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ИНСТИТУТ МЕЖДУНАРОДНЫХ ОТНОШЕНИЙ
ВЫСШАЯ ШКОЛА ИНОСТРАННЫХ ЯЗЫКОВ И ПЕРЕВОДА

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ENGLISH FOR STUDENTS OF MATHEMATICS

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Данное учебное пособие предназначено для бакалавров Института математики и механики им. Н.И. Лобачевского, содержит материалы по английскому языку, способствующие развитию навыков и умений профессиональной коммуникации, чтения и перевода аутентичных текстов. Пособие может быть использовано как для аудиторной работы, так и для самостоятельной работы студентов.

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Предисловие

Настоящее учебное пособие по английскому языку предназначено для бакалавров Института математики и механики им. Н.И. Лобачевского Казанского (Приволжского) федерального университета уровня Intermediate / Upper-Intermediate.

Целью пособия является овладение студентами компетенциями устного и письменного профессионально-ориентированного общения на английском языке. В задачи пособия входит развитие навыков и умений самостоятельно работать с аутентичными текстами на английском языке, коммуникативных навыков для поддержания профессиональных контактов, а также развитие навыков письма.

Пособие состоит из 9 разделов, каждый из которых содержит тематические тексты и задания для их полного и точного понимания, а также задания по развитию коммуникативных компетенций. Предтекстовые задания знакомят студентов с содержанием учебных текстов и способствуют усвоению и запоминанию специальных терминов по направлению обучения, устраняют трудности понимания прочитанного материала. Послетекстовые упражнения позволяют определить уровень усвоения изученного материала, способствуют развитию навыков устного и письменного перевода, монологической и диалогической речи. В пособие также включены творческие задания по подготовке презентаций и докладов.

Пособие содержит тексты для самостоятельной работы студентов, инструкции как составить краткий пересказ текста – *How To Write a Summary,* правила чтения математических выражений и формул – *Basic Arithmetic Expressions, Formulas, Equations and Rules for Reading Them in English* и словарь терминов по математике *English-Russian Dictionary of Mathematical Terms.*

Пособие может быть рекомендовано к использованию для аудиторной и самостоятельной работы студентов.

Материалы пособия прошли апробацию в студенческих группах.

CONTENTS

Unit 1. MATHEMATICS AS A SCIENCE

"Mathematics is the queen of the sciences,
and number theory is the queen of mathematics."
– Carl Friedrich Gauss

Part 1
WHAT IS MATHEMATICS?

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. What is Mathematics from your point of view?
2. Is Mathematics a science?
3. How does Mathematics function in real life?

Task 2. Practise reading the following words.

№	Word	Transcription
1	mathematics	['mæθɪ'mætɪks]
2	arithmetic	['ærɪθ'metɪk]
3	geometry	[ʤɪ'ɔmɪtrɪ]
4	pythagorean	[paɪ'θægə'rɪən]
5	pure	[pjuə]
6	applied	[ə'plaɪd]
7	processes	['prəusesiz]
8	generalization	['ʤen(ə)rəlaɪ'zeɪʃ(ə)n]
9	theorem	['θɪərəm]
10	axiom	['æksɪəm]

Task 3. Study and remember the following words.

№	Word	Transcription	Translation
1	cognition	[kɔg'nɪʃ(ə)n]	познание
2	deduce	[dɪ'dju:s]	выводить (заключение, следствие, формулу)
3	encompass	[ɪn'kʌmpəs]	заключать
4	symbolic	[sɪm'bɔlɪk]	символический, символьный
5	deduction	[dɪ'dʌkʃ(ə)n]	вывод, доказательство
6	inference	['ɪnf(ə)rəns]	вывод, заключение
7	relation	[rɪ'leɪʃ(ə)n]	связь, отношение
8	postulate	['pɔstjuleɪt]	постулат
9	quantity	['kwɔntɪtɪ]	количество, величина
10	magnitude	['mægnɪtju:d]	абсолютная величина
11	property	['prɔpətɪ]	показатель, свойство
12	concise	[kən'saɪs]	краткий, сжатый, сокращенный

13	latter	['lætə]	последний из двух
14	counting	['kauntıŋ]	счет, вычисление, подсчет
15	concrete	['kɔŋkri:t]	определенный

Task 4. Read and translate Text A using a dictionary if necessary.

Text A

WHAT IS MATHEMATICS?

Mathematics is the product of many lands and it belongs to the whole of mankind. We know how necessary it was even for the early people to learn to count and to become familiar with mathematical ideas, processes and facts. In the course of time, counting led to *arithmetic* and measuring led to *geometry*. *Arithmetic* is the study of number, while *geometry* is the study of shape, size and position. These two subjects are regarded as the foundations of mathematics.

It is impossible to give a concise definition of mathematics as it is a multifield subject. Mathematics in the broad sense of the word is a peculiar form of the general process of human cognition of the real world. It deals with the space forms and quantity relations abstracted from the physical world.

Contemporary mathematics is a mixture of much that is very old and still important (e.g., counting, the Pythagorean theorem) with new concepts such as sets, axiomatics, structure. The totality of all abstract mathematical sciences is called *Pure Mathematics.* The totality of all concrete interpretations is called *Applied Mathematics.* Together they constitute *Mathematics* as a science.

One of the modern definitions of mathematics runs as follows: mathematics is the study of relationships among quantities, magnitudes, and properties of logical operations by which unknown quantities, magnitudes and properties may be deduced. In the past, mathematics was regarded as the science of *quantity*, whether of magnitudes, as in geometry, or of numbers, as in arithmetic, or the generalization of these two fields, as in algebra. Toward the middle of the 19th century, however, mathematics came to be regarded increasingly as the science of *relations*, or as the science that draws necessary conclusions. The latter view encompasses mathematical or symbolic logic, the science of using symbols to provide an exact theory of logical deduction and inference based on definitions, axioms, postulates, and rules for combining and transforming positive elements into more complex relations and theorems.

Adopted from *Пушкина Е.Н. English for Mathematicians and Information Technologies Learners = Английский для студентов, изучающих математику и информационные технологии: учебно-методическое пособие [Электронный ресурс] / Е.Н. Пушкина. – Нижний Новгород, ННГУ, 2019. – 88 с.*

AFTER TEXT TASKS

Task 5. Answer the following questions.

1. What two subjects did counting lead to?
2. What is mathematics in the broad sense of the word?
3. What does it deal with?
4. What is *Pure Mathematics?*

5. How is *Applied Mathematics* defined?
6. What is one of the modern definitions of mathematics?
7. How was mathematics interpreted in the past?
8. What is it considered to be now?

Task 6. Give Russian equivalents to these word combinations.

1. foundations
2. concise
3. the study of
4. measuring
5. to deal with
6. applied
7. pure
8. contemporary
9. concept
10. mixture

Task 7. Find the English equivalents to the following word combinations.

1. измерение (действие)
2. изучать
3. преобразовывать
4. множества
5. рассматривать
6. современный
7. количество
8. логический вывод
9. чистая математика
10. прикладная математика

Task 8. Match the terms with their translation.

№	**Term**		**Translation**
1	mankind	**a**	подсчет, вычисление
2	to become familiar with	**b**	привести к
3	counting	**c**	человеческое познание
4	to belong to	**d**	познакомиться с
5	human cognition	**e**	совокупность
6	to lead to	**f**	составлять
7	in the broad sense	**g**	принадлежать
8	totality	**h**	число
9	to constitute	**i**	человечество
10	number	**j**	в широком смысле

Task 9. Mark true (T) or false (F) sentences.

1. Mathematics is the product of many lands.
2. Arithmetic and calculus are regarded as the foundations of mathematics.
3. Geometry is the study of shape, size and position.
4. The totality of all abstract mathematical sciences is called Applied Mathematics.
5. The totality of all concrete interpretations is called Pure Mathematics.
6. Mathematics is the study of relationships among quantities, magnitudes, and properties of logical operations by which unknown quantities, magnitudes and properties may be deduced.
7. In the past, mathematics was regarded as the science of quantity, whether of magnitudes, as in geometry.
8. Toward the middle of the 18^{th} century mathematics came to be regarded as the science of relations.
9. The theory of logical deduction and inference is based on definitions, axioms, postulates, and rules for combining and transforming positive elements into more complex relations and theorems.
10. Contemporary mathematics is a mixture of much that is very old and still important with new concepts such as sets, axiomatics, structure.

Task 10. Insert the necessary word from the chart into the gaps.

science, magnitudes, geometry, number, count, multifield, mankind, measuring, quantity relations, conclusions

1. Mathematics belongs to the whole of … .
2. It was necessary even for the early people to learn to … .
3. In the course of time, … led to geometry.
4. Arithmetic is the study of … .
5. Mathematics is a … subject.
6. Mathematics deals with the space forms and … abstracted from the physical world.
7. Applied mathematics and pure mathematics constitute mathematics as a … .
8. Mathematics is the study of relationships among quantities, … , and properties of logical operations.
9. Mathematics was regarded as the science of quantity, whether of magnitudes, as in … .
10. Mathematics came to be regarded as the science that draws necessary … .

Task 11. Match the beginnings and the endings of the given sentences.

№	Beginnings		Endings
1	It was necessary for the early people to become familiar	**a**	with new concepts such as sets, axiomatics, structure.
2	In the course of time, counting led	**b**	mathematical or symbolic logic.
3	Geometry is the study	**c**	is called Pure Mathematics.
4	Mathematics is a peculiar form of the general process	**d**	with mathematical ideas, processes and facts.

5	Contemporary mathematics is a mixture of much that is very old and still important (e.g., counting, the Pythagorean theorem)	**e**	by which unknown quantities, magnitudes and properties may be deduced.
6	The totality of all abstract mathematical sciences	**f**	of shape, size and position.
7	The totality of all concrete interpretations	**g**	of human cognition of the real world.
8	Mathematics is the study of relationships among quantities, magnitudes, and properties of logical operations	**h**	on definitions, axioms, postulates, and rules for combining and transforming positive elements into more complex relations and theorems.
9	The science that draws necessary conclusions encompasses	**i**	is called Applied Mathematics.
10	The theory of logical deduction and inference is based	**j**	to arithmetic.

Task 12. In pairs, take turns to interview your partner about understanding what mathematics is. What questions do you think are the most relevant?

Task 13. Retell Text A.

Task 14. Write a short essay on the suggested topics. The volume of the essay is 200-250 words. Suggest some other relevant essay topics.

1. Mathematics is the product of many lands.
2. Mathematics as a science.
3. Mathematics: past, present, future.

Task 15. Read the words and try to remember the pronunciation.

1. primitive man ['prɪmɪtɪv 'mæn] – первобытный человек
2. count [kaunt] – считать
3. possessions [pə'zeʃ(ə)nz] – собственность
4. express a number [ɪk'spres ə 'nʌmbə] – обозначить число
5. inseparable part [ɪn'sep(ə)rəb(ə)l] – неотделимая часть
6. everyday life ['evrɪdeɪ 'laɪf] – повседневная жизнь
7. decimal system ['desɪm(ə)l 'sɪstɪm] – десятичная система
8. value ['vælju:] – значение
9. digit ['dɪʤɪt] – цифра
10. ten times as great ['ten ˈtaɪmz æs ˈgreɪt] – в десять раз больше
11. Hindu-Arabic ['hɪndu: 'ærəbɪk] – индоарабская
12. number system ['nʌmbə 'sɪstɪm]– числовая система
13. suffice for [sə'faɪs ˈfɔ:] – быть достаточным для
14. proper place ['prɔpə 'pleɪs] – подобающее место

15. large numbers ['lɑ:ʤ 'nʌmbəz] – многозначные числа
16. separate ['sep(ə)rɪt] – отделять
17. unit ['ju:nɪt] – единица
18. comma ['kɔmə] – запятая
19. billion ['bɪljən] – миллиард

Task 16. Read Text B. Translate it from English into Russian.

Text B

MATHS IN REAL LIFE

Many thousands of years ago this was a world without numbers. Nobody missed them. Primitive men knew only ten number-sounds. The reason was that they counted in the way a small child counts today, one by one, making use of their fingers. The needs and possessions of primitive men were few: they required no large numbers. When they wanted to express a number greater than ten they simply combined certain of the ten sounds connected with their fingers. Thus, if they wanted to express “one more than ten” they said “one-ten” and so on.

Nowadays Maths has become an inseparable part of our lives and whether we work in an office or spend most of our time at home, each one of us uses Maths as a part of our everyday life. No matter where we are as well as whatever we are doing, Maths is always there whether you notice it or not.

The system of numbers we use, called Arabic system, is a decimal system: that is, it is based on tens. In this system the value a digit represents is determined by the place it has in the number; if a digit is moved to the left one place, the value it represents becomes ten times as great.

Our present-day number-symbols are Hindu characters. It is important to notice that no symbols for zero occur in any of these early Hindu number system. They contain symbols for numbers like twenty, forty, and so on. A symbol for zero had been invented in India. The invention of this symbol for zero was very important, because its use enabled the nine Hindu symbols 1, 2, 3, 4, 5, 6, 7, 8 and 9 to suffice for the representation of any number, no matter how great. The work of a zero is to keep the other nine symbols in their proper place.

To make it easier to read large numbers, we separate the figures of the numbers by commas into groups of three, counting from right to left. Each group is called a period and has its own name.

682,000,000,000	847,000,000	136,000	592
Billions	Millions	Thousands	Ones / Units
4 periods	3 periods	2 periods	1 period

These numbers are read: six hundred eighty-two billion, eight hundred forty-seven million, one hundred thirty-six thousand, five hundred and ninety-two.

Adopted from *Назарова Н.А. Профессиональный английский: математика и физика = Professional English in Use: Mathematics and Physics: учебное пособие / Н.А. Назарова, Е.В. Панасенко, О.М. Толстых. – Омск: Изд-во ОмГПУ, 2018. – 124 с.*

AFTER TEXT TASK

Task 17. Answer the following questions on Text B.

1. When did people begin to count?
2. What purposes did the primitive people use numbers for?
3. Why are mathematics and numbers important?
4. What spheres of our life do we use Maths in?
5. What numeration system do we use nowadays?
6. How many digits do we use in our Hindu-Arabic system of numeration?
7. Why do we separate figures of the numbers by commas?
8. How is each group of three figures called?
9. How is the system of numbers we use called?
10. How many digits does a period of a number contain?
11. What is the function of a zero?

Part 2
MAIN BRANCHES OF MATHEMATICS

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. What are the main branches of mathematics?
2. What does each branch of mathematics study?
3. Why is it important to be aware of the specific branches of mathematics?

Task 2. Practise reading the following words.

№	Word	Transcription
1	foundations	[faun'deɪʃ(ə)nz]
2	algebra	['ælʤɪbrə]
3	probability	['prɔbə'bɪlɪtɪ]
4	statistics	[stə'tɪstɪks]
5	trigonometry	[trɪgəˈnɒmɪtrɪ]
6	calculus	[ˈkælkjʊləs]
7	fundamental	['fʌndə'mentl]
8	theory	['θɪ(ə)rɪ]
9	topology	[tə'pɒlədʒi, tə'pɑːl-]
10	concept	['kɔnsept]
11	triangle	['traɪæŋg(ə)l]
12	addition	[əˈdɪʃn]
13	subtraction	[səbˈtrækʃn]
14	multiplication	[mʌltɪplɪˈkeɪʃn]
15	division	[dɪˈvɪʒən]

Task 3. Study and remember the following words.

№	Word	Transcription	Translation
1	interlinked	[ˌɪntə'lɪŋkt]	взаимосвязанный
2	overlapping	['əuvə'læpɪŋ]	перекрывающийся, частично совпадающий
3	pursue	[pə'sju:]	добиваться, стремиться
4	exponent	[ɪk'spəunənt]	экспонент, степень
5	manipulate	[mə'nɪpjuleɪt]	управлять, оперировать
6	equation	[ɪ'kweɪʒ(ə)n]	уравнение, равенство
7	rate of change	['reɪt ɔv 'ʧeɪnʤ]	скорость изменения
8	curve	[kɜ:v]	кривая
9	determine	[dɪ'tɜ:mɪn]	устанавливать, определить
10	indispensable	['ɪndɪ'spensəb(ə)l]	необходимый, обязательный
11	integer	['ɪntɪʤə]	целое число
12	stretching	['stretʃɪŋ]	растяжение
13	crumpling	['krʌmplɪŋ]	комкание, смятие
14	twisting	['twɪstɪŋ]	скручивание, верчение
15	bedding	['bedɪŋ]	наслоение

Task 4. Read and translate Text A using a dictionary if necessary.

Text A

MAIN BRANCHES OF MATHEMATICS

Mathematics is a complex area of study and comprises interlinked topics and overlapping concepts. An extensive analysis of the branches of mathematics helps students in organizing their concepts clearly and develop a strong foundation. Being aware of the specific branches of mathematics also guides students in deciding the branch they would like to pursue as a career.

Here are the main branches of mathematics:

1. Foundations
2. Arithmetic
3. Algebra
4. Geometry
5. Trigonometry
6. Calculus
7. Probability and Statistics
8. Number Theory
9. Topology
10. Applied mathematics

Arithmetic

This is one of the most basic branches of mathematics. Arithmetic deals with numbers and their applications in many ways. Addition, subtraction, multiplication, and division form its basic groundwork as they are used to solve a large number of questions and progress into more complex concepts like exponents, limits, and many other types of calculations. This is one of the most important branches because its fundamentals are used in everyday life for a variety of reasons from simple calculations to profit and loss computation.

Algebra

A broad field of mathematics, algebra deals with solving generic algebraic expressions and manipulating them to arrive at results. Unknown quantities denoted by alphabets that form a part of an equation are solved for and the value of the variable is determined. A fascinating branch of mathematics, it involves complicated solutions and formulas to derive answers to the problems posed.

Geometry

Dealing with the shape, sizes, and volumes of figures, geometry is a practical branch of mathematics that focuses on the study of polygons, shapes, and geometric objects in both two-dimensions and three-dimensions. Congruence of objects is studied at the same time focusing on their special properties and calculation of their area, volume, and perimeter. The importance of geometry lies in its actual usage while creating objects in practical life.

Trigonometry

Derived from Greek words “trigonon” meaning triangle and “metron” meaning “measure”, trigonometry focuses on studying angles and sides of triangles to measure the distance and length. Amongst the prominent branches of mathematics used in the world of technology and science to develop objects, trigonometry is a study of the correlation between the angles and sides of the triangle. It is all about different triangles and their properties!

Calculus

It is one of the advanced branches of mathematics and studies the rate of change. With the advent of calculus, a revolutionary change was brought about in the study of maths. Earlier maths could only work on static objects but with calculus, mathematical principles began to be applied to objects in motion. Used in a multitude of fields, the branch can be further categorized into the differential and integral calculus both starkly different from each other. **Differential calculus** deals with the rate of change of a variable and it is a means of finding tangents to curves. **Integral calculus** is concerned with the limiting values of differentials and is a means of determining length, volume, or area.

Probability and Statistics

The abstract branch of mathematics, probability and statistics use mathematical concepts to predict events that are likely to happen and organize, analyze, and interpret a collection of data. Amongst the relatively newer branches of mathematics, it has become indispensable because of its use in both natural and social sciences. The scope of this branch involves studying the laws and principles governing numerical data and random events. Presenting an interesting study, statistics, and probability is a branch full of surprises.

Number Theory

It is a branch of pure mathematics devoted primarily to the study of integers and integer-valued functions. Number theorists study prime numbers as well as the properties of mathematical objects made out of integers (for example, rational numbers) or defined as generalizations of the integers (for example, algebraic integers). The basic level of Number Theory includes introduction to properties of integers like addition, subtraction, multiplication, modulus and builds up to complex systems like cryptography, game theory and more.

Topology

Topology is a much recent addition into the branches of Mathematics list. It is concerned with the deformations in different geometrical shapes under stretching, crumpling, twisting and bedding. Deformations like cutting and tearing are not included in topologies. Its application can be observed in differentiable equations, dynamical systems, knot theory, and Riemann surfaces in complex analysis.

Applied Mathematics

Applied mathematics is the application of mathematical methods by different fields such as physics, engineering, medicine, biology, finance, business, computer science, and industry. Thus, applied mathematics is a combination of mathematical science and specialized knowledge. The term "applied mathematics" also describes the professional specialty in which mathematicians work on practical problems by formulating and studying mathematical models.

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AFTER TEXT TASKS

Task 5. Answer the following questions.

1. What are the specific branches of mathematics?
2. What does arithmetic deal with?
3. What does algebra involve?
4. Why is geometry called a practical branch of mathematics?
5. What Greek words did the word ‘trigonometry’ derive from?
6. What objects does trigonometry study?
7. Why did calculus bring a revolutionary change in the study of maths?
8. What do differential calculus and integral calculus deal with?
9. Why are probability and statistics called indispensable?
10. What is Number Theory devoted to?
11. What does the basic level of Number Theory include?
12. What is topology concerned with?
13. What does applied maths deal with?

Task 6. Look through the text to search for unfamiliar words and try to understand their general meanings.

Task 7. Write down the transcription and definitions of unfamiliar words, practise reading the words and try to remember them.

Task 8. Give Russian equivalents to these word combinations.

1. to pursue as a career

2. basic groundwork
3. simple calculations
4. congruence of objects
5. the rate of change
6. to find tangents to curves
7. limiting values of differentials
8. to predict events
9. integer-valued functions
10. differentiable equations
11. Riemann surface
12. specialized knowledge
13. stretching
14. twisting
15. professional specialty

Task 9. Find the English equivalents to the following words and word combinations.

1. множество причин
2. прийти к результатам
3. неизвестная величина
4. измерять расстояние и длину
5. в движении
6. резко отличаться
7. дифференциальное исчисление
8. интегральное исчисление
9. набор данных
10. случайные события
11. обобщения целых чисел
12. геометрические фигуры
13. теория узла
14. комкание
15. наслоение

Task 10. Match the terms with their definitions.

№	**Term**		**Defenition**
1	Arithmetic	**a**	deals with the rate of change of a variable and it is a means of finding tangents to curves.
2	Algebra	**b**	use mathematical concepts to predict events that are likely to happen and organize, analyze, and interpret a collection of data.
3	Geometry	**c**	is concerned with the deformations in different geometrical shapes under stretching, crumpling, twisting and bedding.
4	Differential	**d**	deals with numbers and their applications in many

	calculus		ways.
5	Integral calculus	**e**	is the application of mathematical methods by different fields such as physics, engineering, medicine, biology, finance, business, computer science, and industry.
6	Probability and statistics	**f**	is devoted primarily to the study of integers and integer-valued functions.
7	Number Theory	**g**	focuses on the study of polygons, shapes, and geometric objects in both two-dimensions and three-dimensions.
8	Topology	**h**	studies entirely abstract concepts.
9	Pure mathematics	**i**	deals with solving generic algebraic expressions and manipulating them to arrive at results.
10	Applied mathematics	**j**	is concerned with the limiting values of differentials and is a means of determining length, volume, or area.

Task 11. Mark true (T) or false (F) sentences.

1. The term "applied mathematics" also describes the professional specialty in which mathematicians work on practical problems by formulating and studying mathematical models.
2. The application of probability can be observed in differentiable equations, dynamical systems, knot theory, and Riemann surfaces in complex analysis.
3. Deformations like cutting and tearing are not included in topologies.
4. Statisticians study prime numbers as well as the properties of mathematical objects made out of integers.
5. The scope of probability and statistics involves studying the laws and principles governing numerical data and random events.
6. Calculus is one of the advanced branches of mathematics and studies the rate of change.
7. Differential calculus is concerned with the limiting values of differentials and is a means of determining length, volume, or area.
8. Algebra is a study of the correlation between the angles and sides of the triangle.
9. Dealing with the shape, sizes, and volumes of figures, trigonometry is a practical branch of mathematics that focuses on the study of polygons, shapes, and geometric objects in both two-dimensions and three-dimensions.
10. Addition, subtraction, multiplication, and division form the basic groundwork of arithmetic.

