

# Determining the Direction of True Meridian by Micromechanical Gyro

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**Abstract**—The gyrocompassing method based on the parametric excitation of micromechanical gyro (MMG) is proposed. The incoherent mode of parametric excitation of MMG mounted on a horizontal base is considered. A feature of this mode is the presence of “strong resonance”, which enhances the amplitude of oscillations of the rotor with respect to its resonance value, and “weak resonance”, which reduces its amplitude of oscillations. The vibrational shape of the rotor is not preserved in this mode, and instead of harmonic vibration with one spectral component, there is a complex vibration (observed as a quasi-harmonic vibration) consisting of two spectral components with a slight difference in their frequencies. Thus, the rotor vibrations have the shape and frequency of the generated oscillations of the beats. This behavior of the gyroscope is associated with a change in its damping coefficient. Fluctuations of this coefficient with the beat frequency lead to a periodic change in the steepness of the phase-frequency characteristic of the MMG and, accordingly, to the oscillation of the measuring axis of the device regarding its original direction, approximately perpendicular to the direction of the true meridian. Metrological accuracy of the measurer reached by using the amplitude and phase gyrocompassing methods.

**Keywords**—gyrocompass, micromechanical gyro, parametric excitation

## I. INTRODUCTION

Micromechanical gyroscopes (MMG) are increasingly using in various fields of technology due to their small dimensions and lightweight. An overview of MMG models as devices for orientation, stabilization and navigation is given in monographs [1-2]. It was noting that, along with the undoubted advantages, the MMG has insufficiently high measurement accuracy, in connection with which the attention of researchers is focusing on the development of methods for its improvement. The analysis of publications [3-15] shows that when developing devices based on MMG, a promising approach is basing on the use of improving their characteristics by using non-traditional operating modes.

One example of such an approach is the development of a ground gyrocompass based on MMG [16]. In this work, a diagram of the device has proposed, where an MMG with a horizontal measuring axis is mounting on a rotating base. In this case, the useful signal is modulating by the angular velocity of the base, result of which it is possible to separate it from the MMG instrumental errors. Unfortunately, the author of the work does not give specific numerical estimates for improving the accuracy of the device.

Below, a new approach is proposing for determining the direction of the true meridian. The method is basing on the

parametric excitation of MMG, which, as was shown by the authors earlier [17-18], can significantly increase the sensitivity of the device to the measured angular velocity, as well as expand its functionality.

An incoherent mode of parametric excitation of MMG mounted on a horizontal base is considered. A feature of this mode is the presence of “strong resonance”, which increases the amplitude of oscillations of the meter rotor with respect to its resonance value, as well as “weak resonance”, which reduces its amplitude of oscillations. The vibrational shape of the rotor is not preserved in this mode, and instead of harmonic vibration with one spectral component, there is a complex vibration (observed as a quasi-harmonic vibration) consisting of two spectral components with a slight difference in their frequencies. Thus, the rotor oscillations have the shape and frequency of the generated oscillations of the beats.

This behavior of the gyroscope is associated with a change in its damping coefficient. Oscillations of this coefficient with the beat frequency lead to a periodic change in the steepness of the phase-frequency characteristic of the MMG and, accordingly, to the oscillation of the measuring axis of the device relative to its original direction, approximately orthogonal to the true meridian.

## II. METHOD DESCRIPTION

The specificity of the MMG operation allows based on the use of circuit solutions, without changing the design of the mechanical circuit, to increase the sensitivity of the device to the measured angular velocity due to its parametric excitation [19].

The highest sensitivity of the MMG is ensuring with conditions of the maximum amplitude of the primary oscillations, i.e. under the conditions of the implementation of the resonant tuning mode. However, the operation of MMG under conditions of a significant range of ambient temperatures, reaching 100 ° C or more, leads to a deviation of its own parameters and the frequency of the primary oscillation excitation generator, as well as to aging of the material of the sensitive element, which violates the resonant tuning.

