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Physical and Mathematical Foundations Describing Crystallization Process of Melt Cooling on a Moving Wall by Fourier Method and Duhamel Theorem

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Abstract. The process of melt crystallization of the material in a twin-roll mold is considered. An adjoin of heat conduction through variable thickness of wall "rolls wall - hard layer" with a convective heat exchange from two sides problem important for the chemical industry is solved using Fourier method and Duhamel theorem. A theoretical dependence of the thickness of a frozen layer on the physical parameters of the melt and technological parameters of the crystallization process is obtained.

Introduction

Heat flows determination while cooling liquid material with its subsequent crystallization is a complex multiparameter problem. It is necessary to determine the main factors affecting the process and the degree of their influence to solve problem.

Practical application has widely used in the foundry industry, for example in molds in obtaining fine fraction material for subsequent processing [1, 2].

Initially, crystallizer [3] is designed to produce a fine-grained alloy structure throughout its volume [4], thus not provide complete cooling of the melt for the purpose of melt material removal capability like the tape.

In this paper offered method of calculating crystallizer process with a complete cooling of the melt with simultaneous production of fine fraction.

Features of Process of Thin Layer Crystallization on the Rolls

In order to exclude the long-term operation of cooling and energy-consuming step of crushing pigs, proposed to use modified twin roll crystallizer (Fig. 1).

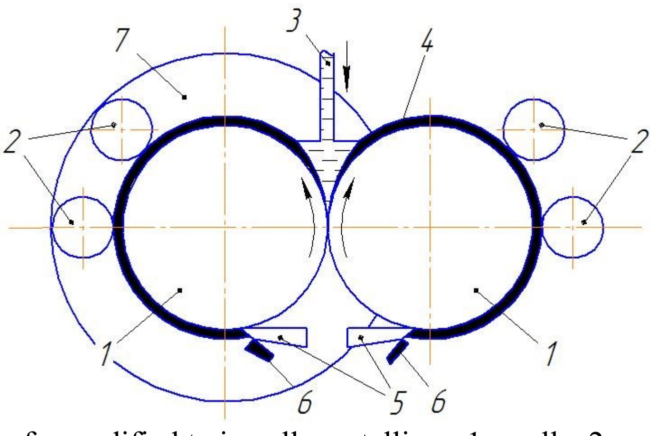

Fig. 1. Scheme of a modified twin-roll crystallizer: 1 – rolls; 2 – rollers; 3 – melt; 4 - crystallized layer; 5 – blades; 6 - fine crystallized fraction; 7 - roll flange.

Crystallizer works as follows: container-crystallizer volume, formed by two rotating rolls 1 with flanges 7, fed continuously with melt 3. In this surfaces of the crystallizer, rotating towards the feed

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melt carry out melt 3 crystallization continuously to form a crust of crystallized layer 4. Further, the crystallized layer cooled and crushed previously by rollers 2. Knives 5 cut fully chilled layer 4 of the rolls 1 to the form fine fraction 6.

For easy clarifying character of crystallization process, two-roll scheme during molding considered on example of the right roll (Fig. 2). Second roll will be same in operating.

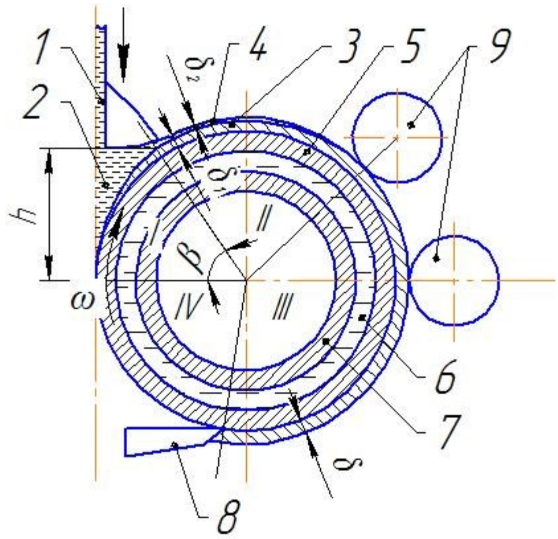

Fig. 2. Scheme of continuous operation roll crystallizer: 1 - melt filing; 2 - container-crystallizer; 3 - crystallized layer; 4 - film liquid melt; 5 - drum, 6 - coolant, 7 - internal drum 8 - blade, I - first cooling zone, II - second cooling zone, III - third cooling zone IV - cooling zone of the roll, δ_1 - thickness of crystallized layer at the outlet of the first zone, δ_2 - layer thickness of liquid melt at the outlet from the first zone, δ - thickness cooled melt, ω - angular rotation velocity of the rolls, β - angle of the first zone, h - the depth of container – crystallizer.

At the moment of cooled roller surface contact zone I and liquid melt at its outer cylindrical surface a thin melt film begins to crystallize, the thickness of which at the exit from container-crystallizer reaches the value δ_1. When exiting the container-crystallizer the outer layer of crystallized melt carries away thin layer of liquid melt δ_2. Layer thickness δ_2 depends on the speed of cylinder rotation, surface tension and melt viscosity of liquid [3,5]. Thus thickness of crystallized and removed layer from roll surface will be:

$$\delta = \delta_1 + \delta_2. \quad (1)$$

Full rotational cycle may be divided into four zones marked as I, II, III and IV [6].

