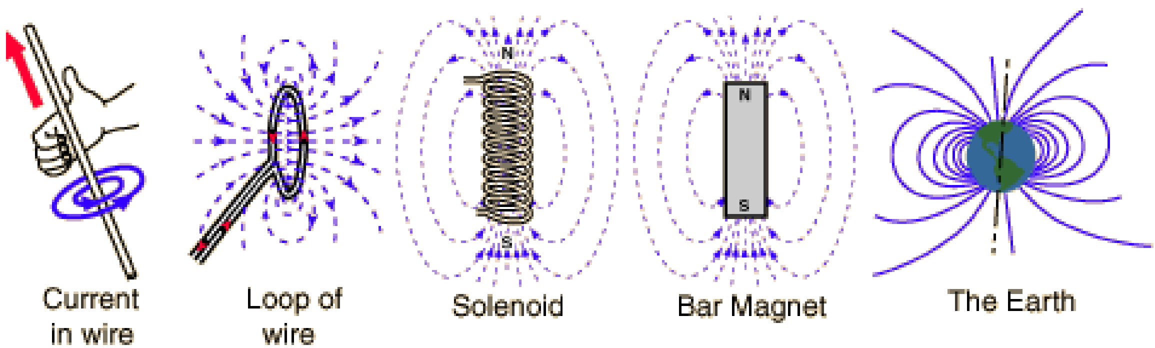
Magnetic Field

Magnetic fields are produced by electric currents, which can be macroscopic currents in wires, or microscopic currents associated with electrons in atomic orbits.

Magnetic field sources are essentially *dipolar* in nature, having a *north* and *south* magnetic pole.

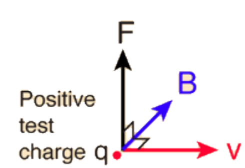
Magnetic field lines are very different from electric field lines because magnetic sources are inherently dipole sources with North and South magnetic poles. The magnetic field lines form closed loops and any closed surface will have a net zero number of lines leaving the surface.



Magnetic Field Sources

The magnetic field B is defined from the Lorentz Force Law, and specifically from the magnetic force on a moving charge:

$$\vec{F} = q(\vec{v} \times \vec{B})$$



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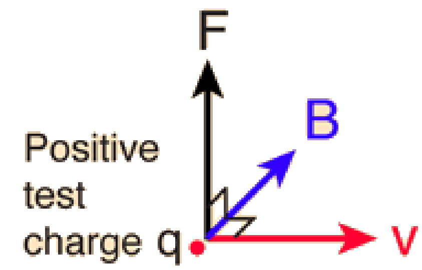
Magnetic Field

$$\vec{F} = q(\vec{v} \times \vec{B})$$

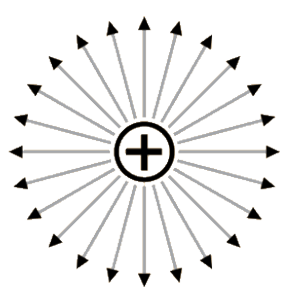


The implications of this expression include:

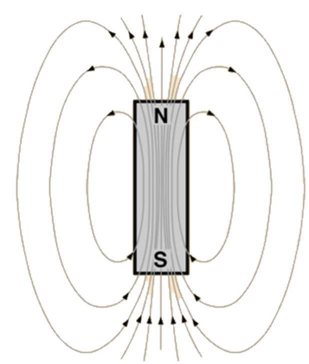
1. The force is perpendicular to both the velocity $\vec{v}$ of the charge q and the magnetic field $\vec{B}$.

2. The magnitude of the force is $F = qvB \sin\vartheta$ where ϑ is the angle < 180 degrees between the velocity and the magnetic field. This implies that the magnetic force on a stationary charge or a charge moving parallel to the magnetic field is zero.

3. The direction of the force is given by the right hand rule. The force relationship above is in the form of a vector product.



The electric field of a point charge is radially outward from a positive charge.



The magnetic field of a bar magnet.

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Lorentz Force Law

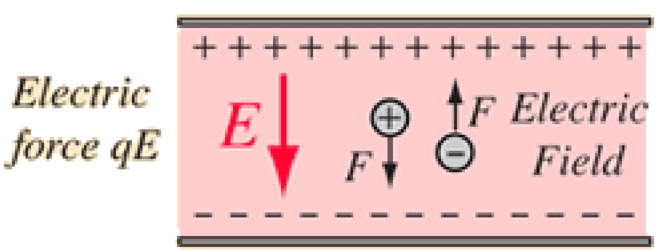
The magnetic field ***B*** is defined in terms of force on moving charge in the ***Lorentz force law***:

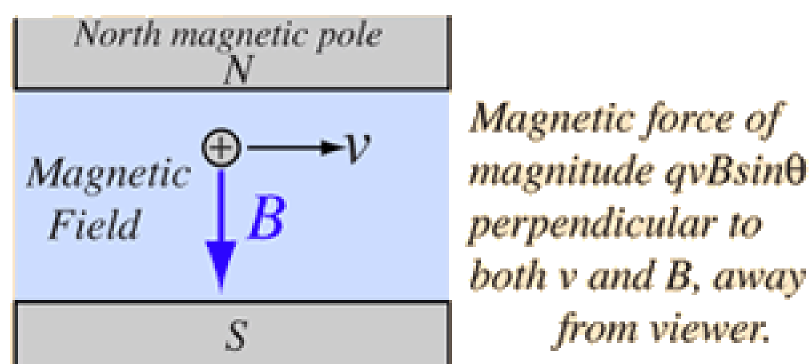
$$\vec{F} = q\vec{\mathbf{E}} + q(\vec{v} \times \vec{\mathbf{B}})$$

Electric force   *Magnetic force*

The SI unit for magnetic field is the ***Tesla***, which can be seen from the magnetic part of the Lorentz force law. A smaller magnetic field unit is the ***Gauss*** (1 Tesla = 10,000 Gauss).

