The values of the D-criterion for two orbits of a comet-meteor, which are found through (eqn. 2), for a range of ejections speeds from 300 – 2100 m/s before and after the comet's perihelion are shown in Figure 1.

In Figure 2 there is a dependence of the D-criterion on the ejection speed of particles in 1348 at the points of the comet orbit with true anomalies of $-60°$, $0°$ and $+60°$. The D-criterion practically does not change for particles ejected with the same speed at different points in the orbit. According to the present day understanding of physical and chemical model of disintegration of the comet nucleus while approaching the Sun, the ejection speeds are not higher than 600 m/s. We can conclude that for the model Perseid stream particles just ejected, the value of the D-criterion is not higher than 0.075 and stays practically equal for the particles ejected before and after perihelion.

The D-criterion does not take into account gravitational and non-gravitational orbit perturbations. But because of planetary perturbations orbits of meteoroids in the steam can be very different from each other. That is why it is interesting to trace the dynamics of the D-criterion values depending on the evolution of the stream. The orbital elements of model particles ejected in 1348 were integrated before 1862 taking into account perturbations from all the planets. The D-criterion

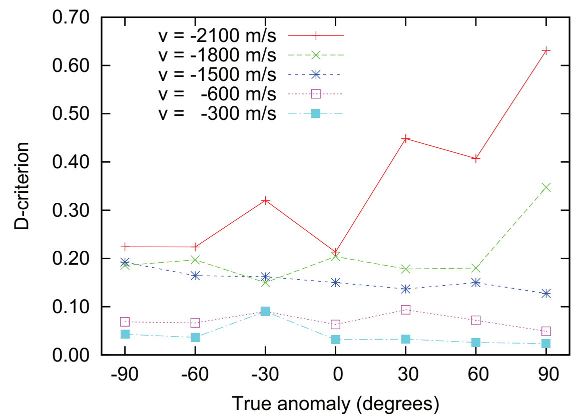

Figure 3 – The D-criterion 500 years after the particle ejection as a function of true anomaly of the ejection point.

value for two orbits of a comet particle, obtained from (eqn. 2) for particle perturbation, are presented in Figure 3.

As we see from Figure 3, for Perseids the D-criterion value of 0.2 is reached only for particles which were both perturbed by planets and were ejected with very high speeds — higher than 1000 m/s. A D-criterion higher than 0.22 is given by a model particle ejected with a speed of 2100 m/s at the point of the orbit with true anomaly $+30°$.

This is connected with the approaches of the particle to Jupiter: one of them is close (the distance between them is 0.27 A.U.) and two of them are on the border of its sphere of influence. With the same ejection speed V_{ej} at the point in the orbit with true anomaly of $-30°$ the main disturbance in the orbit of the particle is caused by the Earth. The number of close approaches to the Earth is rather large — 1469, 1497, 1525, 1590, 1730, 1758, 1786 and 1842; moreover, in these approaches the minimum mutual distance is 0.093 A.U. and the maximum is 0.16 A.U. Such perturbations lead to significant changes of the orbital elements of model meteoroids during 500 years (Table 1).

Let us look at the range of the most probable ejec-

Table 1 – Perturbation of orbital elements of model particles during 500 years.

Orbital element	Comet 109P/ Swift-Tuttle	Model particle orbital elements, V_{ej}=2100 m/s		
		True anomaly:	−30°	+30°
ω	152° (2000.0)		157°	143°
Ω	139°		141°	141°
i	113°		112°	108°
e	0.963		0.790	0.711
a	25.851 A.U.		4.436 A.U.	4.529 A.U.

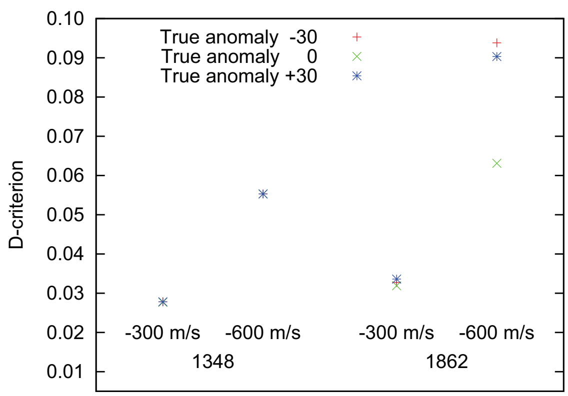

Figure 4 – Variations of the D-criterion values while selecting model particles during the period 1348 – 1862.

tion speeds of Perseid meteoroids out of their parental comet — about 600 m/s. In Figure 4 there are the D-criterion values for model particles ejected in 1348 with a speed of 300–600 m/s in the most probable points of the comet orbit with true anomalies of −30°, 0° and +30° and D-criterion values of the same particles in four orbits around the Sun. The D-criterion values even for perturbed Perseid orbits are not higher than 0.1.

3 Conclusions

The D-criteria are used by many researchers for studying the evolution of meteoroid streams though the upper limit of 0.2–0.3, found empirically for asteroids, has been used for the comet-meteor complex unquestioningly. In his work K.V. Kholshevnikov pointed to the unreliable character of the existing criteria. When identifying two comet orbits or comet and meteor orbits it is suggested that the decision should be based on a comparison of the orbit elements themselves taking into account their possible perturbations, but not their artificial union in some criterion (Kholshevnikov & Besmertny, 2003).

While identifying meteor showers we assume that the most reliable criterion remains the traditional one based on proximity of radiants. Radiant coordinates are found more precisely than the meanings of semimajor axes and perihelion distances included in the different formulas of D-criteria.

It is enough to take any catalogue of meteor orbits and we will see that the spread along the semimajor axes for Perseids is from 3 A.U. to about 40 A.U. This spread is far higher than the real dispersion of orbits in the stream. This is the requirement for the reliability of D-criteria, as the main condition on their usage is that the mistakes of measurements of orbit elements must be a lot smaller than the real dispersion of orbits in the stream.

Only for young meteoroid streams can the D-criterion serve as a reliable instrument for discovering family connections. But at the same time it is necessary to find the individual upper value of D for every meteoroid stream under observation.

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