Task 12. Match the beginnings and the endings of the given sentences.

№	**Beginnings**		**Endings**
1	The basic level of Number Theory builds up to complex systems	**a**	and specialized knowledge.
2	Applied mathematics is a combination of mathematical	**b**	to be applied to objects in motion.

	science		
3	Topology is concerned with the deformations in different geometrical shapes	**c**	from simple calculations to profit and loss computation.
4	Earlier maths could only work	**d**	triangles and their properties.
5	Congruence of objects is studied at the same time focusing on their special properties and	**e**	while creating objects in practical life.
6	With calculus, mathematical principles began	**f**	like cryptography, game theory and more.
7	Trigonometry is all about different	**g**	calculation of their area, volume, and perimeter.
8	The fundamentals of arithmetic are used in everyday life for a variety of reasons	**h**	are solved for and the value of the variable is determined.
9	Unknown quantities denoted by alphabets that form a part of an equation	**i**	on static objects.
10	The importance of geometry lies in its actual usage	**j**	under stretching, crumpling, twisting and bedding.

Task 13. Write out key words from the text.

Task 14. Use the key words of the text to make up the outline of the text.

Task 15. Write out the main idea of the text. Be ready to speak about it.

Task 16. Give the summary of Text A.

Task 17. In pairs, take turns to interview your partner about different branches of mathematics. What questions do you think are the most relevant?

Task 18. Write a short essay on the suggested topics. Suggest some other relevant essay topics.

1. Arithmetic is one of the most basic branches of mathematics.
2. Geometry is a practical branch of mathematics.
3. Topology is a much recent addition into the branches of mathematics list.
4. Calculus is one of the advanced branches of mathematics.
5. Probability and statistics are the abstract branches of mathematics.

Task 19. Read the words and try to remember the pronunciation.

1. mental arithmetic [mentl 'ærɪθ'metɪk] – устный счет в уме
2. artificial intelligence ['ɑ:tɪ'fɪʃ(ə)l ɪn'telɪʤ(ə)ns] – искусственный интеллект
3. superficial part ['s(j)u:pə'fɪʃ(ə)l 'pɑ:t] – поверхностная часть

4. essence ['es(ə)ns] – сущность
5. set tasks ['set 'tɑ:sks] – ставить задачи
6. numerical sequences [nju:'merɪk(ə)l 'si:kwənsiz] – числовые последовательности
7. humanities [hju'mænətiz] – гуманитарные дисциплины
8. social science ['səuʃ(ə)l 'saɪəns] – обществознание
9. soroban ['sɔ:rəbɑ:n] – соробан, японские счеты
10. abacus ['æbəkəs] – абака, счетная доска
11. ancient people ['eɪnʃ(ə)nt 'pi:p(ə)l] – древние люди

Task 20. Read Text B. Translate it from Russian into English.

Text B

МАТЕМАТИКА – ЦАРИЦА НАУК

Зачем нужна устная математика и устный счет в уме? Этот вопрос мучает всех школьников и их родителей. Хитрость ответа заключается в том, что математика – это чистый интеллект, логические операции, установление закономерностей, причинно-следственных связей и систем. Действительно, развитие информационных технологий, искусственного интеллекта делает ненужными самостоятельные человеческие операции, но проблема в том, что без владения математического логического аппарата, невозможно в принципе развитие интеллекта. Да, любой человек может посчитать цену товара, его вес и объем. Но это видимая, поверхностная и формальная часть математического знания.

Сущность математики в том, что человек учится видеть различные варианты решения одной и той же задачи, умеет ставить перед собой задачи и находить на них ответы, ищет доказательства и аргументы. Без числовых последовательностей, закономерностей и способов решения детский ум не способен решать абстрактные и конкретные задачи. Это касается любой области знаний: физика, химия, биология и другие. Как бы это странно не звучало, такие гуманитарные дисциплины как история и русский язык – это теория логических систем, которые лучше всего усваиваются именно на примерах математики. Поэтому неудивительно, что ребенок, не овладевший устным счетом, математикой и быстрыми операциями в уме достаточно слабо развирается и в таких, казалось бы, не связанных с математикой дисциплиной, как русский язык, литература, обществознание.

Это хорошо понимали и люди на востоке, откуда пошли и теория чисел и таблицы сложения, вычитания, умножения и деления. Тогда же и были разработаны специальные системы устного счета под названием соробан или абакус, развивающие логику, память, внимание ребенка и интеллект в целом. Развитие интеллектуальных способностей человека приводит к тому, что он начинает хорошо ориентироваться и в других дисциплинах, там, где требуются эти же навыки. Фактически можно сказать, что математика – это основа человеческого интеллектуального потенциала или, как говорили древние – «царица наук».

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Unit 2. HISTORY OF MATHEMATICS

“Mathematics reveals its secrets only to those who approach it with pure love, for its own beauty.”
– Archimedes

Part 1
COUNTING IN THE EARLY AGES

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. When did the history of mathematics begin?
2. How did people count in the dim and distant past?
3. What are the greatest achievements of early ages?
4. Who are the prominent mathematicians of antiquity?

Task 2. Practise reading the following words.

№	Word	Transcription
1	Arabic	['ærəbɪk]
2	Arabs	['ærəbz]
3	Chinese	['ʧaɪ'ni:z]
4	abacus	['æbəkəs]
5	Columbus	[kə'lʌmbəs]
6	calculator	['kælkjuleɪtə]
7	finger	['fɪŋgə]
8	numeral	['nju:m(ə)rəl]
9	Florence	['flɔrəns]
10	digit	['dɪʤɪt]

Task 3. Study and remember the following words.

№	Word	Transcription	Translation
1	dim	[dɪm]	смутный, темный
2	savage	['sævɪʤ]	дикий
3	tribe	[traɪb]	племя
4	pebble	['peb(ə)l]	булыжник, камешек
5	toe	[təu]	палец ноги
6	apply	[ə'plaɪ]	относиться (to), распространяться
7	instead	[ɪn'sted]	взамен, вместо
8	merchant	['mɜ:ʧ(ə)nt]	купец, торговец
9	manuscript	['mænjuskrɪpt]	рукопись
10	bead	[bi:d]	шарик (со сквозным отверстием)
11	groove	[gru:v]	канавка, прорез
12	carry	['kærɪ]	нести, довозить

13	capture	['kæpʧə]	захват, овладение
14	sign	[saɪn]	знак
15	Calandri	[ka: 'la:ndri]	Каландри

Task 4. Read and translate Text A using a dictionary if necessary.

Text A

COUNTING IN THE EARLY AGES

Counting is the oldest of all processes. How did people count in the dim and distant past, especially when they spoke different languages? Suppose you wanted to buy a chicken from some poor savage tribe. You might point toward some chickens and then hold up one finger. Or, instead of this, you might put one pebble or one stick on the ground. At the same time, you might make a sound in your throat, something like *ung*, and the savages would understand that you wanted to buy one chicken.

But suppose you wanted to buy two chickens or three bananas, what would you do? It would not be hard to make a sign for the number *two*. You could show two fingers or point to two shoes, to two pebbles, or to two sticks.

For *three* you could use three fingers or three pebbles, or three sticks. You see that even though you and the savages could not talk to one another, you could easily make the numbers *one*, *two*, and *three* known. It is a curious fact that much of the story of the world begins right here.

You must have heard about the numerals, or number figures, called *digits*. The Latin word *digiti* means *fingers*. Because we have five fingers on each hand, people began, after many centuries, to count by fives. Later, they started counting by tens, using the fingers of both hands. Because we have ten toes as well as ten fingers, people counted fingers and toes together and used a number scale of twenty. In the English language, the sentence "The days of a man's life are three score years and ten" the word *score* means twenty (so, the life span of humans was considered to be seventy).

Number names were among the first words used when people began to speak. The numbers from one to ten sound alike in many languages. The name *digits* was first applied to the eight numerals from 2 to 9. Nowadays, however, the first ten numerals, beginning with 0, are usually called the digits. It took people thousands of years to learn to write numbers, and it took them a long time to begin using signs for the numbers; for example, to use the numeral 2 instead of the word *two.*

The numerals we use nowadays are known as Arabic. But they have never been used by the Arabs. They came to us through a book on arithmetic which was written in India about twelve hundred years ago and translated into Arabic soon afterward. By chance, this book was carried by merchants to Europe, and there it was translated from Arabic into Latin. This was hundreds of years before books were first printed in Europe, and this arithmetic book was known only in manuscript form.

When people began to use large numbers, they invented special devices to make computation easier. The Romans used a counting table, or abacus, in which units, fives, tens and so on were represented by beads which could be moved in grooves. They called these beads *calculi*, which is the plural of *calculus*, or pebble.

We see here the origin of our word *calculate*. In the Chinese abacus, the calculi slid along on rods. In Chinese, this kind of abacus is called a *suan* – pan; in Japanese it is known as the *soroban* and in the Russian language as the *s'choty.* The operations that could be rapidly done on the abacus were addition and subtraction. Division was rarely used in ancient times. On the abacus, it was often done by subtraction; that is, to find how many times 37 is contained in 74, we see that 74 – 37 = 37, and 37 – 37 = 0, so that 37 is contained twice in 74.

Our present method, often called *long division*, began to be used in the 15th century. It first appeared in print in Calandri's arithmetic, published in Florence, Italy, in 1491, a year before Columbus discovered America.

The first machines that could perform all the operations with numbers appeared in modern times and were called *calculators.* The simplest types of calculators could give results in addition and subtraction only. Others could list numbers, add, subtract, multiply and divide. Many types of these calculators were operated by electricity, and some were so small that they could be easily carried about by the hand.

The twentieth century was marked by two great developments. One of these was the capture of atomic energy. The other is a computer. It may be rightly called the Second Industrial Revolution.

Downloaded from *Lumen Learning Mathematics for the Liberal Arts. URL: https://courses.lumenlearning.com/math4liberalarts/chapter/early-counting-systems/*

AFTER TEXT TASKS

Task 5. Answer the following questions.

1. What is the text about?
2. What signs did people use instead of numerals?
3. What numbers sound alike in many languages?
4. What number names is the word digit applied to?
5. How long has it taken people to learn to use numbers?
6. What is a numeral?
7. What is the role of numerals in our life?
8. How did the first arithmetic book appear in Europe?
9. What numbers were the most important for people in the remote past?
10. What devices did they invent to make computation easier?
11. What operations were done on the abacus?
12. When did long division appear?
13. What were the first counting machines called?
14. Could they perform all basic operations of arithmetic?
15. What development was the next step in counting?

Task 6. Find synonyms for the following words in the text.

1. to make *calculation* easier
2. to *do* operations
3. to *show* one finger
4. the *etymology* of the word calculate

5. to be *quickly* done
6. to be *seldom* used
7. to be marked by two great *achievements*
8. first *printed* in Italy

Task 7. Supply antonyms for the following words.
Subtract, before, hard, unknown, begin, unlikely, multiply, small, addition, ancient times, first, simple, easy, past, rapidly, often, division.

Task 8. Match the terms with their translation.

№	**Term**		**Translation**
1	distant past	**a**	выполнять операции
2	digit	**b**	приспособление
3	abacus	**c**	складывать
4	device	**d**	делить
5	ancient times	**e**	счёты
6	add	**f**	далёкое прошлое
7	subtract	**g**	умножать
8	multiply	**h**	вычитать
9	divide	**i**	однозначное число
10	perform operations	**j**	древние времена

Task 9. Mark true (T) or false (F) sentences.
1. Counting is the oldest of all processes.
2. Suppose you wanted to buy a chicken. You might point toward some chickens and then hold up two fingers.
3. Latin word *digiti* means *toes*.
4. People counted fingers and toes together and used a number scale of twenty.
5. The name *digits* was first applied to the ten numerals from 0 to 9.
6. The numerals came to us through a book on arithmetic which was written in Florence about twelve hundred years
7. In abacus units, fives, tens and so on were represented by beads which could be moved in grooves.
8. The operations that could be rapidly done on the abacus were addition, subtraction and division.
9. Our present method, often called *long division*, began to be used in the 15th century.
10. The first machines that could perform all the operations with numbers appeared in modern times and were called *computers*.

Task 10. Insert the necessary word from the box into the gaps.

Latin, Calandri's, Arabic, Romans, calculators, alike, pebbles, twenty, abacus, developments

1. If you wanted to buy two chickens, you could point to two … .
2. The … word *digiti* means *fingers.*
3. People counted fingers and toes together and used a number scale of … .
4. The numbers from one to ten sound … in many languages.
5. The numerals we use nowadays are known as … .
6. The … used a counting table.
7. The operations that could be rapidly done on the … were addition and subtraction.
8. Our present method, often called *long division*, first appeared in print in … arithmetic.
9. The simplest types of … could give results in addition and subtraction only.
10. The twentieth century was marked by two great … .

Task 11. Match the beginnings and the endings of the given sentences.

№	**Beginnings**		**Endings**
1	The numerals came to us through a book on arithmetic which was written in India about twelve hundred years ago	**a**	which could be moved in grooves.
2	By chance, the book on arithmetic was carried	**b**	they have never been used by the Arabs.
3	The numerals we use nowadays are known as Arabic, but	**c**	the s'choty.
4	In Japanese abacus is known as	**d**	were operated by electricity.
5	In Chinese abacus is called	**e**	and translated into Arabic soon afterward.
6	In the Russian language abacus is known as	**f**	for example, to use the numeral 2 instead of the word two.
7	In abacus units, fives, tens and so on were represented by beads	**g**	by merchants to Europe.
8	It first appeared in print in Calandri's arithmetic, published in Florence, Italy, in 1491,	**h**	a suan – pan.
9	Many types of these calculators	**i**	the soroban.
10	It took people a long time to begin using signs for the numbers;	**j**	a year before Columbus discovered America.

Task 12. In pairs, take turns to interview your partner about understanding what mathematics is. What questions do you think are the most relevant?

Task 13. Retell Text A.

Task 14. Write a short essay on the suggested topics. The volume of the essay is 200-250 words. Suggest some other relevant essay topics.

1. Counting systems of early civilizations.

2. The greatest mathematicians of ancient times.
3. Counting is the oldest of all processes.

Task 15. Read the words and try to remember the pronunciation.
1. Archimedes [ˌɑːkɪ'miːdiːz] – Архимед
2. Syracuse ['s(a)ɪ(ə)rəkju:s] – Сиракузы
3. antiquity [æn'tɪkwɪtɪ] – древний мир
4. phenomenon [fɪ'nɔmɪnən] – явление
5. hydrostatics ['haɪdrə'stætɪks] – гидростатика
6. eureka [ju(ə)'ri:kə] – эврика! озарение
7. immerse [ɪ'mɜ:s] – погружать, окунать
8. genius ['ʤi:nɪəs] – гениальность
9. cylinder ['sɪlɪndə] – цилиндр
10. lever ['li:və] – рычаг
11. buoyancy ['bɔɪənsɪ] – плавучесть (погружённых тел)

Task 16. Read Text B. Translate it from English into Russian.

Text B

ARCHIMEDES

Archimedes was the greatest mathematician, physicist and engineer of antiquity. He was born in the Greek city of Syracuse on the island of Sicily about 287 B.C. and died in 212 B.C. Roman historians have related many stories about Archimedes. There is a story which says that once when Archimedes was taking a bath, he discovered a phenomenon which later became known in the theory of hydrostatics as Archimedes' principle. He was asked to determine the composition of the golden crown of the King of Syracuse, who thought that the goldsmith had mixed base metal with the gold. The story goes that when the idea how to solve this problem came to his mind, he became so excited that he ran along the streets naked shouting "Eureka, eureka!" ("I have found it!"). Comparing the weight of pure gold with that of the crown when it was immersed in water and when not immersed, he solved the problem.

Archimedes was obsessed with mathematics, forgetting about food and the bare necessities of life. His ideas were 2000 years ahead of his time. It was only in the 17th century that his works were developed by scientists.

There are several versions of the scientist's death. One of them runs as follows. When Syracuse was taken by the Romans, a soldier ordered Archimedes to go to the Roman general, who admired his genius. At that moment, Archimedes was absorbed in the solution of a problem. He refused to fulfill the order and was killed by the soldier.

Archimedes laid the foundations of mechanics and hydrostatics and made a lot of discoveries. He added new theorems to the geometry of the sphere and the cylinder and stated the principle of the lever. He also discovered the law of buoyancy.

Downloaded from *Archimedes of Syracuse. URL: https://www.math.tamu.edu/~don.allen/history/archimed/archimed.html*

AFTER TEXT TASK

Task 17. Answer the following questions on Text B.

1. When and where was Archimedes born?
2. How did he discover the famous principle known under his name in the theory of hydrostatics?
3. What was his emotional reaction to the solution of the problem?
4. What was Archimedes ordered to do when Syracuse was taken by the Romans?
5. Why did he refuse to fulfill the order?
6. What happened to him upon the refusal?
7. What were his contributions to science?

Part 2
HISTORY OF MATHEMATICS: 17th – 20th centuries

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. How did mathematics develop in the Middle Ages?
2. How does mathematics evolve in modern times?
3. What do you know about famous mathematicians and their contribution to science?

Task 2. Practise reading the names of mathematicians.

Nicolaus Copernicus [ˈnɪkələs kə'pɜːnɪkəs] – Николай Коперник
Johannes Kepler [jə'hænɪs ˈkeplə] – Иоганн Кеплер
Galileo [ˌgælɪ'leɪəʊ] – Галилей
Isaac Newton ['aɪzək 'nju:t(ə)n] – Исаак Ньютон
John Napier [ʤɔn 'neɪpiə] – Джон Напье
Justus Byrgius ['ʤʌstəs 'bɜːʤəs] – Юстас Бирджес
Gottfried Wilhelm Leibniz [ˈɡɒtfrid ˈwɪlˌhelm ˈliːbˌnɪz] – Го́тфрид Ви́льгельм Ле́йбниц
Isaac Barrow [ˈaɪzək ˈbærəʊ] – Исаак Барроу
Rene Descartes [rəˈneɪ deˈkɑːt] Рене Декарт
Pierre de Fermat [ˈpɪr ˈdeɪ fɛ:(r) ˈmɑː] – Пьер де Ферма
Joseph Louis Lagrange [ˈʤəʊzɪf ˈluɪs ˈlægreɪnʤ] – Жозеф Луи Лагранж
Leonard Euler [ˈlenəd ˈɔɪlər] – Леонард Эйлер
Carl Frederich Gauss [kɑːl fredˈrɪk gaʊs] – Карл Фридрих Гаусс
Augustin Louis Cauchy [ɔːgʌstɪn ˈluɪs ˈkɔːkɪ] – Августин Луи Коши
Karl Weierstrass ['kɑːl wɪˈstreɪs] – Карл Вейерштрасс
Jean Baptiste Fourier [ʤiːn bəptɪst ˈfjʊərɪeɪ] – Жан Баптист Фурье
Georg Cantor [ˈgeɪəg ˈkæntər] – Георг Кантор
Julius Dedekind [ˈʤuːlɪəs ˌdediːˈkɪnd] – Юлиус Дедекинд

Task 3. Study and remember the following words.

№	Word	Transcription	Translation
1	mathematician	[ˌmæθəmə'tɪʃn]	математик
2	integral	['ɪntɪgrəl]	интеграл
3	treatise	['tri:tɪs]	трактат
4	conic	['kɔnɪk]	коническое сечение
5	logarithm	['lɒgərɪðəm]	логарифм
6	root	[ru:t]	корень
7	differential	['dɪfə'renʃ(ə)l]	дифференциальный
8	priority	[praɪ'ɔrɪtɪ]	приоритет
9	vitality	[vaɪ'tælɪtɪ]	жизненность, жизнеспособность
10	celestial	[sɪ'lestɪəl]	небесный
11	contribute	[kən'trɪbju:t]	вносить вклад
12	series	['sɪ(ə)ri:z]	ряд
13	infinite	['ɪnfɪnɪt]	бесконечный
14	rigorous	['rɪgərəs]	строгий
15	succeed	[sək'si:d]	преуспевать

Task 4. Read and translate Text A using a dictionary if necessary.

Text A
MATHEMATICS DEVELOPMENT

17th Century Mathematic

The scientific revolution of the 17th century spurred advances in mathematics as well. The founders of modern science – **Nicolaus Copernicus**, **Johannes Kepler**, **Galileo**, and **Isaac Newton** – studied the natural world as mathematicians, and they looked for its mathematical laws. Over time, mathematics grew more and more abstract as mathematicians sought to establish the foundations of their fields in logic.

The 17th century opened with the discovery of *logarithms* by the Scottish mathematician **John Napier** and the Swiss mathematician **Justus Byrgius**. Logarithms enabled mathematicians to extract the roots of numbers and simplified many calculations by basing them on addition and subtraction rather than on multiplication and division.

Napier, who was interested in simplification, studied the systems of the Indian and Islamic worlds and spent years producing the tables of logarithms that he published in 1614. **Kepler**'s enthusiasm for the tables ensured their rapid spread.

The 17th century saw the greatest advances in mathematics since the time of ancient Greece. The major invention of the century was *calculus.* Although two great thinkers – Sir Isaac Newton of England and **Gottfried Wilhelm Leibniz** of Germany – have received credit for the invention, they built on the work of others. As Newton noted, “If I have seen further, it is by standing on the shoulders of giants.” Major advances were also made in numerical calculation and geometry.

Gottfried Leibniz was born (1st July, 1646) and lived most of his life in Germany. His greatest achievement was the invention of *integral and differential calculus*, the system of notation which is still in use today. In England, Isaac Newton claimed the distinction and accused Leibniz of plagiarism, that is stealing somebody else's ideas but stating that they are original. Modern-day historians, however, regard Leibniz as having arrived at his conclusions independently of Newton. They point out that there are important differences in the writings of both men.

Differential calculus came out of problems of finding tangents to curves, and an account of the method is published in **Isaac Barrow**'s "*Lectiones geometricae*" (1670). Newton had discovered the method (1665-66) and suggested that Barrow include it in his book.

Leibniz had also discovered the method by 1676, publishing it in 1684. Newton did not publish his results until 1687. A bitter dispute arose over the priority for the discovery. In fact, it is now known that the two made their discoveries independently and that Newton had made it ten years before Leibniz, although Leibniz published first. The modern notation of *dy/dx* and the elongated *s* for integration are due to Leibniz.

The most important development in geometry during the 17th century was the discovery of *analytic geometry* by **Rene Descartes** and **Pierre de Fermat**, working in dependently in France. Analytic geometry makes it possible to study geometric figures using algebraic equations. By using algebra, Descartes managed to overcome the limitations of Euclidean geometry. That resulted in the reversal of the historical roles of geometry and algebra.

The French mathematician **Joseph Louis Lagrange** observed in the 18th century, "As long as algebra and geometry proceeded along separate paths, their advance was slow and their applications limited. But when these sciences joined company, they drew from each other fresh vitality and thenceforward marched on at a rapid pace toward perfection."

Descartes' publications provided the basis for Newton's mathematical work later in the century. Pierre de Fermat, however, regarded his own work on what became known as analytic geometry as a reformulation of Appollonius's treatise on conic sections. That treatise had provided the basic work on the geometry of curves from ancient times until Descartes.