The electric force is *straigtforward*, being in the direction of the electric field if the charge q is positive, but the direction of the magnetic part of the force is given by the right hand rule.





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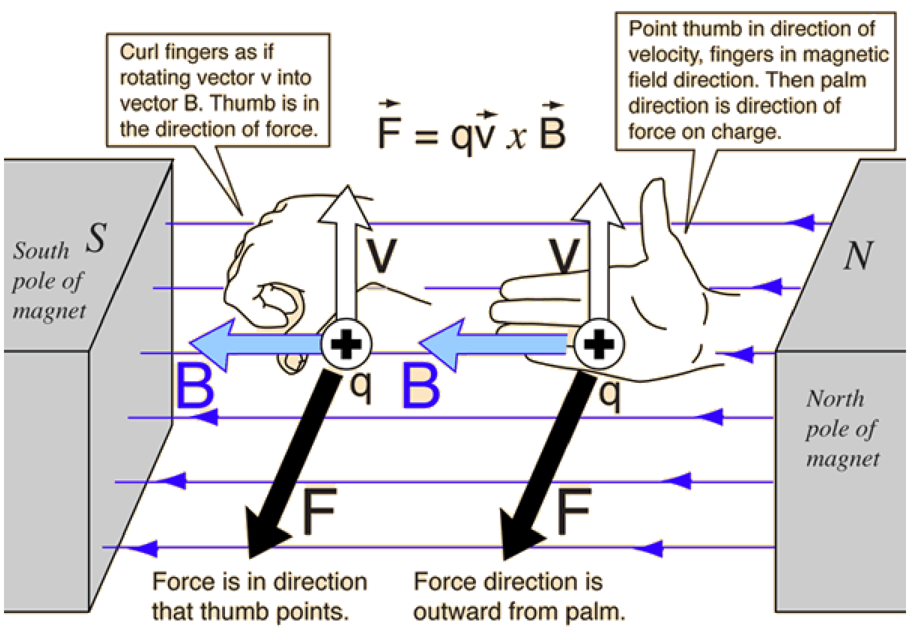
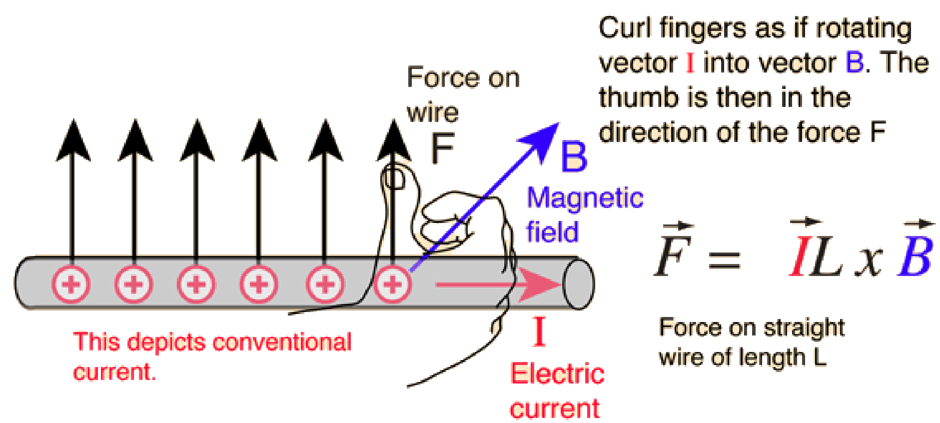
Right Hand Rule

