

1 **Changes of Velocity of Rotation of the Earth, its Figure's**  
2 **Deformation and Long-period Nutational-Precessional**  
3 **Movements of the Instantaneous Pole of Rotation**

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7 **Abstract.** It is shown, that continuous changes of angular velocity of rotation of  
8 the plastically-elastic Earth should cause the continuous coupled deformation of  
9 a crustal layer with redistribution of masses in a sub-crustal layer, and also, as  
10 consequence of these phenomena, a polar pulsation of figure when polar diameter  
11 of the Earth increases and decreases with time. The mechanism of occurrence of  
12 deformations of the planet body under action of the deforming (centrifugal) vari-  
13 able force is found, and the equations of balance are deduced. The calculations  
14 have given the quite real changes of compression and radial displacements of the  
15 Earth's crust and its other shells. The opposite process is also shown, namely:  
16 observed fluctuations of amplitude of the polar compression quite correspond to  
17 real fluctuations of duration of a day. The influence of the rotational regime on  
18 real long-period nutational-precessional motions of the instantaneous pole of the  
19 Earth's rotation is considered. The observed 12-month oscillations of the instan-  
20 tantaneous axis of rotation are explained by the influence of the variable centrifugal  
21 force causing deformation of the figure that is not uniform in different longitudi-  
22 nal zones, without involving the baric-circulational processes as an intermediate  
23 link. Thus, both 14-month free (Chandler) and forced 12-month fluctuations of  
24 the pole location are considered from a single point of view – as a result of the  
25 variability of the rotational regime of the Earth. The analysis of heliogeophysical  
26 data for 1900-2017 confirms the validity of the proposed relationship mechanism.

27 **Keywords:** Earth Rotation, Deformation of the Earth Figure, Polar Compres-  
28 sion, Fluctuations of Angular Velocity, Nutation and Precession, Long-period  
29 Pole Movements, Correlation with Heliogeophysical Factors.

30 **1 Introduction**

31 As known, the rotation of the Earth determines its ellipticity, and the shape of the planet  
32 (its eccentricity) depends on the angular velocity of rotation, the law of density distri-  
33 bution over depth and latitude [1–3]. In this case, the earth's ellipsoid undergoes con-  
34 jugate deformation and, as a consequence of the variability of the angular velocity of  
35 rotation, polar pulsations of the figure should appear [3]. Due to the uneven distribution  
36 of masses in the Earth's body over longitudinal zones, this leads to nutational-preces-  
37 sional oscillations of the Earth's axis or to the movement of the rotation poles [1, 4].

38 We show, that continuous changes of angular velocity of rotation of the plastically-  
 39 elastic Earth cause the continuous coupled deformation of a crustal layer with redistri-  
 40 bution of masses in a sub-crustal layer, and, consequently, a polar pulsation of figure  
 41 when polar diameter of the Earth changes with time. We discuss the mechanism of  
 42 occurrence of deformations of the planet body under action of the centrifugal variable  
 43 force, and deduce the balance equations. We also show the opposite process when ob-  
 44 served fluctuations of amplitude of the polar compression quite correspond to real fluc-  
 45 tuations of duration of a day. The influence of the rotational regime on real long-period  
 46 nutational-precessional motions of the instantaneous pole of the Earth's rotation is also  
 47 discussed. The observed 12-month oscillations of the instantaneous axis of rotation are  
 48 explained by the influence of the variable centrifugal force causing deformation of the  
 49 figure that is not uniform in different longitudinal zones, without involving the baric-  
 50 circulatory processes as an intermediate link. Thus, both 14-month Chandler and  
 51 forced 12-month fluctuations of the pole location are considered from a single point of  
 52 view – as a result of the variability of the rotational regime of the Earth. Note that the  
 53 analysis of heliogeophysical data for 1900-2017 [5] confirms the validity of the pro-  
 54 posed relationship mechanism.

## 55 2 Earth's Rotation and its Figure's Deformation

56 As known, the rotation of the Earth, characterized by angular velocity, determines its  
 57 ellipticity, which is the main consequence resulting from the rotation of the figure itself.  
 58 The shape of the planet – its eccentricity  $e$  (or compression  $\alpha$ ) – depends only on two  
 59 parameters: the angular velocity of rotation  $\omega$  and the law of density distribution over  
 60 depth  $d\rho/dr$ , as well as, as follows from numerous studies, and latitude  $\phi$ , i.e.  
 61  $e = F(\omega, d\rho/dr)$  where  $d\rho/dr = f(r, \phi)$ .

