

Analytic Functions With Polar and Logarithmic Singularities and Locally Convex Boundary Values

F.G. AVKHADIEV*

Kazan (Volga Region) Federal University, 18 Kremlyovskaya str., Kazan 420008, Russia

Abstract. We consider a family of functions which have a pole of order m and a logarithmic term at the infinity and generalize univalent convex functions defined in the exterior of the unit disc. We prove sharp estimates for the derivative and the Schwarzian and describe some geometric properties of such functions.

Keywords: multi-valued analytic functions, conformal mappings, Schwarzian, univalent convex functions, Riemann surface

2000 Mathematics Subject Classification. Primary 30C80

1 INTRODUCTION

Let m be a fixed natural number. We consider a new family $\Sigma_m^0(\log)$ of functions with the expansion of the type

$$F(z) = z^m + a_{m-1}z^{m-1} + \dots + a_1z + \lambda Lnz + \sum_{n=0}^{\infty} \alpha_n z^{-n}, \quad |z| > 1, \quad (1)$$

and satisfying two following conditions:

the function $F(z) - (z^m + a_{m-1}z^{m-1} + \dots + a_1z) - \lambda Lnz$ is holomorphic in the domain $D^* = \{z \in \overline{\mathbb{C}} : |z| > 1\}$ and for all $z \in D^*$ the inequality

$$\Re \left(1 + z \frac{F''(z)}{F'(z)} \right) > 0 \quad (2)$$

is fulfilled.

*E-mail: favhadiev@ksu.ru.

The work is supported by RFBR, grant 11-01-00762-a