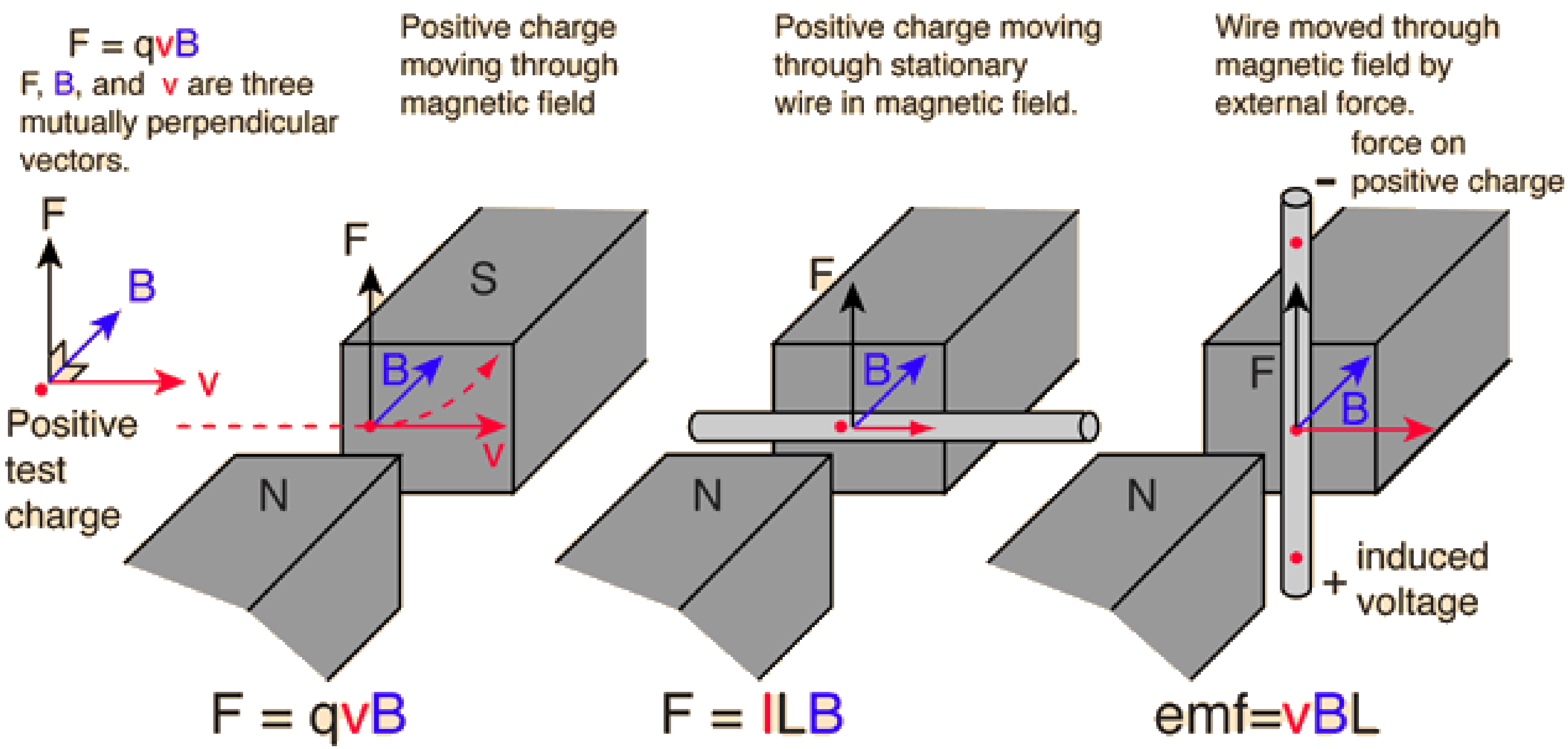
The right hand rule is a useful mnemonic for visualizing the direction of a magnetic force as given by the Lorentz force law.

The force is in the opposite direction for a negative charge moving in the direction shown. One fact to keep in mind is that the magnetic force is perpendicular to both the magnetic field and the charge velocity, but that leaves two possibilities.





Magnetic Interactions with Moving Charge

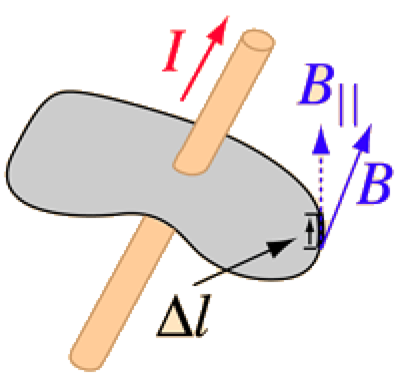


Magnetic Fields from Currents. Ampere's Law

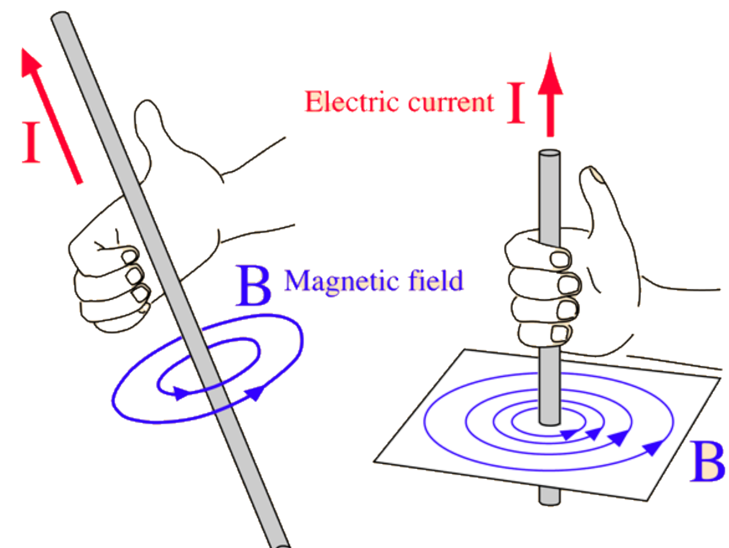
Magnetic fields on a macroscopic scale can be related to electric currents.

The magnetic field in space around an electric current is *proportional* to the electric current which serves as its source, just as the electric field in space is proportional to the charge which serves as its source.

Ampere's Law states that for any closed loop path, the sum of the length elements times the magnetic field in the direction of the length element is equal to the permeability times the electric current enclosed in the loop.