18th – 19th Century Mathematics

During the 18th century, calculus became the cornerstone of mathematical analysis on the European continent. Mathematicians applied the discovery to a variety of problems in physics, astronomy, and engineering. In the course of doing so, they also created new areas of mathematics.

In France, Joseph Louis Lagrange made substantial contributions in all fields of pure mathematics, including differential equations, the calculus of variations, probability theory, and the theory of equations. In addition, Lagrange put his mathematical skills to work in the solution of practical problems in mechanics and astronomy.

The greatest mathematician of the 18th century, **Leonard Euler** of Switzerland, wrote works that covered the entire fields of pure and applied mathematics. He wrote major works on mechanics that preceded Lagrange's work. He won a number of prizes for his work on the orbits of comets and planets, the field known as celestial mechanics. But Euler is best known for his works in pure mathematics. In one of his works, *Introduction to the Analysis of Infinites*, published in 1748, he approached calculus in terms of functions rather than the geometry of curves. Other works by Euler contributed to number theory and differential geometry (the application of differential calculus to the study of the properties of curves and curved spaces).

Mathematicians succeeded in firming the foundations of analysis and discovered the existence of additional geometries and algebras and more than one kind of infinity.

The 19th century began with the German mathematician **Carl Frederich Gauss**. He ranks as one of the greatest mathematicians of the world. His book *Inquiries into Arithmetic* published in 1801 marks the beginning of modern era in number theory.

Gauss called mathematics *the queen of sciences* and number theory *the queen of mathematics.* Almost from the introduction of calculus, efforts had been made to supply a rigorous foundation for it. Every mathematician made some effort to produce a logical justification for calculus and failed. Although calculus clearly worked in solving problems, mathematicians lacked rigorous proof that explained why it worked. Finally, in 1821, the French mathematician **Augustin Louis Cauchy** established a rigorous foundation for calculus with his theory of limits, a purely arithmetic theory.

Later, mathematicians found Cauchy's formulation still too vague because it did not provide a logical definition of *real number*. The necessary precision for calculus and mathematical analysis was attained in the 1850s by the German mathematician **Karl T. W. Weierstrass** and his followers.

Another important advance in analysis came from the French mathematician **Jean Baptiste Fourier**, who studied infinite series in which the terms are trigonometric functions. Known today as Fourier series, they are still powerful tools in pure and applied mathematics.

The investigation of Fourier series led another German mathematician, **Georg Cantor**, to the study of infinite sets and to the arithmetic of infinite numbers. Georg Cantor began his mathematical investigations in number theory and went on to create set theory. In the course of his early studies of Fourier series, he developed a theory of irrational numbers. Cantor and another German mathematician, **Julius W. R. Dedekind**, defined the irrational numbers and established their properties. These explanations hastened the abandonment of many 19th century mathematical principles. When Cantor introduced his theory of sets, it was attacked as a disease from which mathematics would soon recover. However, it now forms part of the foundations of mathematics. The application of set theory greatly advanced mathematics in the 20th century.

Adopted from *Пушкина Е.Н. English for Mathematicians and Information Technologies Learners = Английский для студентов, изучающих математику и информационные технологии: учебно-методическое пособие [Электронный ресурс] / Е.Н. Пушкина. – Нижний Новгород, ННГУ, 2019. – 88 с.*

AFTER TEXT TASKS

Task 5. Answer the following questions.

1. What scholars are considered to be the founders of modern science?
2. Why did mathematics grow more and more abstract?
3. Who were logarithms discovered by?
4. What did logarithms enable mathematicians to do?
5. What was the major invention of the 17th century?
6. What is the essence of analytic geometry?
7. Why did a dispute arise between Leibniz and Newton?
8. What enabled Descartes to overcome the limitations of Euclidean geometry?
9. Whose publications provided the basis for Newton's mathematical work later in the century?
10. What did the discovery of calculus lead to?
11. What was Lagrange's contribution to pure and applied mathematics?
12. What did Euler's works contribute to?
13. What is the essence of differential geometry?
14. What event marked the beginning of modern era in number theory?
15. When was a rigorous foundation for calculus finally supplied?
16. What is the theoretical and practical value of Fourier series?
17. What was Georg Cantor's contribution to mathematical studies?
18. Who were irrational numbers investigated and defined by?
19. What was the first reaction to Cantor's set theory?
20. Was the attitude to the discovery later changed?

Task 6. Look through the text to search for unfamiliar words and try to understand their general meanings.

Task 7. Write down the transcription and definitions of unfamiliar words, practise reading the words and try to remember them.

Task 8. Give Russian equivalents to these word combinations.

1. cornerstone
2. substantial
3. major works
4. to rank as
5. to lack smth.
6. vague
7. precision
8. to attain
9. abandonment
10. to advance
11. simplification
12. to approach

13. to elongate
14. pace
15. tangents to curves

Task 9. Find the English equivalents to the following words and word combinations.

1. первенство
2. сделать открытие
3. извлекать корни
4. упростить
5. плагиат
6. опубликовать
7. интегральные и дифференциальные исчисления
8. система обозначений
9. претендовать (на что-л.)
10. совершенство
11. искривленное пространство
12. теория чисел
13. достичь
14. ускорять
15. несмотря на то, что

Task 10. Match the mathematicians with their contributions to science.

№	Mathematician		Contribution
1	Georg Cantor	**a**	infinite series in which the terms are trigonometric functions
2	Julius W. R. Dedekind	**b**	analytic geometry
3	Augustin Louis Cauchy	**c**	pure mathematics, including differential equations, the calculus of variations, probability theory, and the theory of equations
4	Jean Baptiste Fourier	**d**	a theory of irrational numbers
5	Augustin Louis Cauchy	**e**	tables of logarithms
6	Joseph Louis Lagrange	**f**	integral and differential calculus
7	Rene Descartes and Pierre de Fermat	**g**	irrational numbers and their properties
8	Gottfried Wilhelm Leibniz	**h**	theory of limits, a purely arithmetic theory
9	John Napier	**i**	laws of motion
10	Isaac Newton	**j**	foundation for calculus with his theory of limits, a purely arithmetic theory

Task 11. Mark true (T) or false (F) sentences.

1. The 17^{th} century opened with the discovery of logarithms by the Swiss mathematician John Napier and the Scottish mathematician Justus Byrgius.
2. Kepler's enthusiasm for the tables of logarithms ensured their rapid spread.
3. Gottfried Wilhelm Leibniz of Germany noted, "If I have seen further, it is by standing on the shoulders of giants."
4. The greatest achievement of Carl Frederich Gauss was the invention of integral and differential calculus.
5. Leonard Euler of Switzerland wrote works that covered the entire fields of pure and applied mathematics.
6. In the work "Introduction to the Analysis of Infinites" Euler approached calculus in terms of functions rather than the geometry of curves.
7. During the 18^{th} century, calculus became the cornerstone of mathematical analysis on the European continent.
8. The 19^{th} century began with the mathematician Jean Baptiste Fourier. His book "Inquiries into Arithmetic" marks the beginning of modern era in number theory.
9. The necessary precision for calculus and mathematical analysis was attained in the 1850s by the German mathematician Karl T. W. Weierstrass.
10. The application of set theory greatly advanced mathematics in the 20^{th} century.

Task 12. Complete the sentences below with the words and phrases from the box.

a. Rene Descartes and Pierre de Fermat b. the discovery of calculus c. Kepler d. preceded Lagrange's work e. Newton and Leibniz f. the scientific revolution of the 17^{th} century g. the tables of logarithms h. physics, astronomy, and engineering	i. celestial mechanics j. Fourier series k. Karl T. W. Weierstrass l. mechanics and astronomy m. number theory and differential geometry n. Carl Frederich Gauss o. Cantor and Dedekind

1. The Scottish mathematician Napier spent years producing … .
2. The rapid spread of the tables of logarithms was ensured by … .
3. The development of analytic geometry was beneficial for both … .
4. The invention of calculus is connected with the names of … .
5. A bitter dispute arose over the priority for … .
6. Advances in mathematics were facilitated by … .
7. Euler's major works on mechanics … .
8. Mathematicians applied the discovery of calculus to … .
9. Lagrange managed to solve some practical problems in … .
10. Euler's works contributed to … .
11. Euler won a number of prizes for his work on … .
12. Mathematics was called the queen of sciences by … .
13. Cantor's study of infinite sets became possible due to the study of … .

14. The properties of irrational numbers were established by … .
15. A precise foundation for calculus was supplied by … .

Task 13. Write out key words from the text.

Task 14. Use the key words of the text to make up the outline of the text.

Task 15. Write out the main idea of the text. Be ready to speak about it.

Task 16. Give the summary of Text A.

Task 17. In pairs, take turns to interview your partner about different branches of mathematics. What questions do you think are the most relevant?

Task 18. Write a short essay on the suggested topics. Suggest some other relevant essay topics.
1. Advances in mathematics in the 17th century.
2. The cornerstone of mathematical analysis in the 18th century.
3. The queen of mathematics in the 19th century.
4. Prominence of mathematics in the 20th century.
5. Development of mathematics in the 21st century.

Task 19. Read the words and try to remember the pronunciation.
1. quantum ['kwɔntəm] – квантовый
2. chaos ['keɪɔs] – хаос, неупорядоченность
3. topology [tə'pɒlədʒi] – топология
4. fertile ['fɜ:taɪl] – зд. благодатный
5. Princeton [ˈprɪnstən] – Принстон
6. Chicago [ʃɪ'kɑ:gəu] – Чикаго
7. Cambridge ['keɪmbrɪʤ] – Кэмбридж
8. Bertrand Russel ['bɜ:trənd 'rʌs(ə)l] – Бертранд Рассел
9. premise ['premɪs] – предпосылка
10. Hermann Weyl [ˊhɜ:mən ˊweil] – Герман Вэйль
11. Emmy Noether [ˊemi ˈnɜːtə] –Эмми Нётер
12. econometrics [ɪ'kɔnə'metrɪks] – эконометрика
13. maximize ['mæksɪmaɪz] – предельно увеличить
14. Von Neumann [vɒn ˈnɔɪmən] – фон Нойман
15. series ['sɪ(ə)ri:z] – ряд
16. supply [sə'plaɪ] – зд. разработать
17. profiled ['prəufaɪld] – изображённый (в фильме)
18. the Nobel Prize [ðə nəʊˈbel ˈpraɪz] – Нобелевская премия

Task 20. Read Text B. Translate it from English into Russian.

Text B

20TH CENTURY MATHEMATICS

During the 20th century, mathematics became more solidly grounded in logic and advanced the development of symbolic logic. Philosophy was not the only field to progress with the help of mathematics. Physics, too, benefited from the contributions of mathematicians to relativity theory and quantum theory. Indeed, mathematics achieved broader applications than ever before, as new fields developed within mathematics (*computational mathematics, game theory, and chaos theory*), and other branches of knowledge, including economics and physics, achieved firmer grounding through the application of mathematics. Even the most abstract mathematics seemed to find application, and the boundaries between pure mathematics and applied mathematics grew ever fuzzier.

Until the 20th century, the centres of mathematics research in the West were all located in Europe. Although the University of Göttingen in Germany, the University of Cambridge in England, the French Academy of Sciences and the University of Paris, and the University of Moscow in Russia retained their importance, the United States rose in prominence and reputation for mathematical research, especially the departments of mathematics at Princeton University and the University of Chicago. In some ways, pure mathematics became more abstract in the 20th century, as it joined forces with the field of symbolic logic in philosophy. The scholars who bridged the fields of mathematics and philosophy early in the century were Alfred North Whiteland and Bertrand Russel, who worked together at Cambridge University.

They published their major work, *Principles of Mathematics*, in three volumes from 1910 to 1913. In it, they demonstrated the principles of mathematical knowledge and attempted to show that all of mathematics could be deduced from a few premises and definitions by the rules of formal logic. In the late 19th century, the German mathematician Gottlob Frege had provided the system of notation for mathematical logic and paved the way for the work of Russel and Whitehead.

Mathematical logic influenced the direction of 20th century mathematics, including the work of Hilbert. Speaking at the Second International Congress of Mathematicians in Paris in 1900, the German mathematician David Hilbert made a survey of 23 mathematical problems that he felt would guide research in mathematics in the coming century.

Since that time, many of the problems have been solved. When the news breaks that another *Hilbert problem* has been solved, mathematicians worldwide impatiently await further details. Hilbert contributed to most areas of mathematics, starting with his classic *Foundations of Geometry*, published in 1899. Hilbert's work created the field of functional analysis (*the analysis of functions as a group*), a field that occupied many mathematicians during the 20th century. He also contributed to mathematical physics.

From 1915 on, he fought to have Emmy Noether, a noted German mathematician, hired at Göttingen. When the university refused to hire her because of objections to the presence of a woman in the faculty senate, Hilbert countered that the senate was not the changing room for a swimming pool. Noether later made major contributions to ring theory in algebra and wrote a standard text on abstract algebra. Several revolutionary theories, including relativity and quantum theory, challenged existing assumptions in physics in the early 20th century. The work of a number of mathematicians contributed to these theories.

The Russian mathematician Hermann Minkowski contributed to relativity the notion of the space-time continuum, with time as a fourth dimension. Hermann Weyl, a student of Hilbert's, investigated the geometry of relativity and applied group theory to quantum mechanics. Weyl's investigations helped advance the field of topology. Early in the century, Hilbert quipped, "Physics is getting too difficult for physicists."

Mathematics formed an alliance with economics in the 20th century as the tools of mathematical analysis, algebra, probability, and statistics illuminated economic theories. A specialty called *econometrics* links enormous numbers of equations to form mathematical models for use as forecasting tools.

Game theory began in mathematics, but had immediate applications in economics and military strategy. This branch of mathematics deals with situations in which some sort of decision must be made to maximize a profit – that is, to win. Its theoretical foundations were supplied by von Neumann in a series of papers written during the 1930s and 1940s. Von Neumann and the economist Oscar Morgenstern published the results of their investigations in *The Theory of Games and Economic Behaviour* (1944). John Nash, the Princeton mathematician profiled in the motion picture *A Beautiful Mind*, shared the 1994 Nobel Prize in economics for his work in game theory.

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AFTER TEXT TASK

Task 21. Answer the questions on Text B.

1. The development of what science did mathematics advance in the 20th century?
2. What two famous theories in physics did mathematics contribute to?
3. What new fields developed within mathematics?
4. Was there a great difference between pure and applied mathematics in the 20th century?
5. What role did algebra play in other areas of mathematics?
6. Why did topology become a fertile research field for mathematicians?
7. What universities became centers of mathematical research in the US?
8. What branch of mathematical science influenced the direction of 20th century mathematics?
9. What notion did the Russian mathematician Hermann Minkowski contribute to the theory of relativity?
10. How did mathematics advance economics in the 20th century?
11. What does game theory deal with?
12. Who were the theoretical foundations of game theory supplied by?

Unit 3. NUMERATION SYSTEMS AND NUMBERS

"Without mathematics, there's nothing you can do. Everything around you is mathematics. Everything around you is numbers".
Shakuntala Devi, a prominent Indian mathematician known as the 'Human Computer'

Part 1
TYPES OF NUMBERS

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. What kind of numbers do we use for counting in everyday life?
2. What numbers are called rational?
3. Can you give some examples of irrational numbers?

Task 2. Practise reading the following words.

№	Word	Transcription	Translation
1	natural	[ˈnætʃrəl]	натуральный
2	decimal	[ˈdesɪml]	десятичный
3	whole	[həʊl]	целый, весь
4	integer	[ˈɪntɪdʒə]	целый, целочисленный
5	to encompass	[ɪnˈkʌmpəs]	охватывать
6	to assume	[əˈsuːm]	предполагать
7	rarely	[ˈreəlɪ]	редко
8	π	[paɪ]	число π
9	$\sqrt{2}$	[ˈskweə ˈruːt əv ˈtuː]	квадратный корень из 2
10	completely	[kəmˈpliːtlɪ]	полностью, совершенно

Task 3. Study and remember the following words and expressions.

№	Word	Transcription	Translation
1	to be referred to	[rɪˈfɜːd]	упоминаться, называться
2	to denote	[dɪˈnəʊt]	обозначать
3	layer	[ˈleɪə]	слой, уровень
4	to include	[ɪnˈkluːd]	включать
5	instead of	[ɪnˈsted]	вместо
6	to be expressed	[ɪkˈsprest]	быть выраженным
7	to assume	[əˈsjuːm]	полагать, предполагать
8	fraction	[ˈfrækʃn]	дробь
9	definition	[ˌdefɪˈnɪʃn]	определение
10	to consider	[kənˈsɪdə]	рассматривать
11	imaginary number	[ɪˈmædʒɪnerɪ]	мнимое число
12	since	[sɪns]	здесь: поскольку

Task 4. Read and translate Text A using a dictionary if necessary.

Text A

TYPES OF NUMBERS

Numbers are an integral part of our everyday lives, right from the number of hours we sleep at night to the number of rounds we run around the racing track and much more. In math, numbers can be even and odd numbers, prime and composite numbers, decimals, fractions, rational and irrational numbers, natural numbers, integers, real numbers, rational numbers, irrational numbers, and whole numbers. In this chapter, we'll get an introduction to the different types of numbers and to all the concepts related to it.

The most basic type of classification of numbers are the **natural** numbers. Natural numbers include the symbols as 1,2,3,4,5… and so on. They are often referred to as 'counting numbers'. Natural numbers do not include 0 or any negative numbers, as well as any decimals. An easy way to remember it is to think of it like this: we all naturally count things starting from one, and go on to two, three, four, five, six, and so on. But we rarely count starting from zero. This is the most basic classification of numbers, it can be denoted as **N**.

The next layer of numbers are the whole numbers. Whole numbers can often be denoted using this symbol (**W**). Whole numbers include all natural numbers and it also includes zero (0). Instead of starting from 1, the whole numbers start from 0. The set of whole numbers includes natural numbers, and this means that any natural number is also considered a whole number. But not necessarily, the other way round, since zero is not a natural number.

The next classification of numbers is called **integers.** Integers can often be denoted using symbol **Z**. Integers include all the same numbers called as whole numbers, and they also include all the negatives of them. But integers do not include decimals or fractions of numbers.

The next classification of numbers is called **rational** numbers. It can be denoted using this symbol: **Q.** And again, the set of rational numbers encompasses all sets of numbers that we have mentioned so far, as well as decimals and fractions. However, the decimal numbers must be numbers that can be expressed as a fraction p/q, where p and q are integers, and q is not 0.

If we think of number x, and it is a natural number, can we assume that it is also a rational number? Definitely, and we can also assume that it is a whole number, since that is a bigger set. We can even assume that it is an integer since it is even a bigger set. And finally, we can also assume that it is a rational number, since rational numbers are a bigger set.

We can compare this situation to something like this. If we take some person in Tokyo, can we also assume that this person is in Japan? Obviously, we can! And would it be correct to assume that this person is also in Asia? Absolutely, since Tokyo is in Japan, and Japan is in Asia. And finally, would it be OK to assume that this person is on the Earth? Of course, because the Earth is even a bigger set than Asia.

But there is a completely different set of numbers that is not within any of these. This set of numbers cannot be expressed as a fraction. This set is completely separate from rational numbers altogether. We can call them **irrational** numbers and denote them using symbol **P**. An example of an irrational number would be $\boldsymbol{\pi}$. We know that $\boldsymbol{\pi}$ is a never-ending number that does not repeat with a constant decimal or any pattern fashion. And this is what makes it irrational. $\sqrt{2}$ also turns out to be an irrational number since it cannot be expressed as a fraction.

A **complex** number is a number that can be expressed in the form ($a + bi$) where a and b are real numbers, and i is a solution of the equation $x^2 = -1$. Because no real number satisfies this equation, i is called an **imaginary** number. **Complex numbers** have a real part and an imaginary part.

And lastly, the definition or **real** number is the last classification that we will consider. Real numbers are simply all of the rational and irrational numbers. Positive or negative, large or small, whole numbers or decimal numbers are all real numbers. They are called "Real Numbers" because they are not imaginary numbers. We denote them using symbol **R.**

AFTER TEXT TASKS

Task 5. Answer the following questions.

1. What is the oldest and the most basic type of classification of numbers?
2. How is the set of natural numbers denoted?
3. What is the difference between the classifications of natural and whole numbers?
4. Is the set W included into the set N?
5. How can the set of rational numbers be denoted?
6. What is the definition of a rational number?
7. If we take some person in Japan, can we also assume that this person is inTokyo?
8. Can irrational numbers be represented as the ratio of two integers?
9. What are the examples of irrational numbers?
10. How can real numbers be defined and how are they denoted?

Task 6. Give Russian equivalents to these word combinations.

1. they are often referred to as
2. as well as
3. it can be denoted as
4. the set of whole numbers
5. the other way round
6. since zero is not a natural number
7. something like this
8. cannot be expressed as a fraction
9. a never-ending number
10. this is what makes it irrational

Task 7. Find English equivalents to the following word combinations.

1. числа, используемые для счета
2. самая простая классификация чисел
3. в точности такой, как
4. множество целых чисел
5. но не обязательно бывает наоборот
6. часто обозначается как
7. включает в себя все множества чисел
8. может быть выражено как дробь
9. верно ли предположить, что…
10. именно это делает число π иррациональным

Task 8. Match the terms with their definitions.

№	Term		Definition
1	natural numbers	**a**	a number with no fractional part
2	fraction	**b**	designating a quantity less than zero
3	integer	**c**	any collection of objects (elements)
4	rational	**d**	positive integers used as counting numbers
5	negative	**e**	a numerical quantity that is not a whole number
6	set	**f**	a number that can be expressed as a quotient of two integers
7	imaginary number	**g**	a number that cannot can be expressed as a quotient of two integers
8	irrational	**h**	a number that, when squared, has a negative result

Task 9. Mark the sentences true (T) or false (F).

1. The most basic type of classification of numbers are the whole numbers.
2. Natural numbers do not include any decimals.
3. According to the text, natural numbers include 0.
4. Whole numbers include all natural numbers, zero and negative numbers.
5. Integers include some kinds of fractions.
6. Any rational number can be expressed as a fraction p/q, where p and q are integers.
7. The set of rational numbers is included into natural numbers.
8. If we take some person in Tokyo, we can also assume that this person is in Japan.
9. π is a never-ending irrational number.
10. Real numbers comprise all numbers, including irrational ones.

Task 10. Insert the necessary word from the chart into the gaps. Some words can be used more than once.

positive, decimals, encompasses, natural, zero, rational, integers, Japan, denoted negative, assume, fractions

1. The most basic type of classification of numbers are the (1) … numbers.

2. Natural numbers do not include (2) … or any (3) … numbers, as well as any decimals.
3. Whole numbers include all (4) … numbers and also include (5) … .
4. The set of integers does not include (6) … or (7) … of numbers.
5. The set of (8) … numbers can be denoted using this symbol: Q.
6. The set of rational numbers (9) …all sets of numbers that we have mentioned so far, as well as decimals and (10) … .
7. All (11) … numbers can be expressed as a fraction p/q, where p and q are (12) …, and q is not 0.
8. If we think of number x, and it is a natural number, can we assume that it is also a (13) … number?
9. If we take some person in Tokyo, would it be correct to (14) … that this person is in (15) …?
10. Irrational numbers are (16) … using symbol P.

Task 11. Match the beginnings and the endings of the sentences.

№	**Beginnings**		**Endings**
1	Natural numbers are often	**a**	and the negatives of them.
2	Whole numbers are denoted	**b**	using symbol Z.
3	Integers include whole numbers	**c**	we can assume they are in Japan.
4	All rational numbers can be	**d**	cannot be expressed as a fraction.
5	The set of integers can be denoted	**e**	referred to as 'counting numbers'.
6	If we take some person in Tokyo,	**f**	irrational numbers.
7	$\sqrt{2}$ and π are the examples of	**g**	using symbol W.
8	The set of irrational numbers	**h**	expressed as a fraction p/q.