62 Considering the polar compression of the planets of the Solar System, their angular  
 63 velocities of rotation and average densities, we can conclude that the degree of com-  
 64 pression of a planet mainly depends on its rotation velocity, and therefore, a change of  
 65 the planet's rotation regime<sup>1</sup> should, first of all, affect the change of polar compression.

66 From the law of conservation of angular momentum of the Earth, which is written as

$$67 \quad J\omega = \text{const}, \quad (1)$$

68 it follows that a change of the angular velocity of the Earth's rotation should inevitably  
 69 cause a change of the moment of inertia  $J$ ,  $\delta\omega/\omega = -\delta\tau/\tau = -\delta J/J$ , where  $\tau$  and  $\delta\tau$   
 70 are the length of the day and its change. Moreover, according to the simplest calcula-  
 71 tions, the change of the moment of inertia corresponding to real changes of the day  
 72 duration ( $\Delta\tau \approx 0.0034$  s) should reach

$$73 \quad \delta J/J = 0.0034/86400 = 4 \cdot 10^{-8}. \quad (2)$$

74 According to [8], such a change can occur as a result of a change of the density of  
 75 the subcrustal layer, its “bulging” (as a result of which deformations arise in the crustal  
 76 layer). Moreover, according to the calculations given in [8], if we take the thickness of

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<sup>1</sup> The question of the reasons for the change of the Earth's rotation velocity is not considered in this paper; the reviews of the hypotheses explaining this phenomenon was published in [1, 6, 7].

77 the subcrustal layer where the density redistribution takes place, as 80 km, and the  
 78 thickness of the outer layer that only deforms but does not change its density, as 1 km,  
 79 then to change the moment the inertia of the Earth corresponding to (2), a vertical dis-  
 80 placement of 6-7 m is sufficient.

81 Note that, as a result of the deformation of the Earth's figure resulting from a change  
 82 of  $\omega$ , as it was found in [2, 8, 9], the density redistribution in the subcrustal layer actu-  
 83 ally occurs. Let  $\rho_p^0$  is the density at the point  $P$  in the initial state. After deformation  
 84 due to radial displacement, the density at point  $P$  becomes equal

$$85 \quad \rho_p = (\rho_p^0 - \zeta d\rho_p^0 / dr)(1 - \Theta) = \rho_p^0 - \zeta d\rho_p^0 / dr - \rho_p^0 \Theta \quad (3)$$

86 where  $\zeta$  is the displacement, and  $\Theta = (1/Q)(dQ/dt)$  where  $\Theta = dx dy dz$  is the vol-  
 87 ume. In our case (we believe that the Earth is deformed conjugatedly, without changing  
 88 the volume),  $\Theta = 0$  and Eq. (3) takes form  $\rho_p = \rho_p^0 - \zeta(d\rho_p^0 / dr)$ .

89 Since  $\zeta$  is positive, the substance at point  $P$  becomes denser (since  $d\rho_p^0 / dr < 0$ , and  
 90 therefore  $-\zeta d\rho_p^0 / dr > 0$ ). This corresponds to the above noted considerations (see [4]).

91 The "bulging" of the subcrustal layer should be accompanied by a redistribution of  
 92 the internal masses (that is, their overflow into this region from the regions in which  
 93 the negative radial displacement occurs). As known [8], for any internal point  $P$  in the  
 94 initial state, the Poisson equation is written in form  $\Delta V_p^0 = -4\pi G \rho_p^0$  where  $\Delta V_p^0$  is the  
 95 Laplacian of gravitational potential  $V_p$  in the initial state,  $G$  is the gravitational const-  
 96 ant. For a deformed state we have  $\Delta (V_p^0 + V_p) = -4\pi G (\rho_p^0 - \zeta d\rho_p^0 / dr - \rho_p^0 \Theta)$ .

97 After differentiation, we find  $\Delta V_p = 4\pi G (\rho_p^0 \Theta + \zeta d\rho_p^0 / dr)$  or, if we again assume  
 98 that the substance of the subcrustal layer is incompressible,

$$99 \quad \Delta V_p = 4\pi G \zeta (d\rho_p^0 / dr). \quad (4)$$

100 Equation (4) shows that when  $\omega$  decreases, that is, when the deforming centrifugal  
 101 force is removed, the Earth will return to its original undeformed state due to the oc-  
 102 currence of gravitational effects [given by (4)] caused by the new distribution of masses  
 103 in the Earth's body. However, it should be noted that the change of density and the  
 104 redistribution of masses can, by virtue of Eq. (1), affect the change of the angular ve-  
 105 locity of the Earth's rotation, that is, the opposite effect can occur (and really occurs).