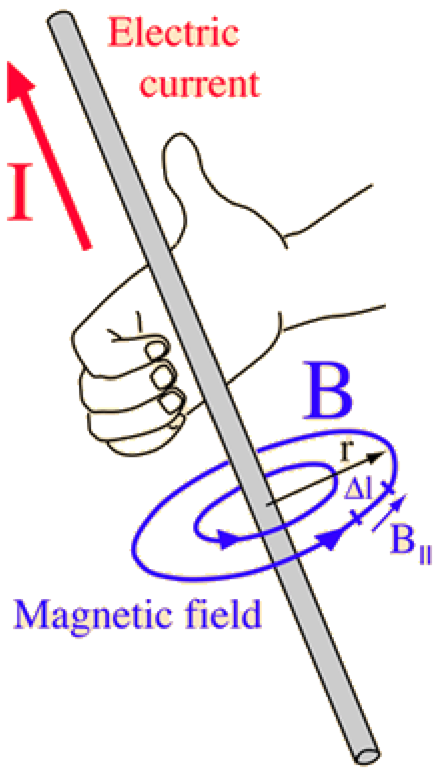


$$\sum B_{||} \Delta l = \mu_0 I$$



In the electric case, the relation of field to source is quantified in Gauss's Law

Magnetic Field of Current



The magnetic field lines around a long wire which carries an electric current form concentric circles around the wire. The direction of the magnetic field is perpendicular to the wire and is in the direction the fingers of your right hand would curl if you wrapped them around the wire with your thumb in the direction of the current.

The magnetic field of an infinitely long straight wire can be obtained by applying Ampere's law. Ampere's law takes the form

$$\sum B_{\parallel}\Delta l = \mu_0 I$$

and for a circular path centered on the wire, the magnetic field is everywhere parallel to the path. The summation then becomes just

$$\sum B_{\parallel}\Delta l = B 2\pi r$$

$$B = \frac{\mu_0 I}{2\pi r}, \qquad \mu_0 = 4\pi \times 10^{-7} T \cdot m/A$$

The constant μ_0 is the ***permeability*** of free space.

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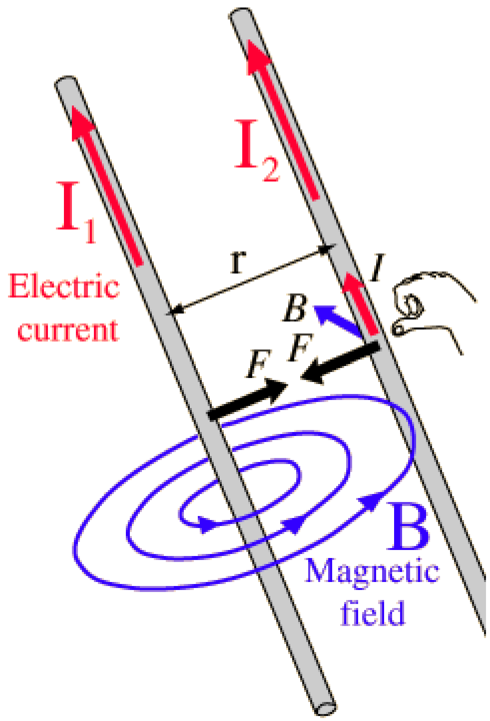
Magnetic Force Between Wires



Magnetic field at wire 2 from current in wire 1:

$$B = \frac{\mu_0 I_1}{2\pi r}$$

Force on a length ΔL of wire 2:

$$F = I_2 \Delta L B$$

Force per unit length in terms of the currents:

$$\frac{F}{\Delta L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

$$F_1 = \frac{\mu_0 I_1 I_2}{2\pi r} l$$

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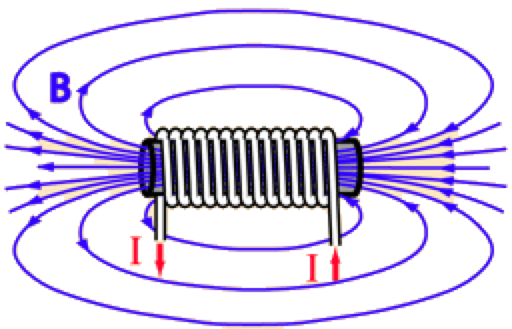
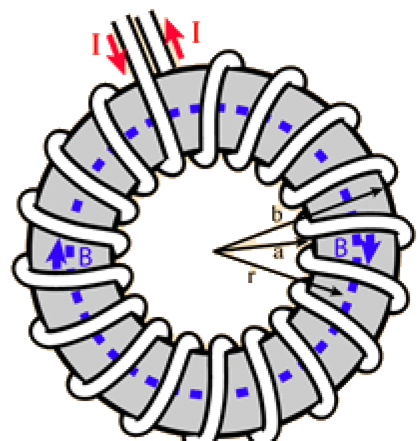
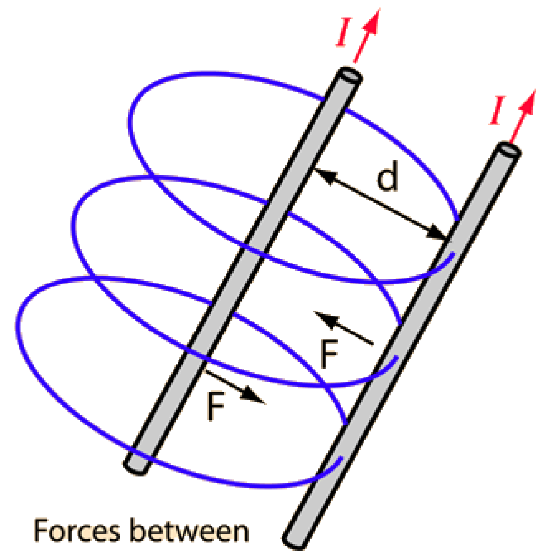
Ampere's Law Applications



