STUDY OF SPECTRUM OF GUIDED WAVES OF DIELECTRIC FIBERS

E. M. Karchevskii
Dept. of Computational Mathematics, Kazan State University, Kazan, Russia
Mailing addres: 74 - 24, Dostoevskii str., Kazan, 420097, Russia
E-mail: Evgenii.Karchevskii@ksu.ru

A mathematical model characterizing the surface and leaky guided waves on dielectric fibers of arbitrary cross-section is studied. The results of [1,2] are essentially used. Analogous approaches for solving the problems of wave propagation on microstrip and slot lines have been developed in [3,4].

The guided waves propagating along the z-axis of the fiber by the law $\exp[i(\beta z - \omega t)]$, where the axial propagation coefficient β is an unknown complex parameter, ω is the given frequency of electromagnetic oscillations, are sought. Under the assumption of nearness of real refractive indices of the fiber (n_1) and environment ($n_2 < n_1$), this phisical problem is reduced, following [5], to the spectral problem for the Helmholtz equation on the plane, with the Reichardt conditions on infinity [6]:

$$\Delta u + \chi_1^2 u = 0, \quad (x,y) \in S_1, \tag{1}$$

$$\Delta u + \chi_2^2 u = 0, \quad (x,y) \in S_2, \tag{2}$$

$$u^+ = u^-, \quad \frac{\partial u^+}{\partial \nu} = \frac{\partial u^-}{\partial \nu}, \quad (x,y) \in \Gamma, \tag{3}$$

$$u(r,\varphi) = \sum_{n=-\infty}^{\infty} \alpha_n H_n^{(1)}(\chi_2 r)\, e^{in\varphi}, \quad \forall r \geq R_0, \tag{4}$$

Here S_1 is a bounded domain, $\Gamma = \partial S_1$, $S_2 = R^2 \setminus \bar{S}_1$, $x = r\cos\varphi$, $y = r\sin\varphi$, $H_n^{(1)}$ is the Hankel function of the first kind and n-th order, $\chi_j = \sqrt{k_0^2 n_j^2 - \beta^2}$, $k_0^2 = \omega^2 \varepsilon_0 \mu_0$, $\beta \in \Lambda = H_1 \bigcap H_2$, H_j is the Riemann surface of the function $\ln \chi_j(\beta)$, $j = 1,2$. Note that both the surface, i.e., exponentially decreasing waves and exponentially growing at infinity leaky waves satisfy conditions (4).

The questions of spectrum localization have been studied. The surface H_j has an infinite number of complex sheets. The "proper" one (H_j^0) is specified by the conditions: $-\pi/2 < \arg \chi_j(\beta) < 3\pi/2$, $\mathrm{Im}\chi_j(\beta) > 0$ if $\mathrm{Im}\beta = 0$, $|\beta| > k_0 n_j$.

Theorem 1 *At* $\Lambda_0 = H_1^0 \bigcap H_2^0$ *the propagation coefficient* β *may belong only to* $G \bigcup \Lambda_2$, $G = \{\beta \in \Lambda_0\colon \mathrm{Im}\beta = 0,\ k_0 n_2 < |\beta| < k_0 n_1\}$, $\Lambda_2 = \{\beta \in \Lambda_0 : \mathrm{Im}\chi_2 < 0\}$.

These results generalize the results of Katsenelenbaum concerning the spectrum localization of the surface and leaky guided waves on the fibers of circular cross-section [7]. We recall that in [7], analysis was done based on the study of characteristic equation obtained by the method of separation of variables.

Original problem (1)-(4) can be reduced to a nonlinear spectral problem for a Fredholm holomorphic operator-valued function

$$A(\beta)y \equiv (I + B(\beta))y = 0 \tag{5}$$

with the aid of potentials of simple layer. Here, I is an identical operator, $B(\beta)$ is a completly continuous holomorphic operator-valued function. We have proved the following

Theorem 2 *Spectral problem (1) - (4) is equivalent to the problem (5) for all $\beta \in \Lambda$, except for some discrete set of points. Spectral problem (1) - (4) is equivalent to the problem (5) for all $\beta \in G$.*

Theorem 3 *The spectrum of the problem (1) - (4) may consist of only isolated points.*

A discretized matrix equation has been derived by Galerkin's method based on trigonometric basis fuinctions, for a numerical solution of the problem (5). Determinant zeros β_n of the matrix $A_n(\beta)$ of this system (n being the number of basis functions) are assumed as the approximation values of the propagation coefficient β. The convergence of this method has been studied.

Theorem 4 *Suppose that $\beta_0 \in \sigma(A) = \Lambda \setminus \{\beta \in \Lambda : \exists A^{-1}(\beta)\}$. Then there exists a sequence $\{\beta_n\}$, $\beta_n \in \sigma(A_n)$, such that $\beta_n \to \beta_0$, if $n \to \infty$. Suppose that there exists a sequence $\{\beta_n\}$, $\beta_n \in \sigma(A_n)$, such that $\beta_n \to \beta_0 \in \Lambda$ if $n \to \infty$. Then $\beta_0 \in \sigma(A)$.*

A practical efficiency of this method has been shown by comparing the solution of some problems of the theory of dielectric fibers with experimental data and results obtained by other methods. The figure compares the Galerkin method results for $n = 2$ with point-matching [8] and constant-field approximation solution [9], for the fundamental mode propagation on a dielectric fiber of the regular triangle cross-section. The dispersion curves of q versus v, where $q = (h^2 - n_2^2)/(n_1^2 - n_2^2)$, $v = 2ak_0\sqrt{n_1^2 - n_2^2}/\sqrt{3}$, $h = \beta/k_0$, $n_1 = 2.31$, $n_2 = 2.25$, a is the side of the triangle, are ploted.

References

1. S. Steinberg, Meromorphic families of compact operators, *Arch. Rational Mathem. Anal.*, 1968, vol. 31, no 5, pp. 372-379.

2. G. M. Vainikko, O. O. Karma, On the convergence rate of the approximate methods in the eigen-value problem with a non-linear entrance of parameter, *URRS J. Comput. Maths. Math. Phys. (Engl. Transl.)*, 1974, vol. 14, no 6.

3. A. S. Illinskii, Y. V. Shestopalov, *Using the methods of spectral theory in the problems of wave propagation*, Moscow: MGU Press, 1989 (in Russian).

4. A. I. Nosich, *Excitation and propagation of waves on cylindrical microstrip and slot lines, and other open waveguides*, D.Sc. dissertation, Dept. Radio Physics, Kharkov State University, 1990 (in Russian).

5. N. N. Voitovich, B. Z. Katsenelenbaum, A. N. Sivov, A. D. Shatrov, Modes of dielectric waveguides of complicated cross section, *Radio Engn. Electron. Physics (Engl. Transl.)*, 1979, vol. 24, no 7.

6. H. Reichardt, Ausstrahlungsbedingungen fur die Wellengleihung, *Abh. Mathem. Seminar*, Univ. Hamburg, 1960, vol. 24, pp. 41-53.

7. B. Z. Katsenelenbaum, Symmetrical exication of an infinite dielectric cylinder, *Zhurnal Tekhnicheskoi Fiziki*, 1949, vol. 19, no 10, pp. 1168-1181 (in Russian).

8. J. R. James, I. N. L. Gallett, Modal analysis of triangular-cored glass-fibre waveguide, *IEE Proc.*, 1973, vol. 120, no 11, pp. 1362-1370.

9 E. F. Keuster, R. C. Pate, Fundamental mode propagation on dielectric fibres of arbitrary cross-section, *IEE Proc.-H*, 1980, vol. 126, no 1, pp. 44-47.