106 To find out the effect of the rotation velocity on the change of the shape of the Earth,  
 107 we consider the relationship of deformations (and displacements) with stresses applied  
 108 to the volume. For this, as is customary in rheology, we first write down the tensors of  
 109 the resulting strains and stresses, and then investigate their relationship with each other.  
 110 As a result, we have obtained the equilibrium equations with three unknowns expressed  
 111 in terms of displacements (equations of motion in displacements in the Lamé form):

$$112 \quad (\lambda + \mu) \frac{\partial \Theta}{\partial x} + \mu \Delta u + \rho X = \rho \frac{\partial^2 u}{\partial t^2}, \quad (\lambda + \mu) \frac{\partial \Theta}{\partial y} + \mu \Delta v + \rho Y = \rho \frac{\partial^2 v}{\partial t^2},$$

$$113 \quad (\lambda + \mu) \frac{\partial \Theta}{\partial z} + \mu \Delta w + \rho Z = \rho \frac{\partial^2 w}{\partial t^2}, \quad (5)$$

113 where  $\Theta = \partial u / \partial x + \partial v / \partial y + \partial w / \partial z$ ;  $u$ ,  $v$  and  $w$  are the displacements;  $\lambda$  and  $\mu$  are the  
 114 Lamé coefficients;  $X$ ,  $Y$ , and  $Z$  are the components of gravitational forces. Equations

115 (5) express the equilibrium state of the Earth's ellipsoid of revolution, which is under  
 116 the action of a centrifugal force, and gravitational and surface tension forces, tending  
 117 to return the Earth to its initial shape, from which, under the influence of a change of  
 118 centrifugal force (of its increase with growth of  $\omega$ ), it passed to a stressed state.

119 Solving Eqs. (5) we showed that the Earth ellipsoid is undergoing conjugate deforma-  
 120 tion at changing the angular velocity of rotation which corresponds to changes of  
 121 compression really observed by geodetic methods, as well as using satellites (which is  
 122  $1/298.25 \pm 0.02$  according to the IUGG/IAG data [10]). Our estimates showed that in the  
 123 zones of latitudes  $\phi = \pm(30 - 40)^\circ$  the radial displacements do not occur with a conju-  
 124 gate change of the figure. Note, that these zones are “belts”, where strong earthquakes  
 125 occur statistically the most frequent, at this, the deformations taking place in the Earth’s  
 126 crust in these belts can generate such precursors of seismic events as the anomalous  
 127 bursts of the electromagnetic radiation in the ELF-VLF frequency range [11].

### 128 **3 Movements of the Instantaneous Pole of the Earth’s Rotation**

129 As known, the instantaneous axis of the Earth's rotation experiences nutational-precres-  
 130 sional oscillations relative to its average position, the main periods are 14-month (the  
 131 Chandler period) and 12-month (forced variations) ones. In addition, these fluctuations  
 132 experience significant long-term changes, the causes of which have not yet been fully  
 133 identified. If for the Chandler variations the fact of their relationship with the unevenness  
 134 of the Earth’s rotation can be considered established (according to the results [12] the  
 135 correlation between the annual variation of the Earth’s rotation period ( $T$ ), on the one  
 136 hand, and the amplitude ( $A$ ) and the duration of the Chandler’s period ( $P$ ), on the other  
 137 hand, are:  $r_{TA} = 0.875$  and  $r_{AP} = 0.910$ ), most researchers consider the 12-month  
 138 forced variations as a consequence of changes in the global baric circulation regime, put-  
 139 ting forward as an argument the observed correlation of pole movements and solar activity  
 140 (see, for example, [1] and references there). Besides, tectonic hypotheses are used to ex-  
 141 plain the long-term changes in the forced part of the oscillations of the Earth's instantane-  
 142 ous axis of rotation (see [4] and references there).

143 We study the role of the Earth's rotation regime in long-term real oscillations of its  
 144 instantaneous axis of rotation, and also to propose a mechanism that qualitatively explains  
 145 the observed long-term forced oscillations of the instantaneous axis of rotation due to  
 146 deformations of the Earth’s figure when the angular velocity changes [3]. Basing on an  
 147 analysis of heliogeophysical data, we show the relationship between real long-period  
 148 poles' motions and changes of the angular velocity of the Earth’s rotation, as well as,  
 149 basing on the hypothesis of the solar-magnetospheric control of the rotation regime [6,  
 150 7], with fluctuations of the Wolf numbers ( $W$ ) and global magnetic disturbance [index  
 151  $M = 10(\sum K_p - 10)$ ]. For the elastically deformable Earth, we obtain the relations that  
 152 relate the period of nutational-precresional pole movements and the deformation of the  
 153 Earth's body with the angular velocity of rotation. We also represent the results showing  
 154 the correlation in cycles of different durations of the parameters of free (Chandler) varia-  
 155 tions of the Earth’s rotation pole with changes of heliogeophysical factors, that can be  
 156 as an indirect confirmation of the hypothesis proposed in [1]. So, we propose a “deforma-  
 157 tional” mechanism determining the influence of changes of the Earth's rotational

158 movement on the forced part of the nutation-precession oscillations of the rotation axis.  
159 Note, that the results of the analysis of heliogeophysical data confirm the validity of the  
160 alleged relationships [5].