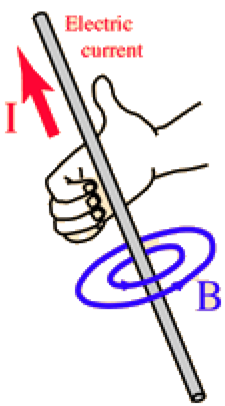
Magnetic field inside a long solenoic.



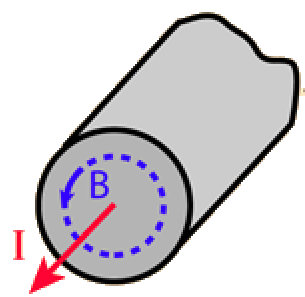
Magnetic field inside a toroidal coil.



Forces between Currents

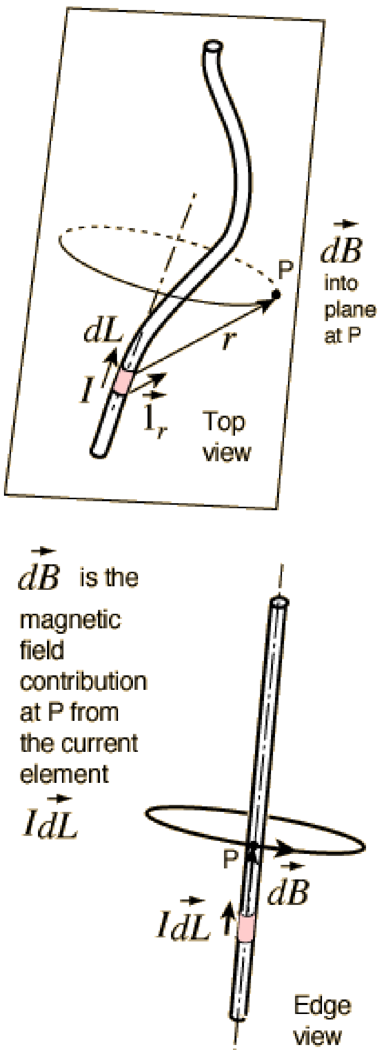


Magnetic field from a long straight wire.



Magnetic field inside a conductor.

Biot-Savart Law



The ***Biot-Savart Law*** relates magnetic fields to the currents which are their sources. In a similar manner, Coulomb's law relates electric fields to the point charges which are their sources. Finding the magnetic field resulting from a current distribution involves the vector product, and is inherently a calculus problem when the distance from the current to the field point is continuously changing.

Magnetic field of a current element:

$$d\vec{B} = \frac{\mu_0 I d\vec{L} \times \vec{1}_r}{4\pi r^2}$$

where

$d\vec{L}$ = infinitesimal length of conductor carrying electric current *I*

$\vec{1}_r$= unit vector to specify the direction of the vector distance r from the current to the field point.

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Magnetic Field Strength H

The magnetic fields generated by currents and calculated from Ampere's Law or the Biot-Savart Law are characterized by the magnetic field *B* measured in Tesla.

But when the generated fields ***pass through magnetic materials*** which themselves contribute ***internal magnetic fields***, ambiguities can arise about what part of the field comes from the external currents and what comes from the material itself. It has been common practice to define another magnetic field quantity, usually called the ***"magnetic field strength"*** designated by *H*. It can be defined by the relationship

$$H = \frac{B_0}{\mu_0} = \frac{B}{\mu_0} - M$$

and has the value of unambiguously designating the driving magnetic influence from external currents in a material, independent of the material's magnetic response.

Magnetic Field Strength H

The relationship for B can be written in the equivalent form

$$B = \mu_0(H + M)$$

H and M will have the same units, amperes/meter. To further distinguish *B* from *H*, *B* is sometimes called ***the magnetic flux density or the magnetic induction***. The quantity *M* in these relationships is called the magnetization of the material.

Another commonly used form for the relationship between *B* and *H* is

$$B = \mu_m H, \text{ where } \qquad \mu = \mu_m = K_m \mu_0$$

μ_0 being the *magnetic permeability* of space and K_m the *relative permeability of the material*. If the material does not respond to the external magnetic field by producing any magnetization, then K_m = 1. Another commonly used magnetic quantity is the magnetic susceptibility which specifies how much the relative permeability differs from one.

$$\text{Magnetic susceptibility } \chi_m = K_m - 1$$

For ***paramagnetic*** and ***diamagnetic*** materials the relative permeability is very close to 1 and the magnetic susceptibility very close to zero. For ***ferromagnetic*** materials, these quantities may be very large.

