

1.

Speed is 36 km/h.

Convert km/h to m/s.

$$1 \text{ km} = 1000 \text{ m}$$

$$1 \text{ h} = 60 \text{ min} \cdot 60 \text{ s}$$

$$v = 36 \text{ (km/h)} = \frac{36 \cdot 1000}{60 \cdot 60} = 10 \text{ (m/s)}.$$

2.

The length of the vectors \vec{A} is 2, \vec{B} is $\sqrt{2}$, the angle between them is 45° .

Find the scalar product.

$$\vec{A} \cdot \vec{B} = |\vec{A}| \cdot |\vec{B}| \cdot \cos(\theta_{AB}) = 2 \cdot \sqrt{2} \cdot \frac{\sqrt{2}}{2} = 2$$

3.

The body moves equally slowly ($a < 0$).

Acceleration is 2 m/s^2 , starting speed is 50 m/s, driving time is 10 s.

Find the final speed.

$$v_f = v_i + a \cdot t = 50 - 2 \cdot 10 = 30 \text{ (m/s)}.$$

4.

The body moves equally accelerated ($a > 0$) and passed 100 m in 2 s.

Starting speed is 30 m/s.

Find the acceleration.

$$S = v_i \cdot t + \frac{a \cdot t^2}{2}$$
$$a = \frac{(S - v_i \cdot t) \cdot 2}{t^2} = \frac{(100 - 30 \cdot 2) \cdot 2}{2^2} = 20 \text{ (m/s}^2\text{)}.$$

5.

The force F_1 is 50 N, the force F_2 is 30 N.

The forces are directed towards each other.

The body mass is 2 kg.

Find acceleration.

Second Newton's law:

$$\sum_i \vec{F}_i = m \cdot \vec{a}$$

$$a = \frac{F_1 - F_2}{m} = \frac{50 - 30}{2} = 10 \text{ (m/s}^2\text{)}.$$

6.

A mass m (10 kg) hangs on a spring with spring constant k (200 N/m).

Find the amount Δx by which the string is stretched when the mass is at rest in static force equilibrium.

Second Newton's law:

$$\sum_i \vec{F}_i = m \cdot \vec{a}$$

$$a=0$$

$$m \cdot g - k \cdot \Delta x = 0$$

$$\Delta x = \frac{m \cdot g}{k} = \frac{10 \cdot 9.8}{200} = 0.49 \text{ (m)}$$

7.

A body (mass m is 5 kg) is at a height h of 2 m.

Find his potential energy U .

$$U = m \cdot g \cdot h = 5 \cdot 9.8 \cdot 2 = 98 \text{ (J)}.$$

8.

A body fell down from a height h of 20 m.

Find its speed in the second degree (v^2) at the moment of collision with the ground.

The principle of the conservation of mechanical energy:

$$K + U = A$$

A is work of conservative forces: $A = 0$.

$$K_i + U_i = K_f + U_f$$

$$K_i = 0$$

$$U_i = m \cdot g \cdot h$$

$$K_f = \frac{m \cdot v^2}{2}$$

$$U_f = 0$$

$$0 + m \cdot g \cdot h = \frac{m \cdot v^2}{2} + 0$$

$$v^2 = 2 \cdot g \cdot h = 2 \cdot 9.8 \cdot 20 = 392 \text{ (m}^2/\text{s}^2)$$

9.

A power is 1492 W.

Convert W to Horspower (HP).

$$1 \text{ HP} = 746 \text{ W}$$

$$P = 1492 \text{ W} = \frac{1492}{746} = 2 \text{ (HP)}$$

10.

A 100-watt lamp work 30 seconds.

Find the energy spent.

$$P = \frac{W}{t}$$

$$W = P \cdot t = 100 \cdot 30 = 3000 \text{ (J)}.$$

11.

A body moves at a speed v (100 m/s) under the action of the force F (5 N).

The directions of velocity and force coincide.

Find the power of this process.

$$P = F \cdot v = 5 \cdot 100 = 500 \text{ (W)}$$

12.

Two bodies are at a distance r_1 and have a gravitational attraction F_1 .

Find the gravitational attraction F_2 between the same bodies, if the distance is increased by 2 times.