Task 12. Look through the text again. Make up a plan for the text.

Task 13. Render Text A according to the plan using mathematical terms.

Task 14. Translate the sentences from Russian into English.
1. Натуральные числа включают символы 1,2,3,4, 5… и так далее.
2. Натуральные числа не включают 0 или отрицательные числа, а также десятичные дроби.
3. Это самая основная классификация чисел, ее можно обозначить как N.
4. Целые числа включают в себя все натуральные числа, а также нуль.
5. Любое натуральное число также считается целым числом, но не обязательно наоборот, поскольку нуль не является натуральным числом.
6. Множество целых чисел часто обозначается с помощью символа Z.
7. В множество целых чисел не включаются дроби.
8. Рациональные числа обозначаются символом Q.
9. Множество рациональных чисел включает в себя все множества чисел, которые мы упомянули до сих пор, а также дроби.

10. Все рациональные числа можно выразить как дробь p / q, где p и q - целые числа, а q не равно 0.
11. Если мы загадаем натуральное число x, то будет ли оно являться рациональным?
12. Эту ситуацию можно сравнить со следующей. Если взять какого-то человека в Токио, можно утверждать, что он находится в Японии, поскольку Токио - столица Японии.
13. Иррациональные числа нельзя выразить с помощью дроби.
14. Примерами иррациональных чисел являются $\sqrt{2}$ и π.
15. Действительные числа включают в себя все рациональные и иррациональные числа.

Task 15. Read the words and try to remember the pronunciation.
1. circumference [səˈkʌmfərəns] – окружность, длина окружности
2. ratio [ˈreɪʃɪəʊ] – соотношение, коэффициент, пропорция
3. Babylonian [ˌbæbɪˈləʊnɪən] – вавилонский
4. hexagon [ˈheksəg(ə)n] – шестиугольник
5. Rhind papyrus [raɪnd pəˈpaɪrəs] – папирус Ахмеса
6. Archimedes [ˈɑːkɪˈmiːdəs] – Архимед
7. accuracy [ˈækjʊrəsɪ] – точность
8. ensuing [ɪnˈsjuːɪŋ] – последующий
9. Srinivasa Ramanujan [ˈsrɪnɪˈvɑːsə ˈrəmənɪˈjɑːn] – Сриниваса Рамануджан
10. pendulum [ˈpendjʊləm] – маятник

Task 16. Read Text B. Translate it from English into Russian.

Text B

NUMBER π

π in mathematics, the ratio of the circumference of a circle to its diameter. The symbol π was devised by British mathematician William Jones in 1706 to represent the ratio and was later popularized by Swiss mathematician Leonhard Euler. Because π is irrational (not equal to the ratio of any two whole numbers), its digits do not repeat, and an approximation such as 3.14 or 22/7 is often used for everyday calculations.

To 39 decimal places, π is 3.141592653589793238462643383279502884197.

The Babylonians (*c.* 2000 BCE) used 3.125 to approximate π, a value they obtained by calculating the perimeter of a hexagon inscribed within a circle and assuming that the ratio of the hexagon's perimeter to the circle's circumference was 24/25. The Rhind papyrus (*c.* 1650 BCE) indicates that ancient Egyptians used a value of 256/81 or about 3.16045. Archimedes (*c.* 250 BCE) took a major step forward by devising a method to obtain π to any desired accuracy, given enough patience. By inscribing and circumscribing regular polygons about a circle to obtain upper and lower bounds, he obtained $223/71 < \pi < 22/7$, or an average value of about 3.1418. Archimedes also proved that the ratio of the area of a circle to the square of its radius is the same constant.

Over the ensuing centuries, Chinese, Indian, and Arab mathematicians extended the number of decimal places known through tedious calculations, rather than improvements on Archimedes' method. By the end of the 17th century, however, new methods of mathematical analysis in Europe provided improved ways of calculating π involving infinite series. For example, Sir Isaac Newton used his binomial theorem to calculate 16 decimal places quickly. Early in the 20th century, the Indian mathematician Srinivasa Ramanujan developed exceptionally efficient ways of calculating π that were later incorporated into computer algorithms. In the early 21st century, computers calculated π to 31,415,926,535,897 decimal places, as well as its two-quadrillionth digit when expressed in binary (0).

π occurs in various mathematical problems involving the lengths of arcs or other curves, the areas of ellipses, sectors, and other curved surfaces, and the volumes of many solids. It is also used in various formulas of physics and engineering to describe such periodic phenomena as the motion of pendulums, the vibration of strings, and alternating electric currents.

AFTER TEXT TASK

Task 17. Answer the following questions on Text B.

1. What does π mean?
2. Who was π devised by?
3. What approximation of π is often used for everyday calculations?
4. What number did Babylonians use to approximate π?
5. Why did the Rhind papyrus indicate?
6. What major step forward did Archimedes take?
7. What did he also prove?
8. What did new methods of mathematical analysis in Europe provide?
9. What did Newton use to calculate 16 decimal places quickly?
10. Who developed exceptioally efficient ways of calculating π early in the 20^{th} century?
11. What progress in calculating π was reached in the early 21st century?
12. In which formulas of physics and engineering π is used?

Part 2
NUMERATION SYSTEMS

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. Why do you think we have a numeration system based on ten digits?
2. What numeration system is used in computers?
3. What kinds of numbers do you know?

Task 2. Practise reading the following words.

№	Word	Transcription	Translation
1	ancient	[ˈeɪnʃənt]	древний
2	awkward	[ˈɔːkwərd]	громоздкий, неуклюжий
3	binary	[ˈbaɪnərɪ]	двоичный
4	digit	[ˈdɪdʒɪt]	1) цифра 2) палец
5	Egyptian	[ɪˈdʒɪpʃn]	египетский
6	hieroglyphics	[ˌhɑɪərəˈglɪfiks]	иероглифы
7	Mesopotamia	[ˌmesəpəˈteɪmɪə]	Месопотамия
8	society	[səˈsaɪətɪ]	общество
9	toe	[təʊ]	палец на ноге
10	worthwhile	[ˌwɜːrθˈwaɪl]	стоящий

Task 3. Study and remember the following words and expressions.

№	Word / Expression	Transcription	Translation
1	ruler	[ˈruːlər]	правитель
2	to record	[ˈrekɔːd]	записывать
3	ceremonial mace	[serɪˈməʊnɪəl]	церемониальная булава
4	the art of counting	[ðiː ˈɑːt əv ˈkaʊntɪŋ]	искусство счета
5	system of numeration	[ˈsɪstəm əv njuːməˈreɪʃ(ə)n]	система счисления
6	crude	[kruːd]	сырой, недоработанный
7	clumsy	[ˈklʌmzɪ]	громоздкий
8	related to	[rɪˈleɪtɪd tə]	относящийся к чему-либо
9	give a deeper insight	[ˈgɪv ə ˈdiːpə ˈɪnsaɪt]	дать более глубокое представление

Task 4. Read and translate Text A using a dictionary if necessary.

Text A

ANCIENT NUMERATION SYSTEMS

More than 5000 years ago an Egyptian ruler recorded, perhaps with a bit of exaggeration, the capture of 120, 000 prisoners, 400, 000 oxen and 1, 422, 000 goats. The event was inscribed on a ceremonial mace which is now a museum in Oxford, England.

The ancient Egyptians developed the art of counting to a high degree, but their system of numeration was very crude. For example, the number 1,000 was symbolized by a picture of lotus flower, and the number 2,000 was symbolized by a picture of two lotus flowers growing out of a bush. Although these systems called hieroglyphics permitted the Egyptians to write large numbers, the numeration system was clumsy and awkward to work with. For instance, the number 999 required 27 individual marks.

In our system of numeration we use ten symbols called digits – 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 – and combinations of these symbols. Our system of numeration is called decimal, or base-ten system. There is little doubt that out ten fingers influenced the development of a numeration system based on ten digits.

Other numeration systems were developed in early cultures and societies. Two of the most common were the base five-system, related to the number of fingers on one hand, and the base twenty-system, related to the number of fingers and toes. In some languages the word for ‘five’ is the same as the word for ‘hand’, and the word for ‘ten’ is the same as the word for ‘two hands’. In English the word ‘digit’ is the synonym for the word ‘finger’ – that is, ten digits, ten fingers.

Another early system of numeration was a base-sixty system developed by Mesopotamians and used for centuries. These ancient people divided the year into 360 days (6 x 60); today we still divide the hour into 60 minutes and the minute to 60 seconds.

Numeration system of current interest include a binary, or base-two system used in electronic computers and a base-twelve, or duodecimal system.
It is worthwhile to become familiar with the principles of the base-twelve numeration system and with those of base-two, base five and other systems. Working with other bases gives you deeper insight into the decimal system you have used since childhood.

Downloaded from *Britannika Kids. URL: https://kids.britannica.com/students/article/numeration-systems-and-numbers/276134*

AFTER TEXT TASKS

Task 5. Answer the following questions.
1. What information did an Egyptian ruler order to record about 5000 years ago?
2. What kind of numeration system did ancient Egyptians have?
3. Did the system permit the Egyptians to write large numbers?
4. How many digits does our numeration system use?
5. What influenced the development of our numeration system?
6. What other numeration systems were developed in early cultures and societies?
7. What systems were the most common of them?
8. Which word is the word ‘digit’ the synonym for in English?
9. What numeration system was developed by Mesopotamians?
10. What field is binary numeration system used nowadays?

Task 6. Give Russian equivalents to these words or word combinations.
1. with a bit of exaggeration
2. to a high degree
3. permitted to write large numbers
4. awkward to work with
5. base-ten system
6. the most common
7. there is little doubt that

8. related to to the number
9. numeration system of current interest
10. to become familiar

Task 7. Find the English equivalents to the following words and word combinations.
1. с некоторой долей преувеличения
2. было высечено на церемониальной булаве
3. изображаться в виде цветка лотоса
4. громоздкая и неудобная в использовании
5. система счисления с основанием 10
6. нет сомнений в том, что
7. шестидесятиричная система счисления
8. двенадцатеричная система счисления
9. представляющий интерес в данный момент
10. следует ознакомиться (с)
11. получить более глубокое представление (о)

Task 8. Match the terms with their definitions.

№	**Term**		**Definition**
1	exaggeration	**a**	constructed in a primitive way
2	numeration system	**b**	any of the five digits at the end of the human foot
3	crude	**c**	a statement that represents something as better or worse than it really is
4	clumsy	**d**	worth of spending time
5	toe	**e**	a mathematical notation for representing numbers
6	duodecimal system	**f**	difficult to handle or use
7	worthwhile	**g**	a system of counting or numerical notation that has twelve as a base

Task 9. Mark the sentences true (T) or false (F).
1. The event was inscribed on a ceremonial mace which is now a museum in Cambridge, England
2. The ancient Egyptian system of numeration was imperfect.
3. This system did not permit the Egyptians to write large numbers.
4. In our numeration system we use nine digits and zero.
5. It’s doubtful that our ten fingers influenced the creation of our numeration system.
6. The two most common systems in the ancient world were the base five-system and base fifteen systems.
7. Base-sixty system developed by ancient Indians.
8. Nowadays we still divide the hour into 60 minutes.
9. Duodecimal system is currently used in electronic computers.
10. It is no use to become familiar with the principles of other numeration systems.

Task 10. Insert the necessary word from the chart into the gaps.

Mesopotamians, related, hieroglyphics, binary, numeration, hand, digit, counting, divide, inscribed, duodecimal

1. The ancient Egyptians developed the art of (1) … to a high degree.
2. A base-sixty system was developed by (2) … and was used for centuries.
3. The event was (3) … on a ceremonial mace.
4. These systems called (4) … permitted the Egyptians to write large numbers.
5. In our system of (5) … we use ten symbols called digits.
6. Base five-system is (6) … to the number of fingers on one hand.
7. In some languages the word for 'five' is the same as the word for (7)'…'.
8. In English the word (8) '…' is the synonym for the word 'finger'.
9. Today we still (9) … the hour into 60 minutes.
10. Numeration system of current interest include a (10) … and a (11) … system.

Task 11. Match the beginnings and the endings of the sentences.

№	**Beginnings**		**Endings**
1	Although hieroglyphics permitted the Egyptians to write large numbers,	**a**	the development of a numeration system based on ten digits.
2	The number 1,000 was symbolized	**b**	the number of fingers on one hand.
3	The event was inscribed on a ceremonial mace	**c**	the synonym for the word 'finger'.
4	There is little doubt that out ten fingers influenced	**d**	which is now a museum in Oxford, England.
5	The base five-system was related to	**e**	to the number of fingers and toes.
6	The base twenty-system was related	**f**	deeper insight into the decimal system.
7	In English the word 'digit' is	**g**	the numeration system was clumsy and awkward to work with.
8	Working with other bases gives you	**h**	by a picture of lotus flower.

Task 12. Translate from Russian into English.

1. Древние египтяне развили искусство счета, но их система счисления была очень примитивной.
2. Например, число 1 000 символизировалось изображением цветка лотоса.
3. Хотя эта система позволяла египтянам писать большие числа, она была неуклюжей и неудобной в работе.
4. В нашей системе счисления мы используем десять символов, называемых цифрами, и комбинации этих символов.
5. Несомненно, что наши десять пальцев повлияли на развитие системы счисления, основанной на десяти цифрах.

6. Двумя наиболее распространенными системами счисления были пятиричная система, связанная с количеством пальцев на одной руке, и двадцатиричная система, связанная с количеством пальцев на руках и ногах.
7. В английском языке слово "digit" является синонимом слова "finger", то есть десять цифр - десять пальцев.
8. Другой ранней системой счисления была шестидесятиричная система, разработанная месопотамцами.
9. Эти древние люди делили год на 360 дней (6 х 60); сегодня мы по-прежнему делим час на 60 минут, а минуту – на 60 секунд.
10. В настоящее время интерес представляет двоичная система счисления, используемая в электронных компьютерах.

Task 13. Write out key words from the text.

Task 14. Use the key words of the text to make up the outline of the text.

Task 15. Write out the main idea of the text. Be ready to speak about it.

Task 16. Retell Text A.

Task 17. In pairs, take turns to interview your partner about different numeration systems. What questions do you think are the most relevant?

Task 18. Write a short essay on the suggested topics.
1. Numeration systems used in the ancient cultures.
2. Binary numeration system.
3. The reasons for the origin of decimal system.

Task 19. Read the words and try to remember the pronunciation.
1. digit [ˈdɪdʒɪt] – цифра, разряд
2. hexadecimal [ˌheksəˈdesɪml] – шестнадцатеричный
3. switch [swɪtʃ] – переключатель, коммутатор
4. digitize [ˈdɪdʒɪtaɪz] – преобразовывать в цифровую форму
5. discrete [dɪˈskriːt] – дискретный, отвлеченный, абстрактный
6. grid [ɡrɪd] – сетка
7. expanded [ɪkˈspandɪd] – расширенный
8. storage space [ˈstɔːrɪdʒ ˈspeɪs] – пространство для хранения

Task 20. Read Text B. Translate it from English into Russian.

Text B

BINARY NUMBER SYSTEM

The binary number system, also called the base-2 number system, is a method of representing numbers that counts by using combinations of only two numerals: zero (0) and one (1). Computers use the binary number system to manipulate and store all of their data including numbers, words, videos, graphics, and music.

The term bit, the smallest unit of digital technology, stands for "**BI**nary digi**T**." A byte is a group of eight bits. A kilobyte is 1,024 bytes or 8,192 bits.

Using binary numbers, 1 + 1 = 10 because "2" does not exist in this system. A different number system, the commonly used decimal or **base-10** number system, counts by using 10 digits (0,1,2,3,4,5,6,7,8,9) so 1 + 1 = 2 and 7 + 7 = 14. Another number system used by computer programmers is the hexadecimal system, **base-16** , which uses 16 symbols (0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F), so 1 + 1 = 2 and 7 + 7 = E. Base-10 and base-16 number systems are more compact than the binary system. Programmers use the hexadecimal number system as a convenient, more compact way to represent binary numbers because it is very easy to convert from binary to hexadecimal and vice versa. It is more difficult to convert from binary to decimal and from decimal to binary.

The advantage of the binary system is its simplicity. A computing device can be created out of anything that has a series of switches, each of which can alternate between an "on" position and an "off" position. These switches can be electronic, biological, or mechanical, as long as they can be moved on command from one position to the other. Most computers have electronic switches.

When a switch is "on" it represents the value of one, and when the switch is "off" it represents the value of zero. Digital devices perform mathematical operations by turning binary switches on and off. The faster the computer can turn the switches on and off, the faster it can perform its calculations.

Bits are a fundamental element of digital computing. The term "digitize" means to turn an **analog** signal—a range of voltages—into a digital signal, or a series of numbers representing voltages. A piece of music can be digitized by taking very frequent samples of it, called sampling, and translating it into **discrete** numbers, which are then translated into zeros and ones. If the samples are taken very frequently, the music sounds like a continuous tone when it is played back.

A black and white photograph can be digitized by laying a fine grid over the image and calculating the amount of gray at each intersection of the grid, called a **pixel** . For example, using an 8-bit code, the part of the image that is purely white can be digitized as 11111111. Likewise, the part that is purely black can be digitized as 00000000. Each of the 254 numbers that fall between those two extremes (numbers from 00000001 to 11111110) represents a shade of gray. When it is time to reconstruct the photograph using its collection of binary digits, the computer decodes the image, assigns the correct shade of gray to each pixel, and the picture appears. To improve resolution, a finer grid can be used so the image can be expanded to larger sizes without losing detail.

A color photograph is digitized in a similar fashion but requires many more bits to store the color of the pixel. For example, an 8-bit system uses eight bits to define which of 256 colors is represented by each pixel (2^8 equals 256). Likewise, a 16-bit system uses sixteen bits to define each of 65,536 colors (2^{16} equals 65,536). Therefore, color images require much more storage space than those in black and white.

AFTER TEXT TASKS

Task 21. Answer the questions on Text B.

1. What is the the binary number system?
2. What does the term ‘bit’ stand for?
3. How many symbols does the hexadecimal system use?
4. What does programmers use the hexadecimal number system for?
5. What 2 values can switches have?
6. What does the term "digitize" mean?
7. How can a piece of music be digitized?
8. What is used to digitize a black and white photograph?
9. How can we improve the resolution?
10. What is the difference between digitizing black and white photo and a color photo?

Task 22. Make up a plan for the text and write a summary.

Unit 4. ARITHMETIC

"Arithmetic is a kind of knowledge in which
the best natures should be trained,
and which must not be given up."
– Plato

Part 1
BASIC ARITHMETIC OPERATIONS

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. Can people live without numerals?
2. What is Arithmetic from your point of view?
3. What are the main operations of arithmetic?
4. What arithmetic properties do you know?

Task 2. Study and remember the following words and expressions.

ADDITION [ə'dɪʃn] – сложение **3 + 2 = 5**		
3 & 2 + 5 =	Addends ['ædendz] Plus sign [plʌs 'saɪn] Sum [sʌm] Equals sign ['iːkwəlz 'saɪn]	слагаемые знак плюс сумма знак равенства
SUBTRACTION [səb'trækʃn] – вычитание **3 – 2 = 1**		
3 – 2 1	Minuend ['mɪnjʊend] Minus sign ['maɪnəs 'saɪn] Subtrahend [ˏsʌbtrə'hend] Difference ['dɪfrəns]	уменьшаемое знак минус вычитаемое разность
MULTIPLICATION ['mʌltɪplɪ'keɪʃ(ə)n] – умножение **3 × 2 = 6**		
3 × 2 6 3 & 2	Multiplicand ['mʌltɪplɪ'kand] Multiplication sign ['mʌltɪplɪ'keɪʃ(ə)n 'saɪn] Multiplier ['mʌltɪplaɪə] Product ['prɒdʌkt] Factors ['fæktəz]	множимое (множитель 1) знак умножения множитель (множитель 2) произведение сомножители
DIVISION [dɪ'vɪʒən] – деление **6 : 2 = 3**		
6 : 2 3	Dividend ['dɪvɪdend] Division sign [dɪ'vɪʒən 'saɪn] Divisor [dɪ'vaɪzə] Quotient ['kwəʊʃ(ə)nt]	делимое знак деления делитель частное

Task 3. Read and translate Text A using a dictionary if necessary.

Text A

FOUR BASIC OPERATIONS OF ARITHMETIC

There are four basic operations of arithmetic. They are *addition, subtraction, multiplication and division*. In arithmetic, an operation is a way of thinking of two numbers and getting one number. An equation like $3 + 5 = 8$ represents an operation of *addition*. Here you add 3 and 5 and get 8 as a result. 3 and 5 are addends (or summands) and 8 is the sum. There is also a plus (+) sign and a sign of equality (=). They are mathematical symbols.

An equation like $7 – 2 = 5$ represents an operation of *subtraction*. Here 7 is the minuend and 2 is the subtrahend. As a result of the operation, you get the difference. There is also the mathematical symbol of the minus (-) sign. We may say that subtraction is the inverse operation of addition since $5 + 2 = 7$ and $7 – 2 = 5$.

The same may be said about division and multiplication, which are also inverse operations.

In *multiplication*, there is a number that must be multiplied. It is the multiplicand. There is also a multiplier. It is the number by which we multiply. If we multiply the multiplicand by the multiplier, we get the product as a result. In the equation $5 \times 2 = 10$ (five multiplied by two is ten) five is the multiplicand, two is the multiplier, ten is the product; ($\times$) is the multiplication sign.

In the operation of *division*, there is a number that is divided and it is called the dividend and the number by which we divide that is called the divisor. When we are dividing the dividend by the divisor, we get the quotient. In the equation $6 : 2 = 3$, six is the dividend , two is the divisor and three is the quotient; (:) is the division sign.

But suppose you are dividing 10 by 3. In this case, the divisor will not be contained a whole number of times in the dividend. You will get a part of the dividend left over. This part is called the *remainder*. In our case, the remainder will be 1. Since multiplication and division are inverse operations, you may check division by using multiplication.

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AFTER TEXT TASKS

Task 4. Answer the following questions.

1. What are the four basic operations of arithmetic?
2. What mathematical symbols are used in these operations?
3. What are inverse operations?
4. What is the remainder?
5. How can division be checked?

Task 5. Match the terms in Column A with their Russian equivalents in Column B.

	Column A		Column B
1	addend (summand)	**a**	уменьшаемое
2	subtrahend	**b**	слагаемое
3	minuend	**c**	частное
4	multiplier	**d**	уравнение
5	multiplicand	**e**	делимое
6	quotient	**f**	множимое
7	divisor	**g**	остаток
8	dividend	**h**	обратное действие
9	remainder	**i**	делить
10	inverse operation	**j**	вычитаемое
11	equation	**k**	разность
12	product	**l**	произведение
13	difference	**m**	множитель
14	subtract	**n**	делитель
15	add	**o**	умножать
16	divide	**p**	вычитать
17	multiply	**q**	складывать

Task 6. The italicized words are all in the wrong sentences. Correct the mistakes.

1. *Multiplication* is an operation inverse of subtraction.
2. The product is the result given by the operation of *addition*.
3. The part of the dividend which is left over is called the *divisor*.
4. *Division* is an operation inverse of addition.
5. The difference is the result of the operation of *multiplication*.
6. The quotient is the result of the operation of *subtraction*.
7. The sum is the result of the operation of *division*.
8. *Addition* is an operation inverse of multiplication.

