QUARTERLY OF APPLIED MATHEMATICS
VOLUME LXIX, NUMBER 4
DECEMBER 2011, PAGES 771–786
S 0033-569X(2011)01242-8
Article electronically published on July 12, 2011

THREE–PHASE ECCENTRIC ANNULUS SUBJECTED TO A POTENTIAL FIELD INDUCED BY ARBITRARY SINGULARITIES

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Abstract. An infinite planar, three-component heterogeneous medium with a pair of circles as interfaces between homogeneous zones forming an eccentric annulus is considered for refraction of a potential field on the two interfaces. The velocity field is generated by an arbitrary system of singularities of arbitrary order, in congruity with the Milne-Thomson case of a two-component medium and a single circular interface. An exact analytical solution of the corresponding $\mathbb{R}$-linear conjugation problem of two Laplacian fields in the eccentrical annulus structure is derived in the class of piecewise meromorphic functions with fixed principal part. Three general cases of loci of the singularities with respect to the interfaces are investigated. Flow nets (isobars and streamlines) are presented.

1. Introduction. Composite (heterogeneous) materials are common in nature (e.g. soils and rocks, Dagan [2]), in modern designed engineering constructions (e.g. tessellations of doped semi-conductors, Milton [15]) and in engineering control (modification) of natural environments where fascinating composite patterns are formed with anthropogenic interventions (De Zwart [4], Obnosov et al. [20]). The induced or existing fields of pressure, hydraulic head, electrical current, concentration, temperature, which are described by the Darcy, Ohm, Fick, Fourier laws, in these composites are amazingly complex, counterintuitive and puzzling in some characteristics (e.g. topology of streamlines). Moreover, if the engineer or designer can control the position of field-inducing singularities and/or conducting properties of the components of the composite, an unexpected optimality of integral characteristics (e.g. total flow rate or total current) has been revealed (Obnosov et al. [20]).

It is remarkable that the same mathematical model describes the different physical fields mentioned above. In what follows we shall use the language of Darcian flows in

Received May 3, 2010.
2010 *Mathematics Subject Classification.* Primary 30E25, 76T30.
Key words and phrases. Refraction, heterogeneous media, $\mathbb{R}$-linear conjugation problem, analytic functions.
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