The method under consideration is basing on modulation of the static rigidity of the MMG suspension [19]. The kinematic scheme of device is showing in Fig. 1. Modulation is providing by a slight change in the amplitude of the alternating current applied to the additional sensor winding of the torque sensor with frequency  $\omega_m$ , at which a parametric excitation of MMG is creating.

To obtain a mathematical model of a parametrically excited MMG we use the variational principle of

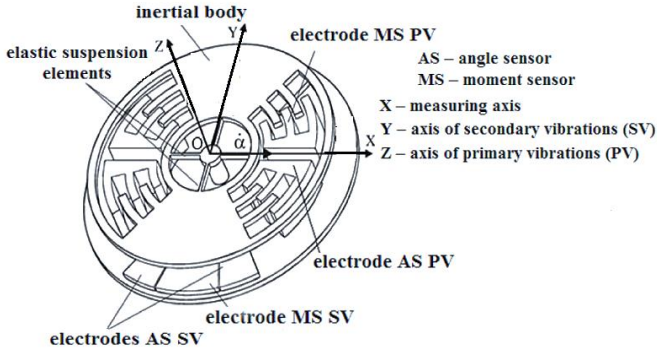


Fig. 1. The kinematic diagram of a micromechanical gyroscope

Ostrogradsky-Hamilton [20]. Differential equations of motion for the case of constant angular velocity of rotation of the base, taking into account the modulation of static rigidity relative to the axis of secondary vibrations after the factorization procedure [21] and subsequent linearization using the Jacobi matrix, will have the form:

$$\ddot{\alpha} + 2a_{\alpha}\dot{\alpha} + \omega_0^2(1 + m \sin(\omega_m t - \varphi_0))\alpha = K\omega_X \sin \Omega t, \quad (1)$$

$$a_{\alpha} = \frac{\mu_{\alpha}}{2A}; \quad \omega_0 = \sqrt{\frac{k_{\alpha}}{A}}; \quad K = \frac{(C+B-A)\theta_0\Omega}{A}.$$

Coefficients  $A$ ,  $B$ ,  $C$  are the main moments of inertia relative to the axes of the coordinate system associated with the rotor. Parameter  $\mu_{\alpha}$  is the coefficient of viscous friction.  $k_{\alpha}$  is the stiffness of the elastic suspension relative to the axis of secondary vibrations.  $\omega_X$  – the projection of the portable angular velocity of the base on the measuring axis.  $\Omega$  is the excitation frequency of the primary oscillations;  $m$  is the modulation coefficient of static stiffness;  $\theta_0$  is the amplitude of the primary oscillations of the gyro rotor;  $\varphi_0$  is the initial phase of the parametric excitation created by the torque sensor.

A feature of the differential equation (1) is the presence of a term associated with a periodic change in the positioning torque

$$\omega_0^2 m \sin(\omega_m t - \phi)\alpha.$$

The presence of such a periodically changing energy-consuming parameter MMG, such as static stiffness, which is part of the corresponding inhomogeneous differential equation, provides favorable conditions for parametric excitation of the mechanical circuit of the gyroscope under consideration at the modulation frequency  $\omega_m = 2\Omega$ .

Fig. 2 shows the location of axes  $OX$  and  $OY$  relative to the true meridian. It is for the case of its resonance tuning (axis  $OXY$ ) and the change in its position under incoherent mode of parametric excitation of MMG (for the case of “strong resonance” – the position of the axes  $OX'Y'$  and for the case of “weak resonance” – the position of the axes  $OX''Y''$ ).