The unit for the magnetic field strength *H* can be derived from its relationship to the magnetic field *B*, $B=\mu H$. Since the unit of magnetic permeability μ is N/A^2, then the unit for the magnetic field strength is:

$$T/(N/A^2) = (N/Am)/(N/A^2) = A/m$$

An older unit for magnetic field strength is the oersted: 1 A/m = 0.01257 oersted

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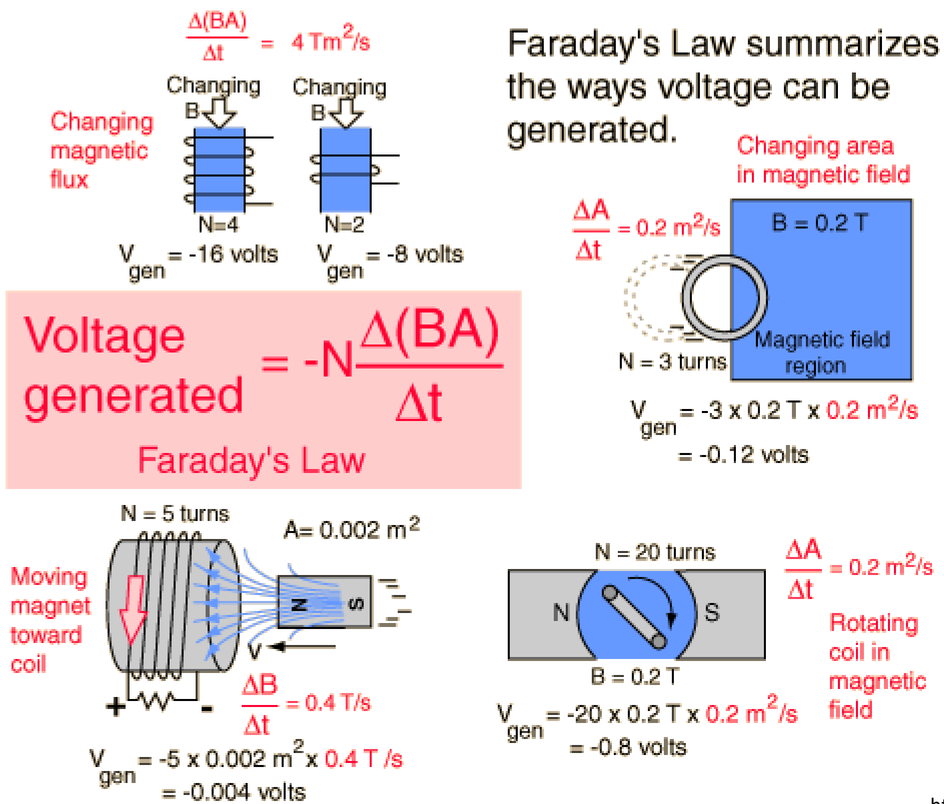
Faraday's Law

Any change in the magnetic environment of a coil of wire will cause a voltage (emf) to be "induced" in the coil. No matter how the change is produced, the voltage will be generated. The change could be produced by changing the magnetic field strength, moving a magnet toward or away from the coil, moving the coil into or out of the magnetic field, rotating the coil relative to the magnet, etc.



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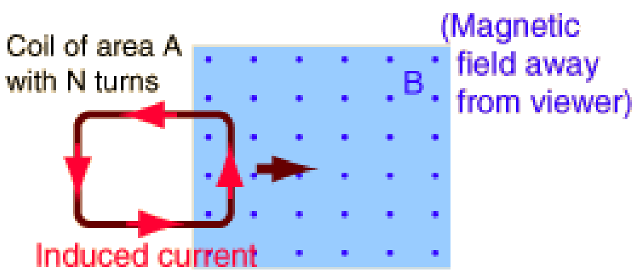
Faraday's Law

Faraday's law is a fundamental relationship and it serves as a succinct summary of the ways a voltage (or emf) may be generated by a changing magnetic environment. The induced emf in a coil is equal to the negative of the rate of change of magnetic flux times the number of turns in the coil. It involves the interaction of charge with magnetic field.



A coil of wire moving into a magnetic field is one example of an emf generated according to Faraday's Law. The current induced will create a magnetic field which opposes the buildup of magnetic field in the coil.

Faraday's Law

$$\text{Emf} = -N\frac{\Delta\Phi}{\Delta t}$$

(− : Lenz's Law)