Universal gravitational law:

$$F = G \cdot \frac{m \cdot M}{r^2}$$

$$F_1 = G \cdot \frac{m \cdot M}{r_1^2}$$

$$F_2 = G \cdot \frac{m \cdot M}{r_2^2}$$

$$\frac{F_2}{F_1} = \frac{r_1^2}{r_2^2}$$

$$r_2 = 2 \cdot r_1$$

$$\frac{F_2}{F_1} = \frac{r_1^2}{(2 \cdot r_1)^2} = 1/4$$

$$F_2 = F_1/4$$

13.

A ball (mass is 1 kg) slides at a speed of 5 m/s.

Find the momentum.

$$p = m \cdot v = 1 \cdot 5 = 5 \text{ (kg} \cdot \text{m/s)}$$

14.

A point particle (mass is 1 kg) moves in a circle with radius of 2 m with angular acceleration 5 rad/s².

Find the torque.

$$\tau = I \cdot \alpha$$

The moment of inertia $I = m \cdot R^2$

$$\tau = m \cdot R^2 \cdot \alpha = 1 \cdot 2^2 \cdot 5 = 20 \text{ (N} \cdot \text{m)}$$

15.

A first goods (mass m_1 is 2 kg) lies on the left shoulder (radius R_1 is 1 m) of see-saw.

A second goods (mass m_2) lies on the right shoulder (radius R_2 is 2 m).

The see-saw is in balance.

Find mass m_2 .

$$m_1 \cdot g \cdot R_1 = m_2 \cdot g \cdot R_2$$

$$m_2 = \frac{m_1 \cdot R_1}{R_2} = \frac{2 \cdot 1}{2} = 1 \text{ (kg)}$$

16.

A child (mass m is 20 kg) descends a 5 m high and reaches the bottom with a speed 2 m/s.

How much thermal energy (Th) due to friction was generated in this process?

Generalized non-conservative work-mechanical energy theorem:

$$U_i + K_i = U_f + K_f + Th$$

$$U_i = m \cdot g \cdot h \quad K_i = 0$$

$$U_f = 0 \quad K_f = \frac{m \cdot v^2}{2}$$

$$m \cdot g \cdot h + 0 = 0 + \frac{m \cdot v^2}{2} + Th$$

$$Th = m \cdot g \cdot h - \frac{m \cdot v^2}{2} = 20 \cdot 9.8 \cdot 5 - \frac{20 \cdot 2^2}{2} = 980 - 400 = 580 \text{ (J)}$$

17.

First ball (mass m_1 is 2 kg) moving east with a speed of 10 m/s collides head-on with second ball (mass m_2 is 1 kg) puck initially at rest.

Find velocity (speed and direction) of each ball after perfectly elastic collision.

$$m_1 \cdot \vec{v}_1 + 0 = m_1 \cdot \vec{U}_1 + m_2 \cdot \vec{U}_2 \quad (1)$$

$$\frac{m_1 \cdot v_1^2}{2} = \frac{m_1 \cdot U_1^2}{2} + \frac{m_2 \cdot U_2^2}{2} \quad (2)$$

Suppose that both balls move east after a collision.

$$U_1 \geq 0, U_2 \geq 0$$

$$\Rightarrow m_1 \cdot v_1 = m_1 \cdot U_1 + m_2 \cdot U_2 \quad (1)$$

$$U_2 = \frac{m_1 \cdot v_1 - m_1 \cdot U_1}{m_2} \rightarrow (2)$$

$$\Rightarrow \frac{m_1 \cdot v_1^2}{2} = \frac{m_1 \cdot U_1^2}{2} + \frac{m_2 \cdot (m_1 \cdot v_1 - m_1 \cdot U_1)^2}{2 \cdot m_2^2}$$

$$v_1^2 = U_1^2 + \frac{m_1}{m_2} \cdot v_1^2 - \frac{m_1}{m_2} \cdot 2 \cdot v_1 \cdot U_1 + \frac{m_1}{m_2} \cdot U_1^2$$

$$3 \cdot U_1^2 - 40 \cdot U_1 + 100 = 0$$

Epy equation solution:

1) $U_1 = 10$; $U_2 = 0$; mathematically only

2) $U_1 = 10/3$; $U_2 = 13 \frac{1}{3}$

Ans.: $U_1 = 10/3$ (m/s) east ; $U_2 = 13 \frac{1}{3}$ (m/s) east.