## 161 **4 Discussion and Conclusion**

162 In conclusion, it was established that since the angular velocity of the Earth's rotation  
163 changes abruptly and continuously, increasing and decreasing on a general tidal back-  
164 ground of its damping, the signs of the displacement vectors change passing through zero  
165 and, therefore, continuous changes of angular velocity of rotation of the plastic-elastic  
166 Earth should cause continuous conjugate deformation of the crustal layer, redistribution  
167 of masses in the subcrustal layer and the associated change of density, and also, as a conse-  
168 quence of all these phenomena, the polar pulsation, when the Earth polar diameter in-  
169 creases and decreases. Free 14-month and forced 12-month oscillations of the Earth's  
170 instantaneous axis of rotation were considered from a single point of view – as a conse-  
171 quence of the variability of the Earth's rotational regime (due to changes in solar activity  
172 indirectly through oscillations global magnetic disturbance). We have attempted to ex-  
173 plain the forced part of real variations of the pole's location by the alternating Earth's  
174 rotational regime arising due to the polar pulsations of the figure, and also by uneven,  
175 along the longitudinal zones, movement of the subcrustal masses in the planetary body.  
176 The reality of the movement of subcrustal masses, their “flowing” from the polar regions  
177 to the equatorial ones and vice versa, at change of the rotational regime follows from the  
178 fact that the substances in the subcrustal layer are in semi-liquid state due to the colossal  
179 pressures and temperatures [9]. Further, based on the fact that the density in the Earth's  
180 body is nonuniformly distributed, we hypothesized the “deformational” nature of the  
181 forced nutational-precessional wave in the oscillations of the instantaneous axis of the  
182 Earth's rotation and confirm it by analyzing the heliogeophysical data for 1900-2017 [5].

183 The main conclusions can be formulated as follows.

184 1. We have clarified the mechanism of the occurrence of deformations of the  
185 planet's body under the action of a time-varying centrifugal force, and based on rheo-  
186 logical equations we have derived equilibrium equations and also calculated the mod-  
187 ulus of variation of polar compression and radial displacements at real fluctuations of  
188 the angular velocity of the Earth's rotation. The results gave quite real changes of com-  
189 pression and radial displacements of the Earth's crust and its underlying shells. The  
190 opposite is also shown: the observed fluctuations of the polar compression, leading to  
191 corresponding changes of the moment of inertia of the Earth, are consistent with real  
192 fluctuations of the day duration.

193 2. Real fluctuations of the location of the instantaneous pole of rotation occur quite  
194 synchronously with fluctuations of the angular velocity of the Earth's rotation, and with  
195 some delay relative to the latter, that allows us to put forward the variability of the rotation  
196 regime as the cause of long-period precession-nutation movements of the rotation axis.

197 3. A deforming force arising when the angular velocity changes, causes a deformation  
198 of the figure nonuniform over the longitudinal zones. With such a nonuniform defor-  
199 mation, the moment of inertia of the Earth changes, that causes oscillations of its instan-  
200 taneous axis of rotation, that is, nutation and precession of the axis (forced wave).

201 4. An analysis of the available factual material confirms the validity of the assumption  
 202 about the decisive role of changes of the angular velocity both for free 14-month (Chan-  
 203 dler) and forced 12-month oscillations of instantaneous rotation axis.

204 5. The observed correlation of cyclic changes of solar activity, magnetic disturbance,  
 205 and oscillations of instantaneous axis of the Earth's rotation is explained, taking into account  
 206 the hypothesis [6, 7], by cyclic changes of the rotation regime, without involving as an in-  
 207 termediate link, the baric circulation processes which have relatively low energy efficiency.

208 6. It is natural to consider the baric circulation regime changes as a consequence of the  
 209 variability of the deflecting (Coriolis) force of the Earth's rotation both in magnitude and  
 210 direction. The Coriolis force vector itself undergoes changes, since the vector of the an-  
 211 gular velocity of the Earth's rotation changes in time. Thus, the variability of the rotational  
 212 regime is put forward as a single reason for the nutational-precessional oscillations of the  
 213 instantaneous axis of rotation, and it is fundamentally new to consider the variability of  
 214 the rotational regime as the cause of forced movements of the planet's rotation pole.

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