Task 7. Complete the following definitions.

Pattern: **The operation, which is the inverse of addition, is <u>subtraction</u>.**

1. The operation, which is the inverse of subtraction, is _______.
2. The quantity, which is subtracted, is ________.
3. The result of adding two or more numbers, is _______.
4. The result of subtracting two or more numbers, is _______.
5. To find the sum is _______.
6. To find the difference is _______.
7. The quantity number or from which another number (quantity) is subtracted is _______.
8. The terms of the sum is _______.

9. A number that is divided is _______.
10. The inverse operation of multiplication is ______.
11. A number that must be multiplied is _______.
12. A number by which we multiply is _______.
13. A number by which we divide _______.
14. A part of the dividend left over after division is _______.
15. The number, which is the result of the operation of multiplication, is ________.

Task 8. Match the answers to the following questions.

№	**Question**		**Answer**
1	What is the result of addition called?	**a**	Remainder
2	What is the result of subtracting whole numbers called?	**b**	Zero
3	What arithmetic operation is usually used to check the answer of addition?	**c**	Product
4	What is the result of multiplication called?	**d**	Meaningless
5	What is the result of division called?	**e**	Sum
6	What is the product of any number multiplied by zero?	**f**	Quotient
7	What is the name of the part that is left over after the dividend has been divided equally?	**g**	Difference
8	What can we say about the following operation "n:0" for all values of n?	**h**	Subtraction

Task 9. Read the following equations aloud.

7 + 5 = 12	is read	seven plus five equals twelve
		seven plus five is equal to twelve
		seven plus five is (are) twelve
		seven added to five makes twelve
7 – 5 = 2	is read	seven minus five equals two
		seven minus five is equal to two
		seven minus five is two
		five from seven leaves two
		the difference between seven and five is two
5 x 2 = 10	is read	five multiplied by two is equal to ten
		five multiplied by two equals ten
		five times two is ten
		two times five make(s) ten
10 : 2 = 5	is read	ten divided by two is equal to five
		ten divided by two equals five
		two into ten goes five times

1. 16 + 22 = 38
2. 280 – 20 = 260
3. 1345 + 15 = 136015
4. 2017 – 1941 = 76

5. 15200 – 1300 + 738 = 14638
6. 70 × 3 = 210
7. 48 : 8 = 6
8. 3419 × 2 = 6838
9. 4200 : 2 = 2100
10. 750 : 10 × 4 = 300

Task 10. Give examples of equations representing the four basic operations of arithmetic and name their constituents.

Task 11. In pairs, take turns to interview your partner about the basic operations of arithmetic. What questions do you think are the most relevant?

Task 12. Speak on the Topic "Four Basic Operations of Arithmetic", give your own examples.

Task 13. Translate into English in writing.

1. Числа, которые нужно сложить, называются слагаемыми, а результат сложения, то есть число, получающееся от сложения, называется суммой.
2. Вычитанием называется действие, посредством которого **(by means of which)** по данной сумме и одному слагаемому отыскивается другое слагаемое.
3. Число, которое умножают, называется множимым; число, на которое умножают, называется множителем.
4. Результат действия, то есть число, полученное при умножении, называется произведением.
5. Число, которое делят, называется делимым; число, на которое делят, называется делителем; число, которое получается в результате деления, называется частным.

Task 14. Translate into Russian in writing.

Signs of Operations Used in Arithmetic

The signs most used in arithmetic to indicate operations with numbers are plus (+), minus (–), multiplication (×), and division (:) signs. When either of these is placed between any two numbers it indicates respectively that the sum, difference, product, or quotient of the two numbers is to be found. The equality sign (=) shows that any indicated operation or combination of numbers written before it (on the left) produces the result or number written after it.

Task 15. Read the words and try to remember the pronunciation.

1. property ['prɔpətɪ] – свойство
2. equation [ɪ'kweɪʒ(ə)n] – уравнение, равенство
3. commutative [kə'mju:tətɪv] – перестановочный; коммутационный
4. associative [ə'səuʃɪətɪv] – сочетаемый
5. distributive [dɪs'trɪbjutɪv] – разделительный

6. quantity ['kwɔntɪtɪ] – величина
7. affect ['æfekt] – влиять, отразиться на
8. involve [ɪn'vɔlv] – включать в себя
9. describe [dɪs'kraɪb] – описывать
10. order ['ɔ:də] – порядок

Task 16. Read Text B. Translate it from English into Russian.

Text B

THE BASIC ARITHMETIC PROPERTIES

Commutative Property

The commutative property describes equations in which the *order* of the numbers involved does not affect the result. Addition and multiplication are commutative operations:

- 2+3=3+2=5
- 5×2=2×5=10

Subtraction and division, however, are not commutative.

Associative Property

The associative property describes equations in which the *grouping* of the numbers involved does not affect the result. As with the commutative property, addition and multiplication are associative operations:

- (2+3)+6=2+(3+6)=11
- (4×1)×2=4×(1×2)=8

Once again, subtraction and division are not associative.

Distributive Property

The distributive property can be used when the sum of two quantities is then multiplied by a third quantity.

- (2+4) ×3=2×3+4×3=18

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https://courses.lumenlearning.com/boundless-algebra/chapter/introduction-to-arithmetic-operations/

AFTER TEXT TASK

Task 17. Answer the following questions on Text B.

1. What are the basic arithmetic properties?
2. What equations does the commutative property describe?
3. What equations does the associative property describe?
4. When is the distributive property used?
5. Which arithmetic operations are not commutative and associative?

Part 2
ARITHMETIC OPERATIONS OF FRACTIONS

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. What does a fraction represent?
2. Who invented fractions?
3. What arithmetic operations of fractions do you know?

Task 2. Study and remember the following words and expressions.

№	Word	Transcription	Translation
1	value	['vælju:]	значение
2	equal	['i:kwəl]	равный
3	term of a fraction	['tɜ:m ɔf ə 'frækʃ(ə)n]	числитель, знаменатель дроби
4	numerator	['nju:məreɪtə]	числитель
5	denominator	[dɪ'nɔmɪneɪtə]	знаменатель
6	mixed number	[mɪkst 'nʌmbə]	смешанное число
7	whole number	[həul 'nʌmbə]	целое число
8	proper fraction	['prɔpə 'frækʃ(ə)n]	правильная дробь
9	improper fraction	[ɪm'prɔpə 'frækʃ(ə)n]	неправильная дробь
10	fraction line	[laɪn 'frækʃ(ə)n]	дробная черта

Task 3. Practise reading the following fractions.

$\frac{1}{2}$ a half, one half

$\frac{1}{3}$ a third, one third

$\frac{1}{4}$ one fourth, a quarter

$\frac{1}{10}$ one tenth

$\frac{2}{3}$ two thirds

$\frac{3}{7}$ three sevenths

$\frac{7}{2}$ seven halves

Task 4. Read and translate Text A using a dictionary if necessary.

Text A
ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION OF FRACTIONS

A fraction represents a part of one whole thing! A fraction indicates that something has been cut or divided into a number of equal parts. For example, a pie has been divided into four equal parts. If you eat one piece of the pie, you have taken one part out of four parts. This part of the pie can be represented by the fraction **1⁄4**.

The remaining portion of the pie, which consists of three of the four equal parts of the pie, is represented by the fraction **3⁄4**.

In a fraction the upper and lower numbers are called the terms of the fraction. The horizontal line separating the two numbers in each fraction is called the fraction line. The top term of a fraction or the term above the fraction line is called the numerator; the bottom term or the term below the fraction line is called the denominator.

To add fractions having the same denominator (like fractions) add their numerators and write the sum over the common denominator (do not add the denominators). Reduce the resulting fraction to lowest terms.

To add fractions having different denominators (unlike fractions) the fractions must be changed to equivalent fractions which have the same or a common denominator. The least number which will be a common denominator, for example, of 2/3 and 3/5 is 15, 15 is the least common denominator, or lowest common denominator of 2/3 and 3/5. The least common denominator is sometimes denoted by the letters L.C.D.

To subtract fractions having the same denominator subtract the numerators and write the difference over the common denominator (do not subtract denominators).

To subtract fractions having different denominators first change the fractions to equivalent fractions having a common denominator. To subtract the fractions when they have a common denominator, subtract the numerators and write the difference over the denominator.

To multiply a mixed number and a fraction: 1) reduce the fraction to its lowest terms; 2) change the mixed number to an improper fraction; 3) multiply the two numerators to obtain the numerator of the answer; 4) multiply the denominators to obtain the denominator of the answer; 5) reduce the fraction obtained when possible. Reduction can be done by dividing a numerator and a denominator by the same number. The numbers that are divided are crossed out, and the quotients are written as the new numerator and the new denominator.

To divide a whole number by a fraction, multiply the whole number by the denominator of the fraction and divide the result by the numerator of the fraction.

Changing Fractions

The numerator and denominator of a fraction may be multiplied by the same number without changing the value of the fraction. The resulting equivalent fraction is actually the same fraction expressed in higher terms.

To change a mixed number to an improper fraction we must: 1) multiply the denominator of the fraction by the whole number; 2) add the numerator of the fraction to the product of the multiplication; 3) write the result over the denominator.
To change an improper fraction to a whole or a mixed number we must divide the numerator by the denominator. If there should be a remainder, write it over the denominator. The resulting fraction should then be reduced to its lowest terms.

To change a whole number to an improper fraction with a specific denominator: 1) multiply the specific denominator and the whole number; 2) write the result over the specific denominator.

Fractions can be compared. To compare unlike fractions we must change them to equivalent fractions so that all have like denominators.
When fractions have different numerators but the same denominator, the fraction having the largest numerator has the greatest value.
When fractions have different denominators but the same numerator, the fraction having the largest denominator has the smallest value.

Adopted from *Назарова Н.А. Профессиональный английский: математика и физика = Professional English in Use: Mathematics and Physics: учебное пособие / Н.А. Назарова, Е.В. Панасенко, О.М. Толстых. – Омск: Изд-во ОмГПУ, 2018. – 124 с.*

AFTER TEXT TASKS

Task 5. Answer the following questions.

1. What does a fraction represent?
2. What do we call “the terms of fractions”?
3. What is the numerator?
4. What is the denominator?
5. What should one do in order to add fractions having the same denominator?
6. What should one do in order to add fractions having different denominators?
7. What should one do in order to subtract fractions having the same denominator?
8. What should one do in order to subtract fractions having different denominators?
9. How do you multiply fractions having the same denominators?
10. How do you multiply fractions having different denominators?
11. How do you multiply a mixed number and a fraction?
12. What is an equivalent fraction?
13. How do you change a mixed number to an improper fraction?
14. How do you change an improper fraction to a whole number or mixed number?
15. How do you change a whole number to an improper fraction with a specific denominator?
16. What must you do to compare unlike fractions?
17. How do you compare fractions?

Task 6. Look through the text to search for unfamiliar words and try to understand their general meanings.

Task 7. Write down the transcription and definitions of unfamiliar words, practice reading the words and try to remember them.

Task 8. Give Russian equivalents to these word combinations.

1. to represent
2. to indicate
4. upper number
5. lower number
6. bottom term
7. top term

8. like fractions
9. unlike fractions
10. common denominator

Task 9. Find the English equivalents to the following words and word combinations.
1. дробная черта
2. значение дроби
3. уменьшать
4. разделить
5. сложить, добавить
6. сравнивать
7. получать, достигать
8. частное
9. наименьший общий знаменатель
10. обозначать

Task 10. Match the terms with their definitions.

№	Term		Definition
1	a mixed number	**a**	the number which is left over in a division in which one quantity does not exactly divide another
2	like fractions	**b**	the process of combining matrices, vectors, or other quantities under specific rules to obtain their product
3	reduction	**c**	fractions which have the same denominator
4	equivalent fractions	**d**	a result obtained by dividing one quantity by another
5	multiplication	**e**	different fractions that name the same number
6	unlike fractions	**f**	a number consisting of an integer and a proper fraction
7	a remainder	**g**	fractions with different numbers in the denominator
8	a quotient	**h**	the process of converting an amount from one denomination to a smaller one, or of bringing down a fraction to its lowest terms
9	numerator	**i**	the horizontal line separating the two numbers in each fraction
10	fraction line	**j**	the top term of a fraction or the term above the fraction line

Task 11. Mark true (T) or false (F) sentences.
1. A fraction represents a part of one whole thing!
2. In a fraction the upper and lower numbers are called the terms of the fraction.
3. The bottom term or the term below the fraction line is called the numerator.
4. The least common denominator is sometimes denoted by the letters L.C.D.
5. The numerator and denominator of a fraction may be multiplied by the same number, but the value of the fraction changes.

6. When fractions have different numerators but the same denominator, the fraction having the largest numerator has the greatest value.
7. When fractions have different denominators but the same numerator, the fraction having the largest denominator has the greatest value.

Task 12. Match the beginnings and the endings of the given sentences.

№	**Beginnings**		**Endings**
1	To add fractions having the same denominator (like fractions) add their numerators and	**a**	so that all have like denominators.
2	To compare unlike fractions we must change them to equivalent fractions	**b**	divide the result by the numerator of the fraction.
3	To change an improper fraction to a whole or a mixed number we must	**c**	fractions which have the same or a common denominator.
4	To divide a whole number by a fraction, multiply the whole number by the denominator of the fraction and	**d**	write the sum over the common denominator (do not add the denominators).
5	To subtract the fractions when they have a common denominator, subtract the numerators and	**e**	write the difference over the common denominator (do not subtract denominators).
6	To subtract fractions having the same denominator subtract the numerators and	**f**	divide the numerator by the denominator.
7	To add fractions having different denominators (unlike fractions) the fractions must be changed to equivalent	**g**	write the difference over the denominator.

Task 13. Fill in the gaps with the words from the box.
A.

subtrahend	*divide*	*denominator*	*product*	*L.C.D.*	*sum*

When fractions have a common (1) _______, they can be added by simply adding the numerators and writing the (2) _______ over the same denominator. Any fractions with a common denominator are subtracted by subtracting the numerator of the (3) _______ fraction from that of the minuend fraction, and writing the remainder over the common denominator to form the remainder fraction. Thus to add or subtract fractions, first change them into ones with the (4) _______, and then add or subtract the numerators, writing the result as the numerator of a fraction with the common denominator. This fraction is the desired sum or difference respectively. To multiply a fraction by a whole number, multiply the numerator by that number, and write the (5) _______ as the numerator of a new fraction with the same denominator. This fraction is the desired product. In order to (6) _______ a fraction by any number, multiply the denominator by that number.

B.

affect	*values*	*principles*	*division*	*same*

When denominators and numerators of different fractions are both different, the (1) _______ of the fractions cannot be compared until they are converted so as to have the (2) _______ denominators. Since fractions indicate (3) _______, all changes in the terms of a fraction (numerator and denominator) will (4) _______ its value (quotient) according to the general principles of division. These relations constitute the general (5) _______ of fractions.

Task 14. Write out key words from the text.

Task 15. Use the key words of the text to make up the outline of the text.

Task 16. Write out the main idea of the text. Be ready to speak about it.

Task 17. Give the summary of Text A.

Task 18. In pairs, take turns to interview your partner about addition, subtraction, multiplication and division of fractions. What questions do you think are the most relevant?

Task 19. Translate the sentences from Russian into English in writing.

A.

1. Чтобы сложить дроби с одинаковыми знаменателями, надо сложить их числители и оставить тот же знаменатель.
2. Чтобы сложить дроби с разными знаменателями, нужно предварительно привести их к наименьшему общему знаменателю, сложить их числители и написать общий знаменатель.
3. Чтобы вычесть дробь из дроби, нужно предварительно привести дроби к наименьшему общему знаменателю, затем из числителя уменьшенной дроби вычесть числитель вычитаемой дроби и под полученной разностью написать общий знаменатель.
4. Чтобы умножить дробь на целое число, нужно умножить на это целое число числитель и оставить тот же знаменатель.
5. Чтобы разделить дробь на целое число, нужно умножить на это число знаменатель, а числитель оставить тот же.

B.

1. Чтобы обратить смешанное число в неправильную дробь, нужно целое число умножить на знаменатель дроби, к произведению прибавить числитель и сделать эту сумму числителем искомой (sought for) дроби, а знаменатель оставить прежним.

2. Чтобы обратить неправильную дробь в смешанное число, нужно числитель дроби разделить на знаменатель и найти остаток.
3. Частное покажет число целых единиц; остаток нужно взять в качестве числителя, а знаменатель оставить прежним.
4. Если числитель дроби уменьшить в несколько раз, не изменяя знаменателя, то дробь уменьшится во столько же раз.
5. Если числитель и знаменатель дроби увеличить в одинаковое число раз, то дробь не изменится.

Task 20. Read the words and try to remember the pronunciation.
1. emergence [ɪ'mɜːʤ(ə)ns] – возникновение
2. relationship [rɪ'leɪʃ(ə)nʃɪp] – отношение
3. measurement ['meʒəmənt] – измерение
4. reliable [rɪ'laɪəb(ə)l] – достоверный
5. Babylon ['bæbɪlən] – Вавилон
6. Egypt ['iːʤɪpt] – Египет
7. approximate [ə'prɔksɪmeɪt] – приближенный, приблизительный
8. nautical ['nɔːtɪk(ə)l] – мореходный, морской
9. impetus ['ɪmpɪtəs] – толчок, импульс

Task 21. Read Text B. Translate it from Russian into English.

Text B

ИСТОРИЯ АРИФМЕТИКИ

История арифметики охватывает период от возникновения счёта до формального определения чисел и арифметических операций над ними с помощью системы аксиом. Арифметика — наука о числах, их свойствах и отношениях — является одной из основных математических наук. Она тесно связана с алгеброй и теорией чисел.

Причиной возникновения арифметики стала практическая потребность в счёте, простейших измерениях и вычислениях. Первые достоверные сведения об арифметических знаниях обнаружены в исторических памятниках Вавилона и Древнего Египта, относящихся к III—II тысячелетиям до н. э. Большой вклад в развитие арифметики внесли греческие математики, в частности пифагорейцы, которые пытались с помощью чисел определить все закономерности мира. В Средние века основными областями применения арифметики были торговля и приближённые вычисления. Арифметика развивалась в первую очередь в Индии и странах ислама и только затем пришла в Западную Европу. В XVII веке мореходная астрономия, механика, более сложные коммерческие расчёты поставили перед арифметикой новые запросы к технике вычислений и дали толчок к дальнейшему развитию.

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UNIT 5. ALGEBRA

The algebraic sum of all the transformations
occurring in a cyclical process can only be positive, or,
as an extreme case, equal to nothing.
– Rudolf Clausius

Part 1
ALGEBRA AS A BROAD FIELD OF MATHEMATICS

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. What do you know about the development of algebra as a field of mathematics?
2. What was characteristic of ancient Mathematics?
3. Where did the history of algebra begin?

Task 2. Practise reading the following words.

№	Word	Transcription
1	generic	[ʤɪˈnerɪk]
2	quantities	[ˈkwɒntɪtɪs]
3	indeterminate equation	[ɪndɪˈtɜːmɪnɪt ɪˈkweɪʒ(ə)n]
4	variable	[ˈve(ə)rɪəb(ə)l]
5	solutions	[səˈluːʃn]
6	derive	[dɪˈraɪv]
7	measurement	[ˈmeʒəmənt]
8	quadratic	[kwɒˈdrætɪk]
9	formulas	[ˈfɔːmjʊlə]

Task 3. Study and remember the following words and proper names.

№	Word	Transcription	Translation
1	ancient	[ˈeɪnʃnt]	древний
2	Mesopotamian	[mesɪpɪˈteɪmɪən]	месопотамский
3	Babylonian	[bæbɪˈləʊnɪən]	вавилонский
4	Egypt	[ˈɪʤɪpt]	Египет
5	Egyptian	[ɪˈʤɪpʃn]	египетский
7	Alexandria	[ælɪgˈzɑːndrɪə]	Александрия
8	Diophantus	[daɪəˈfæntəs]	Диофант
9	Al-Khwarizmi	[æl karizmi]	Аль Каризми
10	Abu Kamil	[abu kəˈmɪl]	Абу Камиль
11	Islamic	[ɪzˈlæmɪk]	исламский
12	Omar Khayyam	[ˈəʊmɑː keɪˈjæm]	Омар Хайям
13	Persian	[ˈpɜːʒən]	персидский
14	polynomial	[pɒlɪˈnəʊmɪəl]	многочлен
15	astronomer	[əsˈtrɒnəmə]	астроном
16	algebraic	[ælʤɪˈbreɪɪk]	алгебраический

17	philosopher	[fɪˈlɒsəfə]	философ
18	Rene Descartes	[rɪˈneɪ dəˈkɑːt]	Рене Декарт
19	equation	[ɪˈkweɪʒ(ə)n]	уравнение

Task 4. Read and translate Text A using a dictionary if necessary.

Text A

HISTORY OF ALGEBRA

A broad field of mathematics, algebra deals with solving generic algebraic expressions and manipulating them to arrive at results. Unknown quantities denoted by alphabets that form a part of an equation are solved for and the value of the variable is determined. A fascinating branch of mathematics, it involves complicated solutions and formulas to derive answers to the problems posed.

The earliest records of advanced, organized mathematics date back to the ancient Mesopotamian country of Babylonia and to the Egypt of the 3rd millennium BC.

Ancient mathematics was dominated by arithmetic, with an emphasis on measurement and calculation in geometry and with no trace of later mathematical concepts such as axioms or proofs. It was in ancient Egypt and Babylon that the history of algebra began. Egyptian and Babylonian mathematicians learned to solve linear and quadratic equations as well as indeterminate equations whereby several unknowns are involved.

The Alexandrian mathematicians Hero of Alexandria and Diophantus continued the traditions of Egypt and Babylon, but Diophantus' book Arithmetica is on a much higher level and gives many surprising solutions to difficult indeterminate equations.

In the 9th century, the Arab mathematician Al-Khwarizmi wrote one of the first Arabic algebras, and at the end of the same century, the Egyptian mathematician Abu Kamil stated and proved the basic laws and identities of algebra.

By medieval times, Islamic mathematicians had worked out the basic algebra of polynomials; the astronomer and poet Omar Khayyam showed how to express roots of cubic equations.

An important development in algebra in the 16th century was the introduction of symbols for the unknown and for algebraic powers and operations. As a result of this development, Book 3 of La geometria (1637) written by the French philosopher and mathematician Rene Descartes looks much like a modern algebra text. Descartes' most significant contribution to mathematics, however, was his discovery of analytic geometry, which reduces the solution of geometric problems to the solution of algebraic ones.

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AFTER TEXT TASKS

Task 5. Answer the following questions.

1. What equations did Egyptian and Babylonian mathematicians learn to solve?
2. Who continued the traditions of Egypt and Babylon?
3. Who was algebra developed by in the 9^{th} century?
4. What mathematicians advanced algebra in medieval times?
5. What was an important development in algebra in the 16^{th} century?
6. What was the result of this development?
7. What was Rene Descartes' most significant contribution to mathematics?

Task 6. Give Russian equivalents to these word combinations.

1. to solve generic algebraic expressions
2. to arrive at results
3. unknown quantities
4. a part of an equation
5. the value of the variable
6. a fascinating branch of mathematics
7. complicated solutions and formulas
8. to derive answers to the problems
9. measurement and calculation in geometry
10. axioms or proofs

Task 7. Find the English equivalents to the following word combinations.

1. решать линейные и квадратные уравнения
2. неопределенные уравнения
3. когда задействовано несколько неизвестных
4. основные законы и тождества алгебры
5. к средневековым временам
6. базовая алгебра многочленов
7. корни кубических уравнений
8. введение символов
9. алгебраические степени и операции
10. значительный вклад в математику

Task 8. Match the terms with their definitions.

