Ecological Modelling 250 (2013) 384–390

Contents lists available at SciVerse ScienceDirect

Ecological Modelling

journal homepage: www.elsevier.com/locate/ecolmodel

Optimal shape of an anthill dome: Bejan's constructal law revisited

R.G. Kasimova [a,*], Yu.V. Obnosov [b], F.B. Baksht [c], A.R. Kacimov [d]

[a] Department of Mathematics, German University of Technology in Oman, Oman
[b] Department of Differential Equations, Kazan Federal University, Russia
[c] Tomsk Polytechnical University, Russia
[d] Department of Soils, Water and Agricultural Engineering, Sultan Qaboos University, Oman

ARTICLE INFO

Article history:
Received 16 July 2012
Received in revised form
23 November 2012
Accepted 25 November 2012

Keywords:
Mathematical modelling
Constructal design
Social insects
Ant nest
Heat transfer
Global minimum

ABSTRACT

An anthill is modelled as a paraboloid of revolution, whose surface (dome) dissipates heat from the interior of the nest to the ambient air according to the Robin boundary condition, which involves a constant coefficient, given temperature jump and dome's area. The total heat loss of the net is one (integral) component of ants' colony expenditures of energy. Ants, populating the paraboloid, spend also energy individually, by hoisting the load from the ground surface to a certain elevation within the paraboloid and by overcoming a Coulombian resistance, proportional to the trajectory length. In order to count the gross colony expenditures for these mechanical activities all trajectories are integrated over the volume. Ants are assumed to move along the shortest straight lines of their regular sorties between the nest and forest. The three-component energy is mathematically expressed as a closed-form function of only one variable, the paraboloid height-to-width ratio. The minimum of this function is found by a routine of computer algebra. The proposed model amalgamates into a single and relatively simple function, tractable by standard calculus, the property of the whole structure (dome area) with labouring of insects-comrades. The ants are sociobiologically analogized with Bejan's builders of ancient pyramids and contemporary designers of man-made "dream-houses" or "dream-prisons".

1. Introduction

"...even if man really were nothing but a piano-key, even if this were proved to him by natural science and mathematics, even then he would not become reasonable, but would purposely do something perverse out of simple ingratitude, simply to gain his point."

F. Dostoevsky, Notes From Underground, 1864

Bejan (2000) constructal law states that for a flow system to persist in time (to survive), its configuration must change (morph) such that it provides easier and easier access to its currents. Numerous engineering and biological systems have been shown to comply with this principle (Bejan, 2007; Bejan and Lorente, 2011). In this paper, we model anthills of *Formica rufa* and *Formica polyctena* (herewith abbreviated as FRaFP), which are common ant species in the forest of the European and Siberian Russia (Baksht, 2005; Dlusskii, 1967), as constructions optimizing mechanical energy fluxes of the insects.

Camazine et al. (2001, p. 345) wrote: "The ants appear to build themselves a nest of just right size" with numerous similar statements on the adaptive sizes and shapes of the nest to ambient environments (see, e.g., Dlusskii, 1967; Sudd, 1982). Design and architecture of insect's nests belongs to the amazing tapestry of biological and other natural patterns (Ball, 1999; Kauffman, 1995). So the main question, which we try to answer by mathematical modelling is: What is the "right size" of the anthill?

We follow the general protocol of "calculating the shape of a structure ... that is in some sense optimal for coping with a given environmental parameter, the criterion used to define optimality ... is typically either the minimization of energy needed to produce the structure ... or the return to the organism (either in terms of energy or reproduction) when the structure is used" (Denny, 1994). Consequently, we consider an anthill as a geometrical structure, which morphs into a certain shape (paraboloid, see Baksht, 2005), whose sizes can be deduced from Bejan's principle.

Ants, as well as termites, wasps and bees, are social insects (Oster and Wilson, 1978; Wilson, 1971; Wilson and Hölldobler, 2005), similar to human beings (e.g., Emery and Thorsrud, 1969) in the following features: large numbers, sterility of cyclostationary moving toilers (≈nuns engaged in agro-cultivation tasks), patterned motion in foraging (≈commuting in urban areas), hierarchical community structure (≈casts of workers–soldiers–monarchs–brachmans), colony's

* Corresponding author at: PO Box 1816, Athaibah PC130, Oman.
Tel.: +968 24493051/24413548/99251302; fax: +968 24495568.
E-mail addresses: rouzalia.kasimova@gutech.edu.om, rkasimova@yandex.ru (R.G. Kasimova), yurii.obnosov@gmail.com (Yu.V. Obnosov), baksht@yandex.ru (F.B. Baksht), anvar@squ.edu.om (A.R. Kacimov).

0304-3800/$ – see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ecolmodel.2012.11.021