where N = number of turns

Φ = BA = magnetic flux

B = external magnetic field

A = area of coil

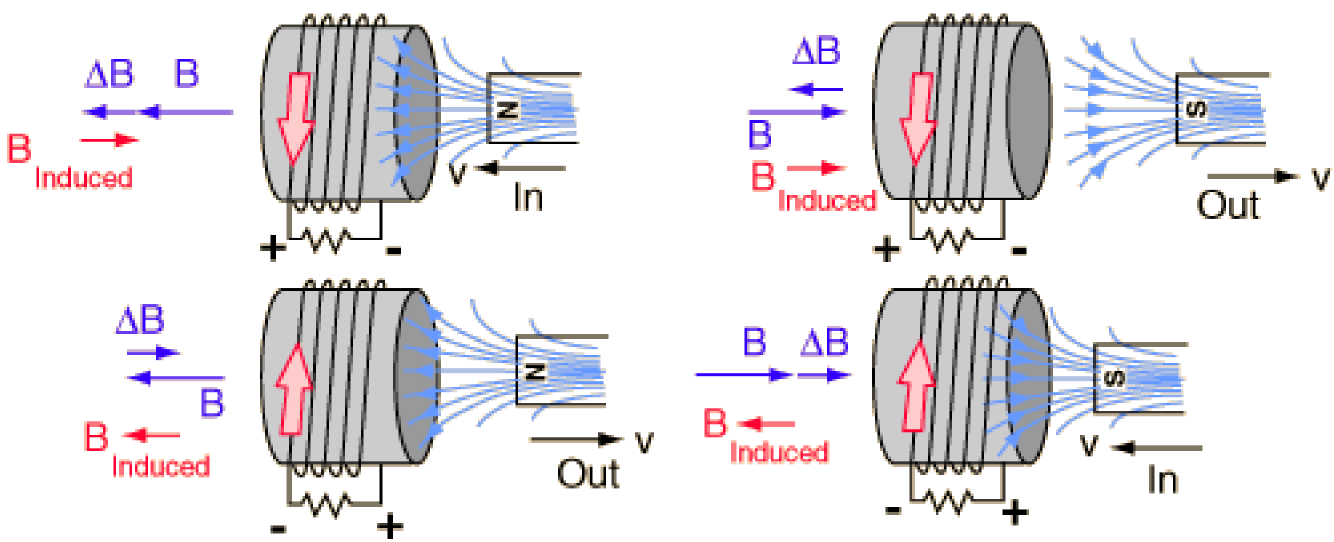
The minus sign denotes Lenz's Law. Emf is the term for generated or induced voltage.

Lenz's Law

When an emf is generated by a change in magnetic flux according to Faraday's Law, the polarity of the induced emf is such that it produces a current whose magnetic field opposes the change which produces it.

The induced magnetic field inside any loop of wire always acts to keep the magnetic flux in the loop constant.

In the examples below, if the B field is increasing, the induced field acts in opposition to it. If it is decreasing, the induced field acts in the direction of the applied field to try to keep it constant.



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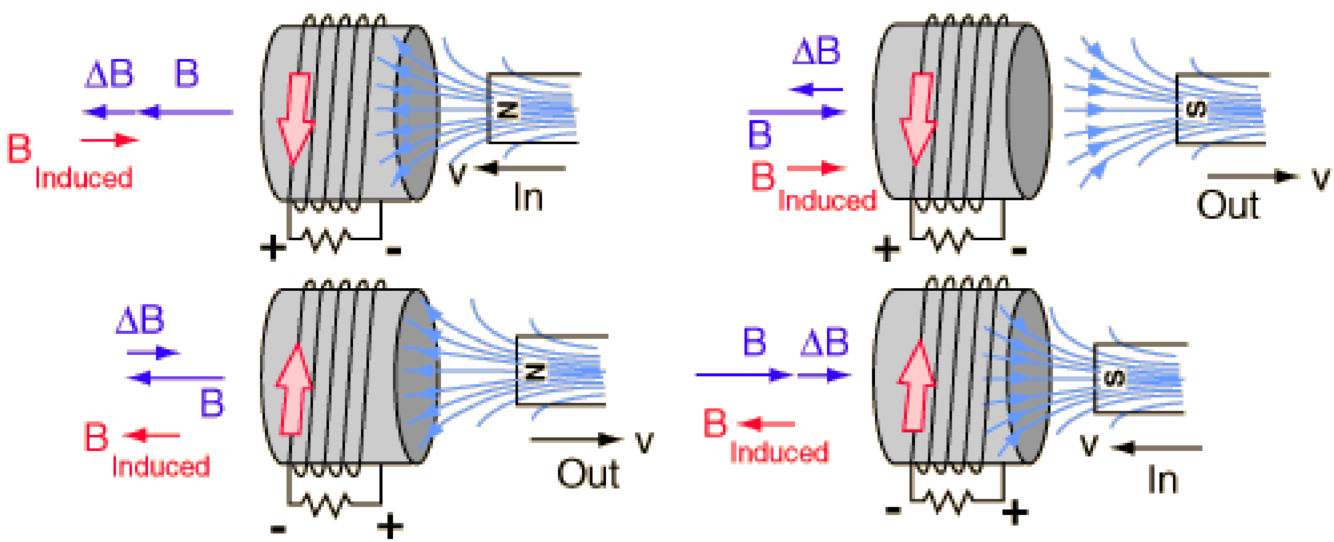