№	**Term**		**Definition**
1	contribution	**a**	решение
2	development	**b**	вклад
3	solution	**c**	достижение
4	records	**d**	степень
5	quadratic	**e**	кубический
6	to work out	**f**	разрабатывать
7	polynomial	**g**	открытие

8	unknown	**h**	многочлен
9	discovery	**i**	неизвестное
10	ancient	**j**	корень
11	indeterminate	**k**	древний
12	identity	**l**	неопределённый
13	root	**m**	тождество
14	power	**n**	письменные материалы
15	cubic	**o**	квадратный

Task 9. Mark true (T) or false (F) sentences.

1. In the 3rd millennium BC, mathematics was dominated by arithmetic.
2. The history of algebra began in Europe.
3. The book Arithmetica was written by Diophantus.
4. One of the first Arabic algebras was written by the Arab mathematician Al-Khwarizmi.
5. The basic algebra of polynomials was worked out by Rene Descartes.
6. Omar Khayyam introduced symbols for the unknown and for algebraic powers and operations.
7. Analytic geometry was discovered by Islamic mathematicians.

Task 10. Insert the necessary word(s) from the chart into the gaps.

polynomials; algebraic expressions; measurement; solutions (2); algebra; equations; the basic laws; roots; mathematics.

1. A broad field of mathematics, algebra deals with solving generic (1) … … and manipulating them to arrive at results.
2. A fascinating branch of mathematics, it involves complicated (2) … and formulas to derive answers to the problems posed.
3. The earliest records of advanced, organized (3) … date back to the ancient Mesopotamian country of Babylonia and to the Egypt of the 3rd millennium BC.
4. Ancient mathematics was dominated by arithmetic, with an emphasis on (4) … and calculation in geometry and with no trace of later mathematical concepts such as axioms or proofs.
5. It was in ancient Egypt and Babylon that the history of (5) … began.
6. Egyptian and Babylonian mathematicians learned to solve linear and quadratic equations as well as indeterminate (6) … whereby several unknowns are involved.
7. The Alexandrian mathematicians Hero of Alexandria and Diophantus continued the traditions of Egypt and Babylon, but Diophantus' book Arithmetica is on a much higher level and gives many surprising (7) … to difficult indeterminate equations.
8. In the 9th century, the Arab mathematician Al-Khwarizmi wrote one of the first Arabic algebras, and at the end of the same century, the Egyptian mathematician Abu
9. Kamil stated and proved (8) … and identities of algebra.

By medieval times, Islamic mathematicians had worked out the basic algebra of (9) … .

10. The astronomer and poet Omar Khayyam showed how to express (10) … of cubic equations.

Task 11. Match the beginnings and the endings of the given sentences.

Beginnings

1. A broad field of mathematics, algebra deals with … .
2. Unknown quantities denoted by alphabets that form a part of an equation are solved for and … .
3. A fascinating branch of mathematics, algebra involves … .
4. The earliest records of advanced, organized mathematics date back … .
5. Ancient mathematics was dominated by arithmetic, with an emphasis on … .
6. Egyptian and Babylonian mathematicians learned to solve linear and quadratic equations as well as … .
7. Diophantus' book Arithmetica gives many … .
8. In the 9^{th} century, the Arab mathematician Al-Khwarizmi wrote … .
9. At the end of the 9th century, the Egyptian mathematician Abu Kamil stated and proved … .
10. By medieval times, Islamic mathematicians had worked out … .
11. The astronomer and poet Omar Khayyam showed how to express … .
12. An important development in algebra in the 16th century was … .
13. Descartes' most significant contribution to mathematics was ... .

Endings

a. his discovery of analytic geometry, which reduces the solution of geometric problems to the solution of algebraic ones.
b. solving generic algebraic expressions and manipulating them to arrive at results.
c. surprising solutions to difficult indeterminate equations.
d. the introduction of symbols for the unknown and for algebraic powers and operations.
e. the value of the variable is determined.
f. roots of cubic equations.
g. complicated solutions and formulas to derive answers to the problems posed.
h. the basic algebra of polynomials.
i. to the ancient Mesopotamian country of Babylonia and to the Egypt of the 3rd millennium BC.
j. measurement and calculation in geometry and with no trace of later mathematical concepts such as axioms or proofs.
k. one of the first Arabic algebras.
l. the basic laws and identities of algebra.
m. indeterminate equations whereby several unknowns are involved.

Task 12. Retell Text A.

Task 13. In pairs, take turns to interview your partner about algebra and its history. What questions do you think are the most relevant?

Task 14. Write a short essay on the suggested topics. The volume of the essay is 200-250 words. Suggest some other relevant essay topics.
1. The Egyptian mathematicians.
2. Abu Kamil.
3. Rene Descartes.
4. The Alexandrian mathematicians.
5. The Arab mathematician Al-Khwarizmi.

Task 15. Read the words and try to remember the pronunciation.
1. non-Euclidean geometry [nɔn-juːˈklɪdɪən ʤɪˈɒmɪtrɪ] – не-Эвклидова геометрия
2. Gauss [´gaʊs] – Гаусс
3. Riemann [ˈriːmən] – Риман
4. Kant [kænt] – Кант
5. decade [ˈdekeɪd] – десятилетие
6. obscure [əbˈskjʊə] – малоизвестный, незаметный
7. unique [juːˈniːk] – уникальный
8. plausibility [´plɔːzəˈbɪlɪtɪ] – очевидность, правдоподобность
9. inherent [ɪnˈhɪərənt] – врождённый, изначальный
10. ingenuity [ɪnʤɪˈnjuːɪtɪ] – оригинальность (мышления), изобретательность
11. convergence [kənˈvɜːʤəns] – сходимость (бесконечного ряда)

Task 16. Read Text B. Translate it from English into Russian.

Text B

N. I. LOBACHEVSKY

Nikolai Ivanovich Lobachevsky was born in 1792 in Nizhny Novgorod. After his father's death in 1797, the family moved to Kazan where Lobachevsky graduated from the University. He stayed in Kazan all his life, occupying the position of dean of the faculty of Physics and Mathematics and president of Kazan University. He lectured on mathematics, physics, and astronomy.

Lobachevsky is the creator of a non-Euclidean geometry. His first book appeared in 1829. Few people took notice of it. Non-Euclidean geometry (as a matter of fact, the name is due to Gauss) remained for several decades an obscure field of science.

Most mathematicians ignored it. The first leading scientist who realized its full importance was Riemann.

There is one axiom of Euclidean geometry whose truth is not obvious. This is the famous postulate of the unique parallel which states that through any point not on a given line, one and only one line can be drawn parallel to the given line. For centuries, mathematicians have tried to find proof of it in terms of the other Euclidean axioms because of the wide-spread feeling that the parallel postulate is of a character essentially different from the others. It lacks the plausibility which an axiom of geometry should possess.

At that time, any geometrical system not in absolute agreement with that of Euclid's would have been considered as obvious nonsense. Kant, the most outstanding philosopher of the period, formulated this attitude in his statement that Euclid's axioms are inherent in the human mind, and, therefore, have no objective validity for *real* space. But, in the long run, there appeared a conviction that the unending failure in the search for a proof of the parallel postulate was due not to any lack scientific character, but rather to the fact that the parallel postulate is really independent of the others.

What does the independence of the parallel postulate mean? Simply that it is possible to construct a consistent system of geometrical statements dealing with points, lines, etc., by deduction from a set of axioms in which the parallel postulate is replaced by a contrary postulate. Such a system is called a non-Euclidean geometry.

It required the intellectual courage of Lobachevsky to realize that such a geometry, based on a non-Euclidean system of axioms, can be perfectly consistent. Lobachevsky settled the question by constructing in all detail a geometry in which the parallel postulate does not hold. Non-Euclidean geometry has developed into an extremely useful instrument for application in the physical world.

After 1840, Lobachevsky published a number of papers on convergence of infinite series and the solution of definite integrals. In modern reference books on definite integrals, about 200 integrals were solved by Lobachevsky.

Non-Euclidean geometry is of great importance in the study of the foundations of mathematics. Lobachevsky was the father of the most famous revolution in mathematics, but the tsarist government erected no monument to commemorate the event. Instead, the government relieved him of his job as head of the University of Kazan at the age of fifty-four – this with no explanation whatsoever, to a mathematician so great and well-known throughout the world. Lobachevsky survived this disgrace, but his health failed and he went blind.

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AFTER TEXT TASK

Task 17. Answer the questions on Text B.

1. What city was Lobachevsky born in?
2. Where did he get his higher education?
3. Where did he live and work all his life?
4. What discovery is Lobachevsky known by in the world of mathematics?
5. Did his first book on non-Euclidean geometry produce a sensation?
6. Who is the term *non- Euclidean* geometry due to?
7. Who was the first great scientist that paid attention to Lobachevsky's work?
8. Why couldn't mathematicians find proof of the parallel postulate?
9. Euclidean geometry was firmly rooted in the scholars' minds, wasn't it?
10. What philosopher contributed to such an attitude?

11. Is the parallel postulate replaced by a contrary postulate in the non-Euclidean geometry?
12. What quality did Lobachevsky reveal when he came out with a new theory?
13. What is the scientific value of Lobachevsky's discovery?
14. What were his other contributions to mathematics?
15. Was Lobachevsky duly appreciated by the tsarist government during his life time?
16. Is he held in high esteem by his descendants at present?

Part 2
WHAT IS ALGEBRA? BASICS, DEFINITION, EXAMPLES

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.
1. Why do people need algebra?
2. What are the main branches of algebra?
3. Why is understanding algebra as a concept more important than solving equations?

Task 2. Practise reading the following words.

№	Word	Transcription
1	representation	[reprɪzenˈteɪʃn]
2	mathematical	[mæθɪˈmætɪkəl]
3	variables	[ˈveərɪəblz]
4	addition	[əˈdɪʃn]
5	subtraction	[səbˈtrækʃn]
6	multiplication	[mʌltɪplɪˈkeɪʃn]
7	division	[dɪˈvɪʒən]
8	trigonometry	[trɪgəˈnɒmɪtrɪ]
9	calculus	[ˈkælkjʊləs]
10	constant	[ˈkɒnstənt]

Task 3. Study and remember the following words and expressions.

№	Word / Expression	Transcription	Translation
1	simplify	[ˈsɪmplɪfaɪ]	упрощать
2	numerous	[ˈnjuːmərəs]	многочисленный
3	complexity	[kəmˈpleksɪtɪ]	сложность
4	various	[ˈve(ə)rɪəs]	различный
5	linear equation	[ˈlɪnɪə ɪˈkweɪʒ(ə)n]	линейное уравнение
6	quadratic equation	[kwɒˈdrætɪk ɪˈkweɪʒ(ə)n]	квадратное уравнение
7	polynomial	pɒlɪˈnəʊmɪəl]	многочлен
8	exponent	[ɪkˈspəʊnənt]	показатель степени
9	logarithm	[ˈlɒgərɪðəm]	логарифм
10	quantity	[ˈkwɒntɪtɪ]	величина

Task 4. Read and translate Text A using a dictionary if necessary.

Text A

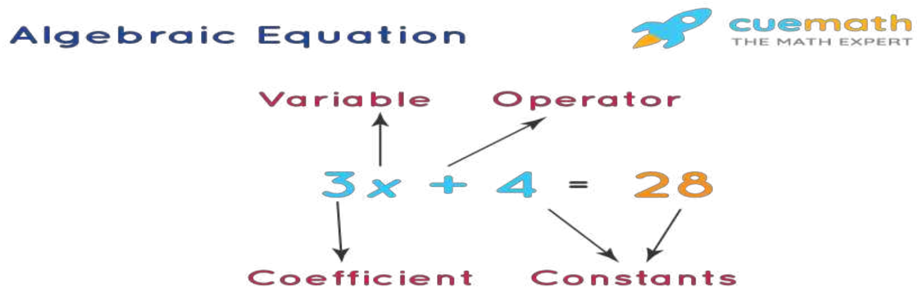
WHAT IS ALGEBRA? BASICS, DEFINITION, EXAMPLES

Algebra helps in the representation of problems or situations as mathematical expressions. It involves variables like x, y, z, and mathematical operations like addition, subtraction, multiplication, and division to form a meaningful mathematical expression. All the branches of mathematics such as trigonometry, calculus, coordinate geometry, involve the use of algebra. One simple example of an expression in algebra is $2x + 4 = 8$.

Algebra deals with symbols and these symbols are related to each other with the help of operators. It is not just a mathematical concept, but a skill that all of us use in our daily life without even realizing it. Understanding algebra as a concept is more important than solving equations and finding the right answer, as it is useful in all the other topics of mathematics that you are going to learn in the future or you have already learned in past.

What is Algebra?

Algebra is a branch of mathematics that deals with symbols and the arithmetic operations across these symbols. These symbols do not have any fixed values and are called variables. In our real-life problems, we often see certain values that keep on changing. But there is a constant need to represent these changing values. Here in algebra, these values are often represented with symbols such as x, y, z, p, or q, and these symbols are called variables. Further, these symbols are manipulated through various arithmetic operations of addition, subtraction, multiplication, and division, with an objective to find the values.



The above algebraic expressions are made up of variables, operators, and constants. Here the numbers 4, 28 are constants, x is the variable, and the arithmetic operation of addition is performed.

Branches of Algebra

The complexity of algebra is simplified by the use of numerous algebraic expressions. Based on the use and the complexity of the expressions, algebra can be classified into various branches that are listed below:

- Pre-algebra
- Elementary Algebra
- Abstract Algebra
- Universal Algebra

Pre-algebra

The basic ways of presenting the unknown values as variables help to create mathematical expressions. It helps in transforming real-life problems into *an algebraic expression* in mathematics. Forming a mathematical expression of the given problem statement is part of pre-algebra.

Elementary Algebra

Elementary algebra deals with solving the algebraic expressions for a viable answer. In elementary algebra, simple variables like x, y, are represented in the form of an equation. Based on the degree of the variable, the equations are called linear equations, quadratic equations, polynomials. *Linear equations* is of the form of $ax + b = c$, $ax + by + c = 0$, $ax + by + cz + d = 0$. Elementary algebra based on the degree of the variables, branches out into quadratic equations and polynomials. A general form of representation of *a quadratic equation* is $ax^2 + bx + c = 0$, and for a polynomial equation, it is $ax^n + bx^{n-1}+ cx^{n-2}+ .....k = 0$.

Abstract Algebra

Abstract algebra deals with the use of abstract concepts like groups, rings, vectors rather than simple mathematical number systems. Rings are a simple level of abstraction found by writing the addition and multiplication properties together. Group theory and ring theory are two important concepts of abstract algebra. Abstract algebra finds numerous applications in computer sciences, physics, astronomy, and uses vector spaces to represent quantities.

Universal Algebra

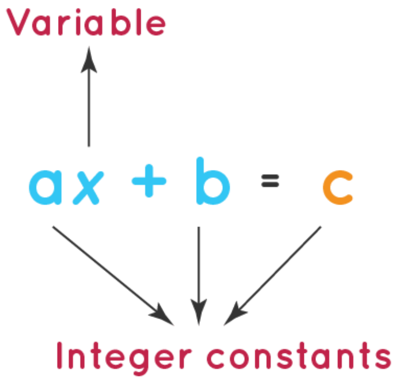
All the other mathematical forms involving trigonometry, calculus, coordinate geometry involving algebraic expressions can be accounted as universal algebra. Across these topics, universal algebra studies mathematical expressions and does not involve the study of models of algebra. All the other branches of algebra can be considered as the subset of universal algebra. Any of the real-life problems can be classified into one of the branches of mathematics and can be solved using abstract algebra.

Algebra Topics

Algebra is divided into numerous topics to help for a detailed study. Here we have listed below some of the important topics of algebra such as algebraic expressions and equations, sequence and series, exponents, logarithm, and sets.

Algebraic Expressions

An algebraic expression in algebra is formed using *integer* constants, variables, and basic arithmetic operations of addition(+), *subtraction*(-), multiplication(×), and *division*(/). An example of an algebraic expression is $5x + 6$. Here 5 and 6 are fixed numbers and x is a variable. Further, the variables can be simple variables using alphabets like x, y, z or can have complex variables like x^2, x^3, x^n, xy, x^2y, etc. Algebraic expressions are also known as polynomials. A *polynomial* is an expression consisting of variables (also called indeterminates), coefficients, and non-negative integer exponents of variables. Example: $5x^3 + 4x^2 + 7x + 2 = 0$.



An equation is a mathematical statement with an 'equal to' symbol between two algebraic expressions that have equal values. Given below are the different types of equations, based on the degree of the variable, where we apply the concept of algebra:

- **Linear Equations:** *Linear equations* help in representing the relationship between variables such as x, y, z, and are expressed in exponents of one degree. In these linear equations, we use algebra, starting from the basics such as the addition and subtraction of algebraic expressions.
- **Quadratic Equations**: *A quadratic equation* can be written in the standard form as $ax^2 + bx + c = 0$, where a, b, c are constants and x is the variable. The values of x that satisfy the equation are called solutions of the equation, and a quadratic equation has at most two solutions.
- **Cubic Equations:** The algebraic equations having variables with power 3 are referred to as cubic equations. A generalized form of a cubic equation is $ax^3 + bx^2 + cx + d = 0$. A cubic equation has numerous applications in calculus and three-dimensional geometry.

Sequence and Series

A set of numbers having a relationship across the numbers is called a sequence. A sequence is a set of numbers having a common mathematical relationship between the number, and a series is the sum of the terms of a sequence. In mathematics, we have two broad number sequences and series in the form of arithmetic progression and geometric progression. Some of these series are finite and some series are infinite. The two series are also called arithmetic progression and geometric progression and can be represented as follows.

Arithmetic Progression: An Arithmetic progression (AP) is a special type of progression in which the difference between two consecutive terms is always a constant. The terms of an arithmetic progression series is a, a+d, a + 2d, a + 3d, a + 4d, a + 5d, ...

Geometric Progression: Any progression in which the ratio of adjacent terms is fixed is a Geometric Progression. The general form of representation of a geometric sequence is a, ar, ar^2, ar^3, ar^4, ar^5, ...

Exponents

Exponent is a mathematical operation, written as a^n. Here the expression a^n involves two numbers, the base a and the exponent or power n. *Exponents* are used to simplify algebraic expressions. In this section, we are going to learn in detail about exponents including squares, cubes, square root, and cube root. The names are based on the powers of these exponents. The exponents can be represented in the form a^n = a x a x a x ... n times.

Logarithms

The logarithm is the inverse function to exponents in algebra. *Logarithms* are a convenient way to simplify large algebraic expressions. The exponential form represented as $a^x = n$ can be transformed into logarithmic form as logaan = x. John Napier discovered the concept of Logarithms in 1614. Logarithms have now become an integral part of modern mathematics.

Sets

A set is a well-defined collection of distinct objects and is used to represent algebraic variables. The purpose of using sets is to represent the collection of relevant objects in a group. Example: Set A = {2, 4, 6, 8}..........(A set of even numbers), Set B = {a, e, i, o, u}......(A set of vowels).

Algebraic Formulas

An algebraic identity is an equation that is always true regardless of the values assigned to the variables. Identity means that the left-hand side of the equation is identical to the right-hand side, for all values of the variables. These formulae involve squares and cubes of algebraic expressions and help in solving the algebraic expressions in a few quick steps. The frequently used algebraic formulas are listed below.

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$
- $(x + a)(x + b) = x + (a + b)x + ab$
- $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
- $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Let us see the application of these formulas in algebra using the following example,

Example: Using the $(a + b)^2$ formula in algebra, find the value of $(101)^2$.

Solution:

Given: $(101)^2 = (100 + 1)^2$

Using algebra formula $(a + b)^2 = a^2 + 2ab + b^2$, we have,

$(100 + 1)^2 = (100)^2 + 2(1)(100) + (1)^2$

$(101)^2 = 10201$

Algebraic Operations

The basic operations covered in algebra are addition, subtraction, multiplication, and division.

- Addition: For the addition operation in algebra, two or more expressions are separated by a plus(+) sign between them.

- Subtraction: For the subtraction operation in algebra, two or more expressions are separated by a minus(-) sign between them.
- Multiplication: For the multiplication operation in algebra, two or more expressions are separated by a multiply(×) sign between them.
- Division: For the division operation in algebra, two or more expressions are separated by a "/" sign between them.

Basic Rules and Properties of Algebra

The basic rules or properties of algebra for variables, algebraic expressions, or real numbers a, b and c are as given below,

- Commutative Property of Addition: $a + b = b + a$
- Commutative Property of Multiplication: $a \times b = b \times a$
- Associative Property of Addition: $a + (b + c) = (a + b) + c$
- Associative Property of Multiplication: $a \times (b \times c) = (a \times b) \times c$
- Distributive Property: $a \times (b + c) = a \times b + b \times c$ or $a \times (b - c) = a \times b - a \times c$
- Reciprocal: Reciprocal of $a = 1/a$
- Additive Identity: $a + 0 = 0 + a = a$
- Multiplicative Identity: $a \times 1 = 1 \times a = a$
- Additive Inverse: $a + (-a) = 0$

AFTER TEXT TASKS

Task 5. Answer the following questions.

1. What does algebra deal with?
2. What is the complexity of algebra simplified by?
3. What are the basic ways of presenting the unknown values as variables?
4. How are simple variables like x, y represented in elementary algebra?
5. How are the equations based on the degree of the variable called?
6. How can we define rings?
7. How can all the other mathematical forms involving trigonometry, calculus, coordinate geometry involving algebraic expressions be accounted?
8. How is an algebraic expression formed?
9. What should be done for the division operation in algebra?

Task 6. Look through the text to search for unfamiliar words and try to understand their general meanings.

Task 7. Write down the transcription and definitions of unfamiliar words, practise reading the words and try to remember them.

Task 8. Give Russian equivalents to these word combinations.

1. coordinate geometry
2. a viable answer

3. the degree of the variables
4. abstract algebra deals with
5. group theory
6. ring theory
7. numerous applications
8. the subset of universal algebra
9. sequence and series
10. integer constants

Task 9. Find the English equivalents to the following words and word combinations.

1. многочлены
2. степень
3. трехмерная геометрия
4. арифметическая прогрессия
5. конечная прогрессия
6. бесконечная прогрессия
7. последовательные члены
8. отношение соседних членов
9. квадраты
10. кубы
11. квадратный корень
12. кубический корень

Task 10. Match the terms with their definitions.

№	**Term**		**Defenition**
1	An algebraic identity	**a**	is a mathematical operation, written as an .
2	A set	**b**	is a special type of progression in which the difference between two consecutive terms is always a constant.
3	Identity	**c**	is an equation that is always true regardless of the values assigned to the variables.
4	The logarithm	**d**	is a set of numbers having a relationship across the numbers.
5	Exponent	**e**	is a well-defined collection of distinct objects and is used to represent algebraic variables.
6	An Arithmetic progression (AP)	**f**	is a mathematical statement with an 'equal to' symbol between two algebraic expressions that have equal values.
7	A sequence	**g**	means that the left-hand side of the equation is identical to the right-hand side, for all values of the variables.
8	Geometric Progression	**h**	are the algebraic equations having variables

			with power 3.
9	Cubic Equations	**i**	is the inverse function to exponents in algebra.
10	An equation	**j**	is any progression in which the ratio of adjacent terms is fixed

Task 11. Match the beginnings and the endings of the given sentences.