Note that in the case of a tuned instrument (when the position of the  $OX$  axis coincides with the direction of the

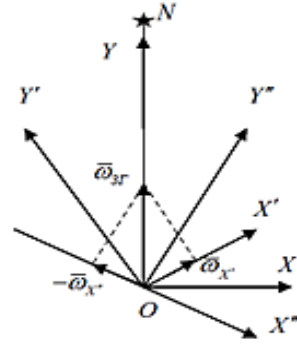


Fig. 2. Changing the position of the axes of the MMG upon parametric excitation

West – East line), the projection of the horizontal component of the Earth’s rotation  $\omega_{3\Gamma}$  on the measuring axis  $OX$  is zero.

With parametric excitation, a periodic change in the magnitude and sign of the  $\omega_{3\Gamma}$  projection is observing. In the case of rotation of the MMG by a certain angle  $\gamma$ , the center of oscillation of the measuring axis  $OX$  is shifted by an amount corresponding to this angle.

It is possible to provide alignment of the axis of secondary oscillations  $OY$  with the direction of the true meridian  $N$ . This is by achieving a rotation of the device around the axis of primary oscillations  $OZ$  (so that the center of oscillation of the measuring axis  $OX$  coincides with the direction of the West – East line that is when the positive and negative values of the amplitude of oscillations of the axis  $OX$  are equal).

Along with the amplitude method for determining the direction of the true meridian, the phase method may also be used. It based on the fact, that with parametric excitation of the gyroscope, a periodic change in the damping coefficient  $a_{\alpha}$  leads to a periodic change in the steepness of the phase-frequency characteristic of the MMG. In this case, the oscillation phase of the gyro rotor also periodically changes with the beat frequency relative to the resonance value equal to  $-\pi / 2$ . Note that for a high-quality oscillatory system, which is MMG parametrically excited, the phase-frequency characteristic in the resonance region has a significant slope. Even at small values of the modulation index  $m$  a change in the slope of the averaged phase-frequency characteristic leads to symmetrical and significant oscillations of the phase of the output signal with respect to the value of  $-\pi / 2$ , which makes it possible to increase the accuracy of determining the direction of the true meridian in comparison with the amplitude method.

### III. THE RESULTS OBTAINED

The numerical simulation of a parametrically excited MMG (the solution of equation (1)) carried out using the Maple 9 mathematical package at the value of the beat frequency  $\Omega = 0.628 \text{ c}^{-1}$  and at the value of modulation parameter  $m = 0.00338$ .

The Fig. 3 shows the time dependences of the amplitude  $\alpha$  (solid line) and the oscillation phase  $\varphi$  (dotted line) of the