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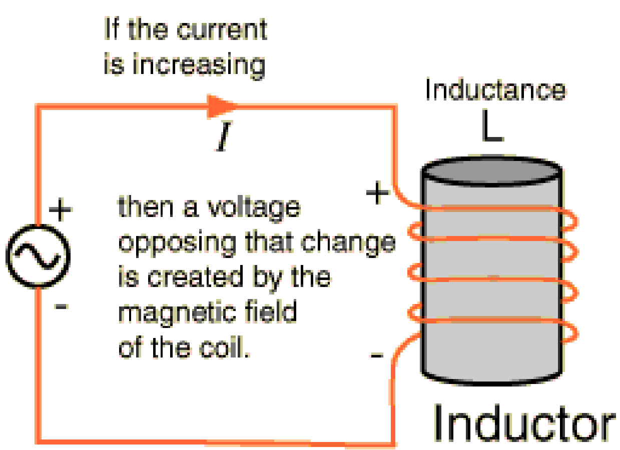
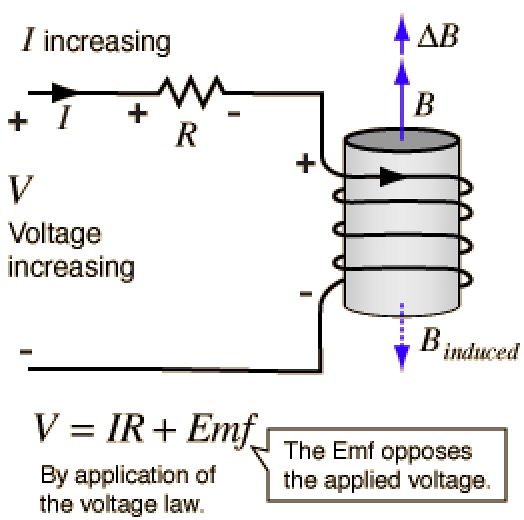
Inductors



Inductance is typified by the behavior of a coil of wire in resisting any change of electric current through the coil.

$$Emf = -L\frac{\Delta I}{\Delta t}$$

Unit for L: $\frac{volt\ second}{ampere} = Henry$



Increasing current in a coil of wire will generate a counter emf which opposes the current. Applying the voltage law allows us to see the effect of this emf on the circuit equation. The fact that the emf always opposes the change in current is an example of Lenz's law.

$$Emf = -N\frac{\Delta \Phi}{\Delta t}$$

The relation of this counter emf to the current is the origin of the concept of inductance.

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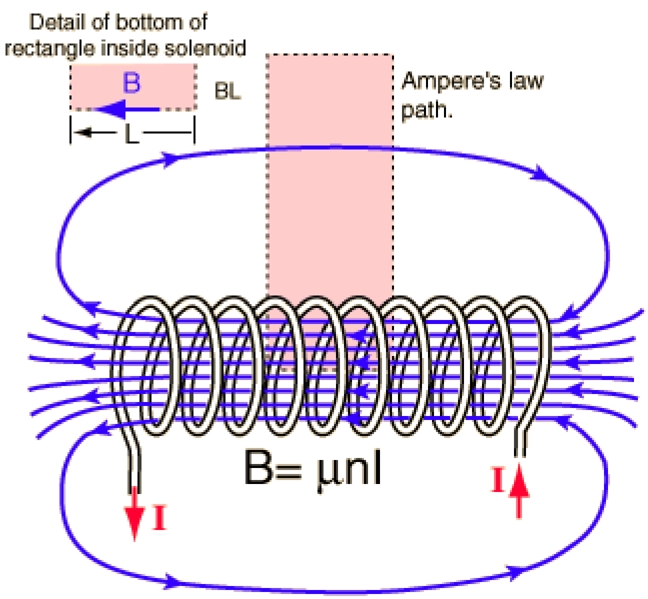
Solenoid Field from Ampere's Law



Taking a rectangular path about which to evaluate Ampere's Law such that the length of the side parallel to the solenoid field is L gives a contribution BL inside the coil. The field is essentially perpendicular to the sides of the path, giving negligible contribution. If the end is taken so far from the coil that the field is negligible, then the length inside the coil is the dominant contribution.

This admittedly idealized case for Ampere's Law gives

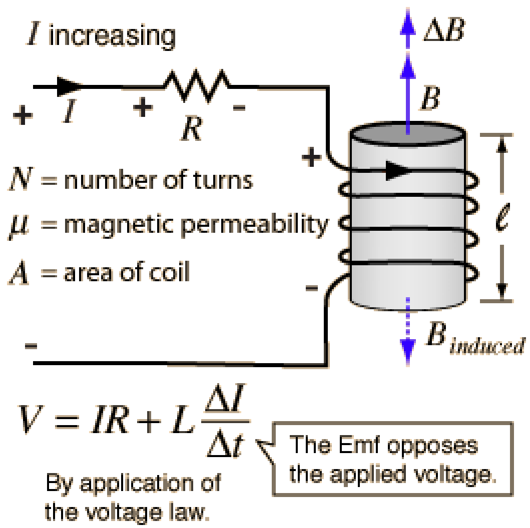
$$BL = \mu NI$$

$$B = \mu \frac{N}{L} I$$

$$B = \mu nI$$

This turns out to be a good approximation for the solenoid field, particularly in the case of an iron core solenoid.

Inductance of a Coil



For a fixed area and changing current, Faraday's law becomes

$$Emf = -N\frac{\Delta\Phi}{\Delta t} = -NA\frac{\Delta B}{\Delta t}$$

Since the magnetic field of a solenoid is

$$B = \mu\frac{N}{l}I$$

then for a long coil the emf is approximated by

$$Emf = -\frac{\mu N^2 A}{l}\frac{\Delta I}{\Delta t}$$

From the definition of inductance

$$Emf = -L\frac{\Delta I}{\Delta t}$$

we obtain

$$L = \frac{\mu N^2 A}{l},$$

l = length of solenoid, A = cross-sectional area

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