Beginnings

1. Algebra is a branch of mathematics that deals with …
2. The complexity of algebra is simplified … .
3. The basic ways of presenting the unknown values as variables help …
4. Elementary algebra deals with …
5. In elementary algebra, simple variables like x, y, are represented …
6. Based on the degree of the variable, the equations are called …
7. Abstract algebra deals with …
8. Rings are a simple level of abstraction found by …
9. All the other mathematical forms involving trigonometry, calculus, coordinate geometry involving algebraic expressions can be accounted as …
10. An algebraic expression in algebra is formed using …
11. For the division operation in algebra, two or more expressions are separated …

Endings

a. integer constants, variables, and basic arithmetic operations of addition, subtraction, multiplication, and division.
b. symbols and the arithmetic operations across these symbols.
c. universal algebra.
d. by the use of numerous algebraic expressions.
e. writing the addition and multiplication properties together.
f. to create mathematical expressions.
g. the use of abstract concepts like groups, rings, vectors rather than simple mathematical number systems.
h. solving the algebraic expressions for a viable answer.
i. linear equations, quadratic equations, polynomials.
j. in the form of an equation.
k. by a "/" sign between them.

Task 12. Write out key words from the text.

Task 13. Use the key words of the text to make up the outline of the text.

Task 14. Write out the main idea of the text. Be ready to speak about it.

Task 15. Give the summary of Text A.

Task 16. In pairs, take turns to interview your partner about algebra as a branch of mathematics. What questions do you think are the most relevant?

Task 17. Write a short essay on the suggested topics. Suggest some other relevant essay topics.

1. What is algebra? Why do we need it?
2. Branches of algebra.
3. The future of algebra.

Task 18. Read the words and try to remember the pronunciation.

1. Isaak Newton [ˈaɪzək njuːtn] – Исаак Ньютон
2. Cambridge [ˈkeɪmbrɪʤ] – Кэмбридж
3. astronomy [əsˈtrɒnəmɪ] – астрономия
4. gravity [ˈgrævɪtɪ] – притяжение
5. spectrum [ˈspektrəm] – спектр
6. spectroscopy [spekˈtrɒskəpɪ] – спектроскопия

Task 19. Read Text B. Translate it from Russian into English.

Text B
ИСААК НЬЮТОН

Исаак Ньютон, один из величайших людей в истории науки, родился в маленькой деревушке в Англии в 1642 году. Его отец был фермером и умер еще до рождения Исаака. Ферма была расположена в уединенном месте, где не было школ, и Ньютон получил образование в школе в соседней деревне. В возрасте двенадцати лет его отправили в среднюю школу. Вскоре он стал лучшим учеником в своей школе. Ньютон не принимал участия в играх, как его одноклассники, он тратил много времени на построение моделей. Он сделал модель ветряной мельницы, деревянные часы, приводимые в движение водой, и другие вещи. Мать хотела, чтобы ее сын стал фермером, поэтому, когда ему исполнилось четырнадцать, он начал работать на ферме. Но вскоре его мать поняла, что учить его работе на ферме бесполезно, потому что он всегда был занят чтением книг, конструированием моделей или наблюдением за различными явлениями в природе. В возрасте девятнадцати лет он стал студентом Кембриджского университета. Он начал изучать физику, астрономию и математику. Вскоре он стал там одним из лучших студентов.

Однажды, когда юный Ньютон сидел в саду своего дома, ему на голову упало спелое яблоко. Ньютон взял яблоко и подумал: “Почему яблоко падает [перпендикулярно земле]? Почему бы ему вместо этого не упасть (второе предложение кажется незаконченным)?” Итак, он пришел к выводу, что яблоко и Земля притягивают друг друга, и начал думать, что одни и те же законы гравитации простираются далеко за пределы Земли. Ньютон вывел и рассчитал силу тяжести, действующую между Солнцем и планетами, установив, таким образом, закон тяготения в его наиболее общей форме.

Он изучал природу света и цвета и пришел к выводу, что белый свет состоит из множества различных цветов, известных нам как спектр. Такое явление было совершенно неизвестно до работ Ньютона. Эти результаты заложили основу современной спектроскопии и значительно обогатили область оптики. Ньютон разработал математический метод, незаменимый во всех вопросах, касающихся движения. Этот метод известен под названием дифференциального и интегрального исчисления. Он открыл законы движения, которые до сих пор считаются основой всех расчетов, касающихся движения.
Вклад Ньютона в науку настолько велик, что его можно считать основоположником современной математики, физики и спектроскопии. Пока живо человечество, Исаак Ньютон, величайший из людей науки, никогда не будет забыт. Ньютон умер в 1727 году в возрасте восьмидесяти четырех лет и был похоронен в Вестминстерском аббатстве.

Adopted from *Пушкина Е.Н. English for Mathematicians and Information Technologies Learners = Английский для студентов, изучающих математику и информационные технологии: учебно-методическое пособие [Электронный ресурс] / Е.Н. Пушкина. – Нижний Новгород, ННГУ, 2019. – 88 с.*

UNIT 6. GEOMETRY

The description of right lines and circles, upon which geometry is founded, belongs to mechanics. Geometry does not teach us to draw these lines, but requires them to be drawn.
- *Isaac Newton*

Part 1
GEOMETRY AS A PRACTICAL BRANCH OF MATHEMATICS

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. Have you ever been interested in geometry? Why?
2. Why do people need geometry?
3. What does geometry deal with?

Task 2. Practise reading the following words.

№	Word	Transcription
1	Monge	[mɒnʤ]
2	Carl Frederich Gauss	[kɑːl fredˈrɪk gaʊs]
3	Janos Bolyai	[jaːnoʃ boːjɒi]
4	Euclid	[ˈjuːklɪd]
5	Euclidean	[juːˈklɪdɪən]
6	infinite	[ˈɪnfɪnɪt]
7	Riemann	[ˈriːmən]
8	Einstein	[ˈaɪnstaɪn]
9	Pierre de Fermat	[pɪə diː fəˈmæt]
10	Euler	[ˈɔɪlər]

Task 3. Study and remember the following words and expressions.

№	Word	Transcription	Translation
1	methodology	[meθəˈdɒləʤɪ]	методология
2	trial-and-error	[ˈtraɪəl-ænd-ˈerə]	метод проб и ошибок
3	approximation	[əprɒksɪˈmeɪʃn]	приближение
4	axiomatic	[æksɪəʊˈmætɪk]	аксиоматичный
5	external	[ɪkˈstɜːn(ə)l]	внешний
6	paradigm	[ˈpærədaɪm]	парадигма
7	trigonometry	[trɪgəˈnɒmɪtrɪ]	тригонометрия
8	Muslim	[ˈmʊslɪm]	мусульманский
9	approximation	[əprɒksɪˈmeɪʃn]	приближенное значение
10	deduction	[dɪˈdʌkʃn]	логический вывод

Task 4. Read and translate Text A using a dictionary if necessary.

Text A

GEOMETRY AND ITS HISTORY

Do you often wonder about the shapes and sizes of various objects? Then geometry is the branch you must explore. Dealing with the shape, sizes, and volumes of figures, geometry is a practical branch of mathematics that focuses on the study of polygons, shapes, and geometric objects in both two-dimensions and three-dimensions. Congruence of objects is studied at the same time focusing on their special properties and calculation of their area, volume, and perimeter. The importance of geometry lies in its actual usage while creating objects in practical life.

Geometry (Greek; geo = earth, metria = measure) arose as the field of knowledge dealing with spatial relationships. For the ancient Greek mathematicians, geometry was the crown jewel of their sciences, reaching a completeness and perfection of methodology that no other branch of their knowledge had attained. They expanded the range of geometry to many new kinds of figures, curves, surfaces, and solids; they changed its methodology from trial-and-error to logical deduction; they recognized that geometry studies “external forms”, or abstractions, of which physical objects are only approximations; and they developed the idea of an “axiomatic theory” which, for more than 2000 years, was regarded to be the ideal paradigm for all scientific theories.

The Muslim mathematicians made considerable contributions to geometry, trigonometry and mathematical astronomy and were responsible for the development of algebraic geometry. The 17th century was marked by the creation of analytic geometry, or geometry with coordinates and equations, associated with the names of Rene Descartes and Pierre de Fermat. In the 18th century, differential geometry appeared, which was linked with the names of L. Euler and G. Monge. In the 19th century, Carl Frederich Gauss, Janos Bolyai and Nikolai Ivanovich Lobachevsky, each working alone, created non-Euclidean geometry. Euclid’s fifth postulate states that through a point outside a given line, it is possible to draw only one line parallel to that line, that is, one that will never meet the given line, no matter how far the lines are extended in either direction. But Gauss, Bolyai and Lo bachevsky demonstrated the possibility of constructing a system of geometry in which Euclid’s postulate of the unique parallel was replaced by a postulate stating that through any point not on a given straight line an infinite number of parallels to the given line could be drawn. Their works influenced later researchers, including Riemann and Einstein.

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AFTER TEXT TASKS

Task 5. Answer the following questions.

1. What is the origin of the term geometry?
2. What was the contribution of Greek mathematicians to the science of geometry?
3. Who contributed to the development of algebraic geometry?

4. Who was analytic geometry created by?
5. Whose names was differential geometry associated with?
6. Whose names was the creation of non-Euclidean geometry linked with?
7. Whose works were later influenced by non-Euclidean geometry?

Task 6. Give Russian equivalents to these word combinations.
1. the shapes and sizes of various objects
2. the branch you must explore
3. focuses on the study of polygons
4. in both two-dimensions and three-dimensions
5. congruence of objects
6. focusing on their special properties
7. its actual usage
8. creating objects in practical life
9. field of knowledge
10. spatial relationships

Task 7. Find the English equivalents to the following word combinations.
1. жемчужина в короне их наук
2. достичь полноты и совершенства методологии
3. расширить диапазон геометрии
4. от метода проб и ошибок до логического вывода
5. физические объекты являются только приближениями
6. считалось идеальной парадигмой
7. внес значительный вклад в геометрию
8. были ответственны за развитие алгебраической геометрии
9. через точку вне заданной линии
10. можно провести только одну линию

Task 8. Match the terms with their translations.

№	**Term**		**Definition**
1	branch	**a**	двумерный
2	polygons	**b**	соответствие
3	shapes	**c**	цифры
4	two-dimensions	**d**	твердое тело
5	congruence	**e**	кривая линия
6	figures	**f**	формы
7	curve	**g**	уравнение
8	solid	**h**	многоугольник
9	equation	**i**	чертить
10	draw	**j**	раздел

Task 9. Mark the sentences T (true), F (false) or DS (doesn't say)
1. People always wondered about the shapes and sizes of various objects.

2. Dealing with the shape, sizes, and volumes of figures, geometry is a theoretical branch of mathematics
3. Practical branch of mathematics focuses on the study of polygons, shapes, and geometric objects in both two-dimensions and three-dimensions.
4. Congruence of objects is studied at the same time focusing on their special properties and calculation of equations.
5. The importance of geometry lies in complicated solutions and formulas.
6. Geometry arose as the field of knowledge dealing with solving generic algebraic expressions and manipulating them to arrive at results.
7. For the ancient Greek mathematicians, geometry was not so important.
8. The Arab mathematician Al-Khwarizmi expanded the range of geometry to many new kinds of figures, curves, surfaces, and solids.
9. The ancient Greek mathematicians changed methodology from trial-and-error to logical deduction.
10. The Alexandrian mathematicians made considerable contributions to geometry, trigonometry and mathematical astronomy and were responsible for the development of algebraic geometry.

Task 10. Insert the necessary word from the chart into the gaps.

trigonometry; properties; spatial relationships; analytic geometry; trial-and-error; differential geometry; the crown jewel; actual usage ; polygons, shapes, and geometric; axiomatic theory; branch; figures, curves, surfaces, and solids.

1. Then geometry is the (1) … you must explore.
2. Dealing with the shape, sizes, and volumes of figures, geometry is a practical branch of mathematics that focuses on the study of (2) … objects in both two-dimensions and three-dimensions.
3. Congruence of objects is studied at the same time focusing on their special (3) … and calculation of their area, volume, and perimeter.
4. The importance of geometry lies in its (4) … while creating objects in practical life.
5. Geometry arose as the field of knowledge dealing with (5) …. .
6. For the ancient Greek mathematicians, geometry was (6) … of their sciences, reaching a completeness and perfection of methodology that no other branch of their knowledge had attained.
7. The ancient Greek mathematicians expanded the range of geometry to many new kinds of (7) … , … , … and … .
8. The ancient Greek mathematicians changed its methodology from (8) … to logical deduction; they recognized that geometry studies “external forms”, or abstractions, of which physical objects are only approximations.
9. The ancient Greek mathematicians developed the idea of an (9) “…” which, for more than 2000 years, was regarded to be the ideal paradigm for all scientific theories.

10. The Muslim mathematicians made considerable contributions to geometry, (10) … and mathematical astronomy and were responsible for the development of algebraic geometry.
11. The 17th century was marked by the creation of (11) …, or geometry with coordinates and equations, associated with the names of Rene Descartes and Pierre de Fermat.
12. In the 18th century, (12) … appeared, which was linked with the names of L. Euler and G. Monge.

Task 11. Match the beginnings and the endings of the given sentences.

Beginnings

1. Geometry is a practical branch of mathematics that focuses on
2. Congruence of objects is studied at the same time focusing on
3. The importance of geometry lies in
4. Geometry arose as the field of knowledge dealing
5. Ancient Greek mathematicians expanded the range of geometry to
6. Ancient Greek mathematicians changed its methodology from
7. The Muslim mathematicians made considerable contributions to
8. The Muslim mathematicians were responsible for
9. The 17th century was marked by the creation of

Endings

a. its actual usage while creating objects in practical life.
b. geometry, trigonometry and mathematical astronomy.
c. trial-and-error to logical deduction.
d. the development of algebraic geometry.
e. analytic geometry, or geometry with coordinates and equations.
f. with spatial relationships.
g. many new kinds of figures, curves, surfaces, and solids.
h. their special properties and calculation of their area, volume, and perimeter.
i. the study of polygons, shapes, and geometric objects.

Task 12. In pairs, take turns to interview your partner about the future development of Geometry. What questions do you think are the most relevant?

Task 13. Retell Text A.

Task 14. Write a short essay on the suggested topics. Suggest some other relevant essay topics.

1. Rene Descartes
2. Pierre de Fermat
3. L. Euler
4. G. Monge
5. Carl Frederich Gauss
6. Janos Bolyai

7. Nikolai Ivanovich Lobachevsky

Task 15. Read the words. Try to remember the pronunciation.
1. Pierre de Fermat [pɪə de fəˈmæt] – Пьер Ферма
2. Toulouse [tuːˈluːz] – Тулуза
3. magistrate [ˈmæʤɪstreɪt] – судья
4. impact [ˈɪmpækt] – влияние
5. foreshadow [fɔːˈʃædəʊ] – предвосхищать
6. hyperbola [haɪˈpɜːbələ] – гипербола
7. parabola [pəˈræbələ] – парабола
8. spiral [ˈspaɪərəl] – спираль
9. conjecture [kənˈʤekʧə] – предположение
10. Blaise Pascal [bleɪz pəˈskæl] – Блез Паскаль
11. Diophantus [daɪəˈfæntəs] – Диофант

Task 16. Read Text B. Translate it from English into Russian.

Text B
Pierre de Fermat

Pierre de Fermat was born in Toulouse, France, on the 17th of August, 1601, and died on the 12th of January, 1665. He came from a wealthy family and studied law in Orleans. After graduating, he began to practise law. By 1652, he had become the chief magistrate of the criminal court. Magistrates in those days spent large amounts of time on their own. It was during this time that de Fermat worked in the field of mathematics. In fact, his devotion to this science was so great, that he spent as much free time as he could, working on mathematical problems and solutions. Although de Fermat published very little in his lifetime, he is still considered to be one of the greatest mathematicians of all times.

Pierre de Fermat made his greatest contribution to mathematics in number theory, and it had an important impact on the study of calculus. His works foreshadowed the later analytic geometry of Descartes and allowed him to define such important curves as hyperbola and parabola, the spiral of Fermat, and the cubic curve, known as the witch of Agnesi. In optics, Fermat formulated the principle of least time. Together with the great French mathematician and inventor of the first calculating machine Blaise Pascal, Fermat also laid the foundation of probability theory.

Fermat’s methods were so advanced that many of his results were not proved for a century after his death, and Fermat’s last theorem took more than three hundred years to prove. He made his most important conjecture in number theory while reading the *Arithmetica* by Diophantus. He stated the problem, but added that there was too little room in the margin for his proof (he used to make notes in the margin of the books he was reading). His theorem was finally proved in 1994.

Adopted from *Пушкина Е.Н. English for Mathematicians and Information Technologies Learners = Английский для студентов, изучающих математику и информационные технологии: учебно-методическое пособие [Электронный ресурс] / Е.Н. Пушкина. – Нижний Новгород, ННГУ, 2019. – 88 с.*

AFTER TEXT TASK

Task 17. Answer the questions on Text B.
1. Where and when was Pierre de Fermat born?
2. What was the social status of his family?
3. What was his qualification?
4. How did he spend his spare time working as a judge?
5. Did Fermat publish much in his lifetime?
6. What was his greatest contribution to mathematics?
7. Were Fermat's results easily proved?
8. The work of what great mathematician helped him to develop number theory?
9. Where did he use to make notes and write proofs?
10. When was his last theorem finally proved?

Part 2
CONCEPTS IN GEOMETRY

Task1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.
1. Where and when did the history of geometry begin?
2. Why did people need geometry?
3. What does geometry include today?

Task 2. Practise reading the following words.

№	Word	Transcription
1	geometry	[ʤɪˈɒmɪtrɪ]
2	engineer	[enʤɪˈnɪə]
3	architect	[ˈɑːkɪtekt]
4	Babylonia	[bæbɪˈləʊnɪə]
5	Egypt	[ˈɪʤʒɪpt]
6	branch	[brɑːnʧ]
7	Egyptian	[ɪˈʤʒɪpʃn]
8	Greek	[griːk]
9	Euclid	[ˈjuːklɪd]
10	intriguing	[ɪnˈtriːgɪŋ]

Task 3. Study and remember the following words.

№	Word	Transcription	Translation
1	dimensions	[ˌdaɪˈmenʃnz, dɪˈmenʃnz,]	размеры
2	measuring	[ˈmeʒərɪŋ]	измерение
3	property	[ˈprɒpətɪ]	собственность

4	fundamental	[fʌndəˈmentl]	основополагающий
5	mysterious	[mɪsˈtɪərɪəs]	таинственный
6	sequence	[ˈsiːkwəns]	последовательность
7	earth	[ɜːθ]	земля
8	figure	[ˈfɪgə]	фигура, рисунок, цифра
9	segment	[ˈsegmənt]	(зд.) отрезок
10	investigate	[ɪnˈvestɪgeɪt]	исследовать

Task 4. Read and translate Text A.

Text A
GEOMETRY: THE IDEA OF A POINT

Engineers, architects and people of many other professions use lines and figures in their daily work. The study of lines and closed figures made by lines is called geometry. Geometry is the branch of mathematics which investigates the relations, properties and measurements of solids, surfaces, lines and angles.

Geometry is a very old subject. It probably began in Babylonia and Egypt. Men needed practical ways for measuring their land, for building pyramids, and for defining volumes. The Egyptians were mostly concerned with applying geometry to their everyday problems. Yet, as the knowledge of Egyptians spread to Greek they found the ideas about geometry very intriguing and mysterious. The Greek began to ask “Why? Why is that true?” in 300 B. C. All the known facts about Greek geometry were put into a logical sequence by Euclid. His book called *Elements*, is one of the most famous books of mathematics. In recent years men have improved on Euclid’s work.

Today geometry includes not only the study of the shape and size of the earth and all things on it, but also the study of relations between geometric objects.

The most fundamental idea in the study of geometry is the idea of a point. We will not try to define what a point is, but instead discuss some of its properties. Think of a point as an exact location in space. You cannot see a point, feel a point, or move a point, because it has no dimensions. There are points (locations) on the earth, in the earth, on the sun, and everywhere in space. When writing about points, you represent the points by dots. Remember the dot is only a picture of a point and not the point itself. Points are commonly referred to by using capital letters. The dots below mark points and are referred to as point A, point B, and point C.

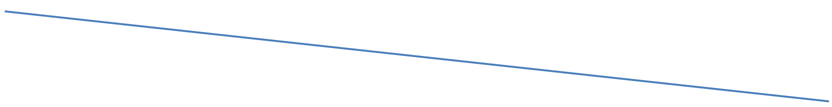
● ***B***

● ***A***

● ***C***

If you mark two points on your paper and, by using a ruler, draw a straight line between them, you will get a figure. The figure below is a picture of a line segment.

D ●



Points ***D*** and ***E*** are referred to as endpoints of the line segment. The line segment includes point ***D***, point ***E***, and all the points between them.
Imagine extending the segment indefinitely. It is impossible to draw the complete picture of such an extension but it can be represented as follows.

Let us agree on using the word line to mean a straight line. The figure above is a picture of line ***DE*** or line ***ED***.

Adopted from *Назарова Н.А. Профессиональный английский: математика и физика = Professional English in Use: Mathematics and Physics: учебное пособие / Н.А. Назарова, Е.В. Панасенко, О.М. Толстых. – Омск: Изд-во ОмГПУ, 2018. – 124 с.*

AFTER TEXT TASKS

Task 5. Answer the following questions.
1. Is geometry an old subject?
2. What is geometry?
3. Did geometry begin in England?
4. Were Egyptians mostly concerned with the practical use of geometry?
5. Did the knowledge of Egyptians spread to Greece?
6. Is Euclid's book called *Elements* famous?
7. Does geometry include only the study of the shape and size objects?
8. Is the idea of a point fundamental in geometry?
9. Can one feel, see, move or hold a point?
10. Has a point any dimensions?
11. How do we represent a point in geometry?
12. Are points represented by dots?
13. How many lines can be drawn through one point?
14. What is a segment?
15. Does a line segment include its endpoints?
16. Can you draw a straight line by using a ruler?
17. How many lines can be drawn between two points?
18. What kind of lines do you know?

Task 6. Look through the text to search for unfamiliar words and try to understand their general meanings.

Task 7. Write down the transcription and definitions of unfamiliar words, practise reading the words and try to remember them.

Task 8. Give Russian equivalents to these word combinations.

1. the branch of mathematics
2. investigates the relations, properties and measurements
3. solids, surfaces, lines and angles
4. needed practical ways
5. practical ways for measuring
6. defining volumes
7. were mostly concerned with applying geometry
8. the knowledge of Egyptians spread to Greek
9. found the ideas very intriguing and mysterious
10. were put into a logical sequence

Task 9. Find the English equivalents to the following word combinations.

1. изучение формы и размеров земли
2. изучение отношений между геометрическими объектами
3. самая фундаментальная идея
4. идея точки
5. чтобы определить, что такое точка
6. обсудить некоторые свойства
7. точное местоположение в пространстве
8. у нее нет измерений
9. везде в пространстве
10. точка – это всего лишь изображение точки

Task 10. Match the terms with their definitions.

№	**Term**		**Defenition**
1	a straight line	**a**	конечная точка
2	an extension	**b**	линейка
3	endpoint	**c**	точка
4	line segment	**d**	заглавные буквы
5	a ruler	**e**	собственность
6	capital letters	**f**	последовательность
7	dot	**g**	расширение
8	sequence	**h**	применение геометрии
9	property	**i**	прямая линия
10	applying geometry	**j**	отрезок линии

Task 11. Insert the necessary word from the chart into the gaps.

Elements; Why is that true; measuring; investigates; applying; geometry; intriguing and mysterious; sequence; Babylonia and Egypt; figures.

