

# The Riemann–Hilbert boundary value problem for matrices on non-smooth arc \*

B.A. Kats, S.R. Mironova and A.Yu. Pogodina

## Abstract

We consider the Riemann–Hilbert boundary value problem for holomorphic matrices (the Fokas–Its–Kitaev version) on certain class of non-smooth arcs. The main result is sufficient condition for its solvability.

Key words: Riemann–Hilbert boundary value problem, countable set of curves, non-smooth arc

Subject classification 30E25

## 1 Introduction.

Let  $\Gamma$  be a Jordan arc on complex plane  $\mathbb{C}$ . We consider boundary value problem on evaluation of holomorphic in  $\mathbb{C} \setminus \Gamma$  matrix

$$Y(z) = \begin{pmatrix} Y_{11}(z) & Y_{12}(z) \\ Y_{21}(z) & Y_{22}(z) \end{pmatrix}$$

such that

$$Y^+(t) = Y^-(t)G(t), t \in \Gamma \setminus \{a_1, a_2\}, \quad (1)$$

where  $Y^+(t)$  and  $Y^-(t)$  stand for boundary values of matrix  $Y$  at a point  $t \in \Gamma \setminus \{a_1, a_2\}$  from the left and from the right correspondingly,  $a_1$  and  $a_2$  are beginning and end points of  $\Gamma$ , and  $G(t)$  is defined on  $\Gamma$  triangular matrix

$$G(t) = \begin{pmatrix} 1 & w(t) \\ 0 & 1 \end{pmatrix}.$$

---

\*The results were reported at the Fifth International Conference in Harmonic Analysis and Approximations (September 10 - 17, 2011, Tsaghkadzor, Armenia).

## References

- [1] A.S. Fokas, A.R. Its and A.V. Kitaev, Discrete Painleve equations and their appearance in quantum gravity, *Comm. Math. Phys.*, 1991, V.142, no 2, P.313–344
- [2] P. Deift, *Orthogonal Polynomials and Random Matrices : A Riemann-Hilbert Approach*, Courant Lecture Notes, Vol. 3, New York University, 1999.
- [3] A.I. Aptekarev, Exact constants for rational approximations of analytic functions, *Mathem. Sbornic*, 2002, V.193, no 9, P.3-72
- [4] E.M. Stein. *Singular integrals and differential properties of functions*, Princeton University Press, Princeton, 1970.
- [5] I.N. Vekua. *Generalized analytical functions*, Nauka publishers, Moscow, 1988.
- [6] E.M. Dynkin. Smoothness of the Cauchy type integral, *Zapiski nauchn. sem. Leningr. dep. mathem. inst. AN USSR*, 92 (1979) 115–133.
- [7] T.S. Salimov. A direct estimate for a singular Cauchy integral over a closed curve, *Azerbaidzhan. Gos. Univ. Uchen. Zap.*, no.5 (1979), 59-75 (Russian)
- [8] L.I. Chibrikova, *The main boundary value problems for analytical functions*, Kazan: Publishers of Kazan University, 1977.
- [9] B.A. Kats. The Cauchy type integral and the Riemann problem on countable set of closed curves. *Izvest, vuzov. Mathem.*, 1985, no 3, P. 20–29.
- [10] F. D. Gakhov. *Boundary value problems*, Nauka publishers, Moscow, 1977.
- [11] N. I. Muskhelishvili. *Singular integral equations*, Nauka publishers, Moscow, 1962.