First zone - crystallization time t_1 layer thickness δ_1, that is the contact time of roller surface element with a liquid melt, which is located in a container-crystallizer.

Second area - further crystallization time t_2, during it imposition of liquid film thickness δ_2 is completely solid.

Third zone - cooling time of "rolls wall - hard layer" system till full crystallized layer at which the destruction of the material under a permanent load by shear knifes 8. This zone is limited to the area of passage crystallized surface layer element by the moment of complete solidification imposition films to chopping knifes.

In the fourth zone, only wall of the roll is cooled till come to entrance of container-crystallizer zone 1.

Is necessary to mark that the process of melt crystallization and cooling liquid melt phase on the roll with δ thickness occurs at variable temperature of the rolls wall. Characteristic diagram of rolls wall temperature variation close to its outer surface during a turn in the steady thermal conditions shown in Fig. 3. Specific values of the rolls wall temperatures depend on many parameters such as the rotational speed of the rolls ω, depth of container-crystallizer h and physical characteristics of melt.

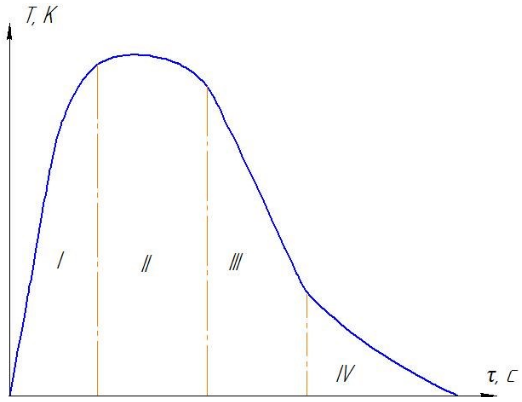

Fig. 3. Characteristic diagram of temperature variation (T) from a separate element of the outer roll surface during time (τ).

At the moment of immersion chilled rolls wall in liquid melt there is a rapid increase in the wall temperature (heat stroke) due to the removal of heat of phase transition [7]. On the moving rolls wall in container-crystallizer freezes the layer of melt. With the growth of crystallized layer thickness, increases the thermal resistance of the growing solid layer that reduces heat flow through the solid layer and reduce intensity increasing wall temperature. Approximately, at the end of crystallization imposed layer δ_2 wall temperature stabilized, and further decrease of wall temperature to initial value.

Heat Transfer in Thin-Layer Crystallization on the Rolls

Considered adjoin problem of heat conduction through variable thickness of wall "rolls wall - hard layer" with a convective heat exchange from two sides [8, 9], is solved using equations of thermal conductivity for rolls wall, solid layer with release heat of phase transition.

In practice, wall thickness of the roll is much smaller than roll radius of curvature, so rolls wall admitted as unlimited flat plate. Problem solved in a Cartesian coordinate system moving together with the wall of the roll [10].

Heat transfer problem for the drum wall comes to the solution of the heat equation [11, 12]:

$$\frac{\partial T_C}{\partial t} = \alpha_{CT} \frac{\partial^2 T_C}{\partial x^2}, \; 0 \le x \le l; \tag{2}$$

where T_C is layer temperature, α_{CT} is heat transfer coefficient from freezing layer to the rolls wall, x is coordinate of the rolls wall layer beginning from inside of rolls wall, washed by coolant, l is thickness of the rolls wall, t is contact time of cooled roll surface with the melt.

In boundary conditions [13, 14]:

$$\begin{aligned} &\frac{\partial T_C}{\partial x} = \frac{\alpha_2}{\lambda_{CT}}\left(T_{CT} - T_2\right), x = 0; \\ &T_{CT} = T_0\,(t),\; x = l; \\ &T_C = f(x),\; t = 0; \end{aligned} \tag{3}$$

where λ_{CT} is rolls wall heat transfer coefficient, α_2 is rolls wall heat transfer coefficient to the coolant, T_2 is coolant temperature, T_{CT} is outside rolls wall temperature.

Temperature profile T_P of solid layer of melt described by the heat equation [15]:

$$\frac{\partial T_p}{\partial t} = a_1 \frac{\partial^2 T_p}{\partial \xi^2}, \; 0 \le \xi \le \delta_1; \tag{4}$$

where a_1 is thermal conductivity of liquid melt, ξ is (coordinate) freezed layer thickness at the beginning on outside rolls wall.

With in view of boundary and initial conditions.

$$T_p = T_0(t),\ \xi = 0;$$
$$T_p = T_{kr},\ \xi = \delta_1;$$
$$\lambda_1 \frac{\partial T_p}{\partial \xi} - c_{kr}\rho_1 \frac{\partial \delta}{\partial t} = \alpha_1 (T_1 - T_{kr}),\ \xi = \delta_1; \quad (5)$$
$$\delta_1 = 0,\ t = 0;$$

where T_{kr} is melt crystallization temperature; λ_1 is melt coefficient of thermal conductivity, c_{kr} is specific heat of melt crystallization, ρ_1 is melt density, T_1 is melt temperature, α_1 is coefficient of heat transfer from the melt to rolls wall.