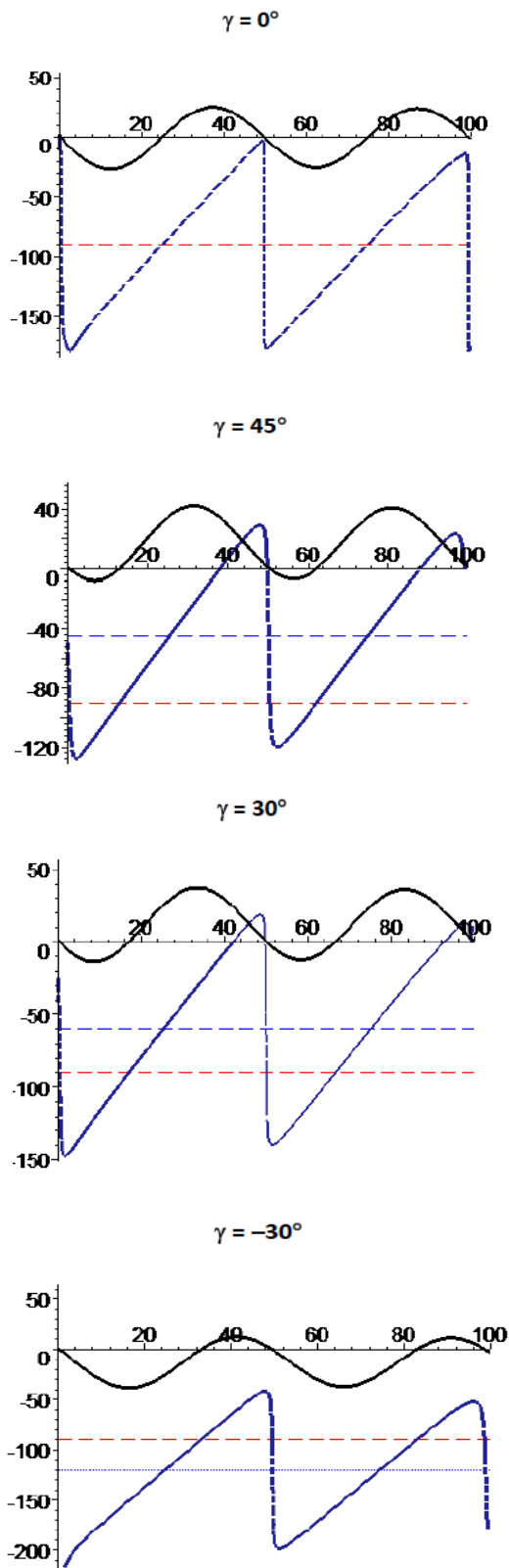


Fig. 3. Changes in the amplitude and phase of secondary oscillations of a parametrically excited MMG

secondary oscillations of a mounted horizontally parametrically excited MMG for various values of the angle  $\gamma$  of its rotation around the  $OZ$  axis, which coincides with the plane vertical.

From the above solutions it follows that, along with periodic oscillations, a constant component appears that is proportional to the angle  $\gamma$  (blue line), both in the information signal  $\alpha$  and in the phase value  $\varphi(t)$ . Using this constant component, you can automatically orientate the MMG mounted on the gyro platform in the direction of the true meridian.

#### IV. CONCLUSIONS

It follows from the graphs in Fig.3 that the modulation of the static stiffness of the MMG provides an increase in its sensitivity in the steady state (differential gyroscope mode) by several tens of times compared to a device in which there is no parametric excitation, which significantly increases the value of its transfer coefficient. This is due to the fact, that modulation of static stiffness significantly reduces the amount of viscous friction. This increases the duration of the linear part of the increase in the vibration amplitude, which corresponds to a significant increase in the time constant of the device and, accordingly, to a narrowing of its bandwidth.

It should be noted, that in the coherent excitation mode, an increase in the MMG sensitivity is of decisive importance for the magnitude of the phase shift between the periodically varying static stiffness of the torsion suspension and the external gyroscopic moment created by the transferred angular velocity of rotation of the device base.

An analysis of the results showed that using the parametrically excited MMG, we can determine the direction of the true meridian. Moreover, this problem can be solved both on the base of measuring amplitude fluctuations, and on the base of measuring phase oscillations.

The determination of the position of the meridian by amplitude is producing by zero values of the amplitude.

The determination of the position of the meridian by the phase of oscillations is producing by achieving equality of phase oscillations relative to the value  $\varphi = -\pi/2$  (the red line in Fig. 3). It is corresponding to the position of the instrument body when the direction of the measuring axis  $OX$  coincides with the West - East line, and the axis of the secondary oscillations  $OY$  coincides with the direction of the true meridian.

The novelty of the results and conclusions is as follows:

- the new method is proposed for determining the direction of the true meridian based on the parametric excitation of a micromechanical gyroscope;
- the numerical simulation of a parametrically excited MMG was carried out;
- it was shown that the presented method allows you to automatically orientate the MMG installed on the gyro platform in the direction of the true meridian, while significantly increasing the sensitivity and quality of the device.

- the operation of the device in the parametric excitation mode allows the meter to be held in resonance mode over a wide range of operating temperatures;
- the parametric excitation mode expands the functionality of the MMG, turning it from a single-component to a two-component measurer;
- the proposed method significantly (by 1-2 orders of magnitude) increases the accuracy of measurements compared to the typical mode of operation of the device, which allows the use of MMG as an inertial measurer (for example [16]).

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