Conjugation conditions include equality of temperatures and heat flux on the contact surface of wall with crystallized melt layer:

$$T_C\big|_{x=l} = T_p\big|_{\xi=0} = T_0(t); \quad (6)$$

$$\lambda_{CT} \frac{\partial T_C}{\partial x}\bigg|_{x=l} = \lambda_2 \frac{\partial T_p}{\partial \xi}\bigg|_{\xi=0}; \quad (7)$$

where λ_2 is thermal conductivity of the coolant.

To solve this problem of the inhomogeneous heat on roller wall used Fourier method [16] and Duhamel theorem. The task of freezing melt layer in our formulation was solved by multivariable L.G. Loitsiansky, common to the heat conduction problem with moving boundary E.M. Smirnov [17].

After the transformation equations to new dimensionless variables and then integrate a series expansion in powers of the parameters, we obtain a system of equations for the determination of $T = T_0\ (t)$ and $\delta_1 = \delta(t)$, describing the process of freezing the liquid product on the surface of the roll wall. Similarly, solve the problem of freeze imposed δ_2 layer (second zone), while in the equation instead of the coefficient of heat transfer to the liquid melt α_1, located in the tank-crystallizer, substituted the heat transfer coefficient from making films to the surrounding air α_{okr} and of liquid melt temperature T_1, it is replaced by the ambient temperature T_{okr}.

For the calculation of the melt film imposition δ_2 used dependence [18]:

$$\delta_2 = 0.94\left(\frac{\mu u}{\sigma}\right)^{\frac{1}{6}} \left(\frac{\mu u}{\rho g \sin\frac{\beta}{2}}\right)^{\frac{1}{2}} \frac{\rho}{\rho_{tv}}, \quad (8)$$

where: μ is dynamic viscosity of the of liquid melt, u is linear outer surface of the rotational speed of the roll, σ is the surface tension of liquid melt, ρ is melt density at a temperature close to the temperature of crystallization, g is acceleration due to gravity, β is angle of first zone, ρ_{tv} is frozen melt density.

Determination of the temperature field in the system of the roll wall - crystallized melt (the third zone) is the task of the composite body of transient heat conduction with boundary conditions of the third kind on the external surfaces.

Problem is formulated as follows:

$$\begin{cases} \dfrac{\partial T_C}{\partial t} = \alpha_{CT} \dfrac{\partial^2 T_C}{\partial x^2}; t \ge 0; 0 \le x \le l \\ \dfrac{\partial T_C}{\partial x} = \dfrac{\alpha_2}{\lambda_{CT}} (T_{CT} - T_2); x = 0 \\ T_C = f(x); t = 0 \end{cases} , \tag{9}$$

$$\begin{cases} \dfrac{\partial T_p}{\partial t} = a_P \dfrac{\partial^2 T_p}{\partial \xi^2}; t \ge 0; 0 \le \xi \le \delta \\ \dfrac{\partial T_P}{\partial \xi} = \dfrac{\alpha_{okr}}{\lambda_P} (T_{okr} - T_P); \xi = \delta \\ T_P = f(x); t = 0 \end{cases} , \tag{10}$$

where λ_P is coefficient of thermal conductivity of the frozen melt.

The conjugation conditions on the contact surfaces have the form:

$$T_P(0, t) = T_{CT}(l, t) = T_0(t); \tag{11}$$

$$\lambda_P \left. \frac{\partial T_P}{\partial \xi} \right|_{\xi=0} = \lambda_{CT} \left. \frac{\partial T_C}{\partial x} \right|_{x=l} . \tag{12}$$

Solving these problems is also done with the help of the Fourier method and Duhamel theorem.

In the fourth area to solve the problem of non-stationary heat conduction, roll wall with boundary conditions of third kind with both sides. The final temperature distribution in the wall of the drum in this zone is the initial distribution of temperature condition for first zone.

As a result of solving the problems described above to determine the dependence of the film thickness, freezed on the surface of the roll, during his stay in container-crystallizer [19,20]:

$$\delta_1 = 6.5 \cdot 10^{-5} \frac{\left(\lambda_P \dfrac{T_{kr} - T_2}{c_{kr} \rho a_P} \tau \right)^{\frac{2}{3}}}{\left(\lambda_P \left(\dfrac{1}{\alpha_2} + \dfrac{\delta_{ct}}{\lambda_{CT}} \right) \right)^{\frac{1}{2}} \left(\alpha_1 \dfrac{T_1 - T_{kr}}{T_{kr} - T_2} \right)^{\frac{1}{6}}} ; \tag{13}$$

where δ_{st} is the wall thickness of roll.

Conclusion

As a result of thermal calculations of the crystallizer were obtained according to the definition of a crystallized layer thickness on rolls wall. These calculations will help in the design melt crystallizer of different materials. Will be useful to designers and researchers in determining the cooling process in working with similar equipment.

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