1. Engineers, architects and people of many other professions use lines and (1) … in their daily work.
2. The study of lines and closed figures made by lines is called (2) … .

3. Geometry is the branch of mathematics which (3) … the relations, properties and measurements of solids, surfaces, lines and angles.
4. It probably began in (4) … … … .
5. Men needed practical ways for (5) … their land, for building pyramids, and for defining volumes.
6. The Egyptians were mostly concerned with (6) … geometry to their everyday problems.
7. Yet, as the knowledge of Egyptians spread to Greek they found the ideas about geometry very (7) … … … .
8. The Greek began to ask (8) "… … … …?" in 300 B.C.
9. All the known facts about Greek geometry were put into a logical (9) ... by Euclid.
10. His book called (10) …, is one of the most famous books of mathematics.

Task 12. Match the beginnings and the endings of the given sentences.

Beginnings

1. Engineers, architects and people of many other professions use
2. The study of lines and closed figures made by lines is called
3. Geometry is the branch of mathematics which investigates
4. It probably began in
5. Men needed practical ways for
6. The Egyptians were mostly concerned with
7. Yet, as the knowledge of Egyptians spread to Greek, they found the ideas about geometry
8. The Greek began to ask "Why? Why is that true?" in
9. All the known facts about Greek geometry were put into a logical sequence
10. Today geometry includes not only the study of the shape and size of the earth and all things on it, but also

Endings

a. the relations, properties and measurements of solids, surfaces, lines and angles.
b. very intriguing and mysterious.
c. applying geometry to their everyday problems.
d. 300 B.C.
e. by Euclid.
f. Babylonia and Egypt.
g. measuring their land, for building pyramids, and for defining volumes.
h. Geometry.
i. the study of relations between geometric objects.
j. lines and figures in their daily work.

Task 13. Write out key words from the text.

Task 14. Use the key words of the text to make up the outline of the text.

Task 15. Write out the main idea of the text. Be ready to speak about it.

Task 16. Give the summary of Text A.

Task 17. In pairs, take turns to interview your partner about the most significant works in modern geometry. What questions do you think are the most relevant?

Task 18. Write a short essay on the suggested topics. Suggest some other relevant essay topics.

1. Geometry in Babylonia and Egypt.
2. Euclid and his book called *Elements*.
3. Geometry and your future profession.

Task 19. Read the words. Try to remember the pronunciation.

1. Euclid [ˈjuːklɪd] – Эвклид
2. Egypt [ˈɪʤɪpt] – Египет
3. survive [səˈvaɪv] *зд.* дойти до наших дней
4. Alexandria [ælɪgˈzɑːndrɪə] – Александрия
5. fundamental [fʌndəˈmentl] – фундаментальный (труд)
6. “Elements [ˈelɪmənts] – «Начала»
7. geometrical [ʤɪəˈmetrɪkəl] – геометрический

Task 20. Read Text B. Translate it from Russian into English.

Text B
EUCLID

Нам мало что известно о жизни Евклида. Очень немногие из его работ сохранились до наших дней. Считается, что Евклид жил в Египте примерно в 330-275 д.н.э. Когда была основана знаменитая Александрийская библиотека, его пригласили открыть математическую школу. Его самая известная книга по геометрии, которая называлась “Элементы”, была написана им между 330 и 320 годами н.э. Эта фундаментальная книга, написанная более 2000 лет назад, до сих пор считается лучшим введением в математические науки. Книга была переведена на многие языки. “Элементы” Евклида до сих пор используются в Великобритании в качестве учебника по геометрии. Говорят, что когда Евклида спросили, есть ли более простой способ овладеть геометрией, чем изучение “Элементов”, он ответил: “Нет королевской дороги к геометрии”. Помимо “Элементов”, существует коллекция его геометрических теорем “Данные”. Первое печатное издание книг Евклида появилось в 15 веке.

Adopted from *Пушкина Е.Н. English for Mathematicians and Information Technologies Learners = Английский для студентов, изучающих математику и информационные технологии: учебно-методическое пособие [Электронный ресурс] / Е.Н. Пушкина. – Нижний Новгород, ННГУ, 2019. – 88 с.*

UNIT 7. TRIGONOMETRY

"If all art aspires to the condition of music,
all the sciences aspire to the condition of mathematics".
– George Santayana, American philosopher

Part 1
HISTORY OF TRIGONOMETRY

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. Did you use to like trigonometry when you studied at school?
2. What does trigonometry deal with?
3. Do you know any prominent mathematicians who wrote works on trigonometry?

Task 2. Practise reading the following words.

№	Word	Transcription	Translation
1	trigonometry	[ˌtrɪgəˈnɒmɪtrɪ]	тригонометрия
2	triangle	[ˈtraɪæŋgl]	треугольник
3	Hipparchus	[hiˈpɑːrkəs]	Гиппарх (древнегреческий астроном и математик II века до н.э.)
4	Ptolemy	['tɒləmɪ]	Птолемей Александрийский, древнегреческий астроном и математик
5	Menelaus	[ˌmenɪ'leɪəs]	Менелай Александрийский, древнегреческий математик и астроном
6	Regiomontanus	[ˈriːdʒiəmən'teɪnəs]	Региомонтан, немецкий астроном и математик
7	John Napier	[dʒɒn ˈneɪpɪə]	Джон Непер — шотландский математик, один из изобретателей логарифмов
9	Egyptian	[ɪˈdʒɪpʃn]	египетский
9	Babylonian	[ˌbæbɪˈləʊnɪən]	вавилонский

Task 3. Study and remember the following words and word combinations.

№	Word / Expression	Translation
1	enclosed angle	здесь: заключенный между ними
2	to distinguish (from)	отличать(ся), различать(ся)
3	qualitative	качественный
4	It was not until the 16^{th} century that…	Только в 16 веке… (усилительный оборот)
5	a prelude to trigonometry	предпосылка, шаг на пути к тригонометрии

6	inscribed in a circle	вписанный в круг
7	angle that subtends	угол, который образует…
8	essence	суть, сущность
9	a consequence of the predominance of astronomy	следствие преобладания астрономии
10	Euclid's propositions for planar triangles	предложения Евклида для треугольников на плоскости
11	congruent	конгруэнтный, совпадающий
12	the law of sines	теорема синусов
13	to study thoroughly	изучить досконально
14	to facilitate	способствовать, облегчать
15	were hailed as…	были восприняты как…

Task 4. Read and translate Text A using a dictionary if necessary.

Text A
HISTORY OF TRIGONOMETRY

The word *trigonometry* comes from the Greek words *trigonon* ("triangle") and *metron* ("to measure"). Until the 16th century, trigonometry was mainly concerned with computing the numerical values of the missing parts of a triangle (or any shape that can be dissected into triangles) when the values of other parts were given. For example, if the lengths of two sides of a triangle and the measure of the enclosed angle are known, the third side and the two remaining angles can be calculated. Such calculations distinguish trigonometry from geometry, which mainly investigates qualitative relations. It was not until the 16th century that the two became separate branches of mathematics.

Several ancient civilizations — in particular, the Egyptian, Babylonian, Hindu, and Chinese — possessed a considerable knowledge of practical geometry, including some concepts that were a prelude to trigonometry.

Trigonometry in the modern sense began with the Greeks. Hipparchus (*c.* 190–120 BCE) was the first to construct a table of values for a trigonometric function. He considered every triangle (planar or spherical) as being inscribed in a circle, so that each side becomes a chord (that is, a straight line that connects two points on a curve or surface, as shown by the inscribed triangle *ABC* in the figure). To compute the various parts of the triangle, one has to find the length of each chord as a function of the central angle that subtends it—or, equivalently, the length of a chord as a function of the corresponding arc width. This became the chief task of trigonometry for the next several centuries.

The first major ancient work on trigonometry was the *Almagest* by Ptolemy (*c.* 100–170 CE). He lived in Alexandria and developed the world picture—the essence of which was a stationary Earth around which the Sun, Moon, and the five known planets move in circular orbits — for this work Ptolemy had to use some elementary trigonometry.

Until the 16th century it was chiefly spherical trigonometry that interested scholars—a consequence of the predominance of astronomy among the natural sciences. The first definition of a spherical triangle is contained in Book 1 of the *Sphaerica*, a three-book treatise by Menelaus of Alexandria (*c.* 100 CE) in which Menelaus developed the spherical equivalents of Euclid's propositions for planar triangles. There are several fundamental differences between planar and spherical triangles. For example, two spherical triangles whose angles are equal in pairs are congruent (identical in size as well as in shape), whereas they are only similar (identical in shape) for the planar case.

The doctrine of trigonometric quantities was further developed in the 9th-15th centuries in the countries of the Middle East in the works of a number of mathematicians, who not only took advantage of the achievements in this area that existed at that time, but also made their significant contribution to science.

The famous Muhammad ibn Musa al-Khwarizmi (IX century) compiled tables of sines and cotangents. Al-Habash or (Ahmed ibn Abdallah al-Marwazi) calculated tables for tangent, cotangent and cosecant.

Abu Rayhan Muhammad ibn Ahmad-al-Beruni (another transcription is Biruni (973-1048)) generalized and specified the results achieved by his predecessors in the field of trigonometry. In his work "Canon of Mas'ood" he set forth all the provisions of trigonometry known at that time and considerably supplemented them.

The first modern book devoted entirely to trigonometry appeared in the Bavarian city of Nürnberg in 1533 under the title *On Triangles of Every Kind*. Its author was the astronomer Regiomontanus (1436–76). *On Triangles* contains all the theorems needed to solve triangles, planar or spherical, although these theorems are expressed in verbal form, as symbolic algebra had yet to be invented. In particular, the law of sines is stated in essentially the modern way. *On Triangles* was greatly admired by future generations of scientists; the astronomer Nicolaus Copernicus (1473–1543) studied it thoroughly, and his annotated copy survives.

The final major development in classical trigonometry was the invention of logarithms by the Scottish mathematician John Napier in 1614. His tables of logarithms greatly facilitated the art of numerical computation—including the compilation of trigonometry tables – and were hailed as one of the greatest contributions to science.

Downloaded from *BRITANNICA. URL:*
https://www.britannica.com/science/trigonometry

AFTER TEXT TASKS

Task 5. Answer the following questions.

1. Which Greek words does the word *trigonometry* come from?
2. What was trigonometry mainly concerned with until the 16th century?
3. What distinguishes trigonometry from geometry?
4. What ancient civilizations possessed a considerable knowledge of practical geometry?
5. What ancient scientist was the first to construct a table of values for a *trigonometric function*?
6. What kind of triangle did *Hipparchus* consider?

7. Who wrote the first major ancient work on trigonometry and what was the name of this work?
8. What kind of trigonometry were scholars chiefly interested in until the 16th century?
9. In which book was the first definition of a *spherical triangle* contained?
10. What example of differences between planar and spherical triangles is given in the text?
11. When and where did the first modern book devoted entirely to trigonometry appear?
12. What did the book *On Triangles* contain?
13. Which famous astronomer admired and studied the book *On Triangles?*
14. What was the final major development in classical trigonometry?

Task 6. Give Russian equivalents to these word combinations.
1. the word *trigonometry* comes from…
2. trigonometry was mainly concerned with…
3. it was not until the 16th century that…
4. Hipparchus was the first to construct…
5. one has to find the length of each chord…
6. it was chiefly spherical trigonometry that…
7. a three-book treatise by Menelaus of Alexandria…
8. the spherical equivalents of Euclid's propositions…
9. as symbolic algebra had yet to be invented…
10. greatly facilitated the art of numerical computation…

Task 7. Find the English equivalents to the following word combinations.
1. вычисление числовых значений
2. недостающие части треугольника
3. в основном заниматься
4. длины двух сторон треугольника
5. отдельные разделы математики
6. в частности
7. обладать значительными знаниями
8. тригонометрия в современном понимании
9. таблица значений для тригонометрической функции
10. вписанный треугольник
11. длина хорды как функция угла
12. разработать картину мира
13. определение сферического треугольника
14.Два сферических треугольника, углы которых равны попарно, конгруэнтны
15. теоремы, необходимые для решения треугольников
16. большое достижение в классической тригонометрии

Task 8. Match the terms with their definitions.

№	Term		Definition
1	triangle	**a**	the space (usually measured in degrees) between two intersecting lines or surfaces at or close to the point where they meet
2	trigonometry	**b**	the branch of mathematics concerned with the properties and relations of points, lines, surfaces, solids, and higher dimensional analogues
3	angle	**c**	a straight line that connects two points on a curve or surface
4	geometry	**d**	a plane figure with three straight sides and three angles
5	chord	**e**	a triangle formed by three arcs of great circles on the surface of a sphere
6	spherical triangle	**f**	the branch of mathematics dealing with the relations of the sides and angles of triangles and with the relevant functions of any angles

Task 9. Match the beginning and the endings of the sentences.

№	Beginnings		Endings
1	The word *trigonometry* comes from	**a**	to construct a table of values for a trigonometric function.
2	The first major ancient work on trigonometry	**b**	the third side and the two remaining angles can be calculated.
3	Hipparchus was the first	**c**	is contained in Book 1 of the *Sphaerica*.
4	Ptolemy lived in Alexandria and	**d**	trigonometry that interested scholars.
5	If the lengths of two sides of a triangle and the measure of the enclosed angle are known,	**e**	the Greek words *trigonon* (“triangle”) and *metron* (“to measure”).
6	The first definition of a spherical triangle	**f**	was the *Almagest* by Ptolemy.
7	The final major development in classical trigonometry was	**g**	developed the world picture.
8	Until the 16th century it was chiefly spherical	**h**	the invention of logarithms by the Scottish mathematician John Napier.

Task 10. Insert the missing words from the chart into the gaps.

enclosed, triangle (2), chord, from, definition, concerned, distinguish, centre, inscribed, remaining, values, world picture, invention, circular, spherical, theorems

1. Until the 16th century, trigonometry was mainly (1) … with computing the numerical values of the missing parts of a (2) … .

2. If the lengths of two sides of a triangle and the measure of the (3) … angle are known, the third side and the two (4) … angles can be calculated.
3. Such calculations (5) … trigonometry (6) … geometry, which mainly investigates qualitative relations.
4. Hipparchus was the first to construct a table of (7) … for a trigonometric function.
5. He considered every triangle (planar or spherical) as being (8) … in a circle.
6.To compute the various parts of the (9) …, one has to find the length of each(10) ….
7. Ptolemy lived in Alexandria and developed the (11) … .
8. The (12) … of his world picture was a stationary Earth around which the Sun, Moon, and the five known planets move in (13) … orbits.
9. Until the 16th century it was chiefly (14)… trigonometry that interested scholars.
10. The first (15) … of a spherical triangle is contained in Book 1 of the *Sphaerica* written by Menelaus of Alexandria.
11. *On Triangles* contains all the (16) … needed to solve triangles, planar or spherical.
12. The final major development in classical trigonometry was the (17) … of logarithms by John Napier in 1614.

Task 11. Mark the sentences true (T) or false (F).
1. Until the 17^{th} century, trigonometry was mainly concerned with computing the numerical values of the missing parts of a triangle.
2. If the lengths of two sides of a triangle and the measure of the enclosed angle are known, the third side and the two remaining angles can be calculated.
3. Trigonometry in the modern sense began with the Chinese.
4. Hipparchus was the first to construct a table of values for a trigonometric function.
5. He considered every triangle (planar or spherical) as being inscribed in an ellipse, so that each side becomes a chord.
6. The first major ancient work on trigonometry was the *Almagest* by Ptolemy.
7. Ptolemy developed the world picture – the essence of which was a stationary Sun around which the Earth, Moon, and the five known planets move in circular orbits.
8. The first definition of a spherical triangle is contained in Book 1 of the *Sphaerica* by Menelaus of Alexandria.
9. There are no differences between planar and spherical triangles.
10. The first book devoted entirely to trigonometry appeared in Nürnberg in 1533 under the title *On Triangles* of Every Kind.
11. On Triangles was strongly criticized by future generations of scientists.
12. One of the major developments in classical trigonometry was the invention of logarithms by John Napier in 1614.

Task 12. Look through the text again. Make up a plan to the text.

Task 13. Render Text A according to the plan using mathematical terms.

Task 14. Translate the sentences from Russian into English.

1. До 16 века тригонометрия в основном занималась вычислением числовых значений недостающих частей треугольника, когда были даны значения других частей.
2. Такие вычисления отличают тригонометрию от геометрии, которая в основном исследует качественные отношения.
3. Несколько древних цивилизаций, в частности египетская, вавилонская, индийская и китайская, обладали значительными знаниями практической геометрии.
4. Тригонометрия в современном понимании началась с трудов древнегреческих ученых.
5. Гиппарх был первым, кто построил таблицу значений для тригонометрической функции.
6. Чтобы вычислить различные части треугольника, нужно найти длину каждой хорды как функцию центрального угла, который его образует.
7. Первым крупным древним трудом по тригонометрии был «*Альмагест»* Птолемея.
8. Птолемей разработал картину мира, центром которой была неподвижная Земля, вокруг которой Солнце, Луна и пять известных планет двигались по круговым орбитам.
9. До 16 века ученых интересовала в основном сферическая тригонометрия.
10. Первое определение сферического треугольника содержится в книге *Sphaerica*, написанной Менелаем Александрийским примерно в 100 г. н.э.
11. В своем трактате Менелай разработал сферические эквиваленты утверждений Евклида для плоских треугольников.
12. Первая книга, полностью посвященная тригонометрии, была написана Региомонтаном и была издана в Нюрнберге в 1533 году под названием «О треугольниках всех видов».
13. В «Треугольниках» содержатся все теоремы, необходимые для решения треугольников.
14. Последним крупным достижением в классической тригонометрии было изобретение логарифмов шотландским математиком Джоном Напиером в 1614 году.
15. Его таблицы логарифмов значительно облегчили искусство вычислений и были признаны одним из величайших вкладов в науку.

Task 15. Read the words. Try to remember the pronunciation.

1. Pythagoras [paɪ ˈθægərəs] – Пифагор
2. hypotenuse [haɪˈpɒtənjuːz] – гипотенуза
3. most commonly quoted [məʊst ˈkɒmənlɪ ˈkwəʊtɪd] – чаще всего цитируемый
4. Pythagorean triples [pæɪˌθægəˈriːən ˈtrɪpəlz] – пифагоровы тройки
5. a multiple of [ə ˈmʌltɪpl əv] – кратное (какого-л.числа)
6. well before [wel bɪˈfɔː] – задолго до чего-л.
7. merely[ˈmɪəlɪ] – только, просто

8. achievement [əˈtʃiːvmənt] – достижение
9. equation [ɪˈkweɪʒn] – уравнение
10. investigations [ɪnˌvestɪˈgeɪʃənz] – исследования

Task 16. Read Text B. Translate it from English into Russian.

Text B

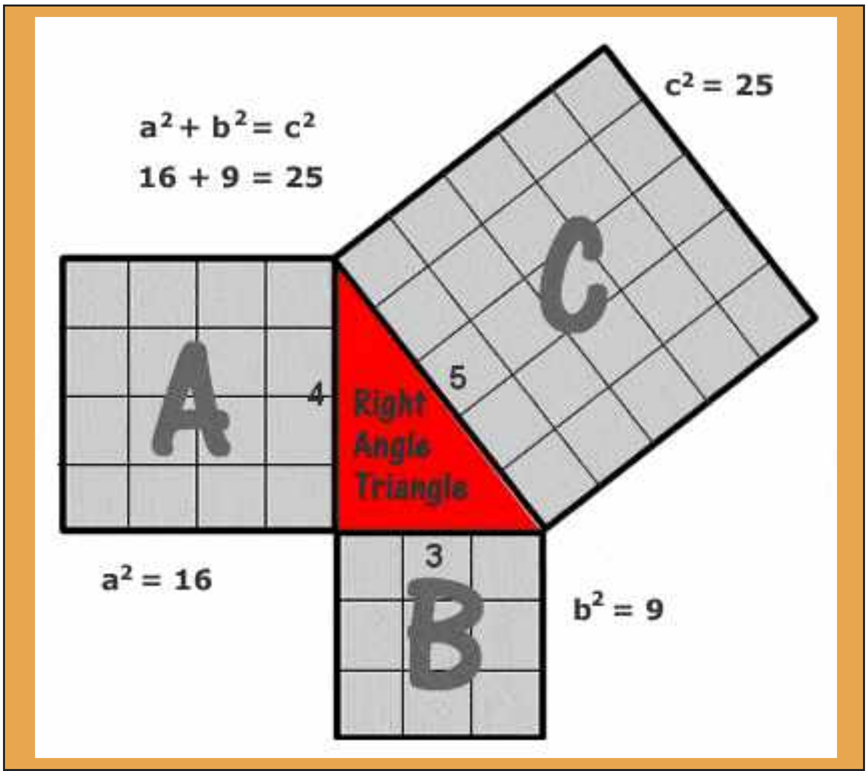
THE PYTHAGOREAN THEOREM

Pythagoras is mainly remembered for what has become known as **Pythagoras' Theorem (or the Pythagorean Theorem)**: that, for any right-angled triangle, the square of the length of the hypotenuse (the longest side, opposite the right angle) is equal to the sum of the square of the other two sides (or "legs").

Written as an equation: $a^2 + b^2 = c^2$.

What Pythagoras and his followers did not realize is that this also works for any shape: thus, the area of a pentagon on the hypotenuse is equal to the sum of the pentagons on the other two sides, as it does for a semi-circle or any other regular (or even irregular) shape.

The simplest and most commonly quoted example of a Pythagorean triangle is one with sides of 3, 4 and 5 units ($3^2 + 4^2 = 5^2$), as can be seen by drawing a grid of unit squares on each side as in the diagram at right), but there are a potentially infinite number of other integer "**Pythagorean triples**", starting with (5, 12 13), (6, 8, 10), (7, 24, 25), (8, 15, 17), (9, 40, 41), etc. It should be noted, however that (6, 8, 10) is not what is known as a "primitive" Pythagorean triple, because it is just a multiple of (3, 4, 5).



Pythagoras' (Pythagorean) Theorem

Pythagoras' Theorem and the properties of right-angled triangles seems to be the most ancient and widespread mathematical development after basic arithmetic and geometry, and it was touched on in some of the most ancient mathematical texts from **Babylon** and **Egypt**, dating from over a thousand years earlier. One of the simplest proofs comes from ancient **China**, and probably dates from well before Pythagoras' birth. It was Pythagoras, though, who gave the theorem its definitive form, although it is not clear whether Pythagoras himself definitively proved it or merely described it. Either way, it has become one of the best-known of all mathematical theorems, and as many as 400 different proofs now exist, some geometrical, some algebraic, some involving advanced differential equations, etc.

Among his other achievements in geometry, Pythagoras (or at least his followers, the Pythagoreans) also realized that the sum of the angles of a triangle is equal to two right angles (180°), and probably also the generalization which states that the sum of the interior angles of a polygon with n sides is equal to $(2n – 4)$ right angles, and that the sum of its exterior angles equals 4 right angles. They were able to construct figures of a given area, and to use simple geometrical algebra, for example to solve equations such as $a(a – x) = x^2$ by geometrical means.

The Pythagoreans also established the foundations of number theory, with their investigations of triangular, square and also perfect numbers (numbers that are the sum of their divisors). They discovered several new properties of square numbers, such as that the square of a number n is equal to the sum of the first n odd numbers (e.g. $4^2 = 16 = 1 + 3 + 5 + 7$).

AFTER TEXT TASKS

Task 17. Answer the questions on Text B.

1. What is Pythagoras mainly remembered for?
2. What didn't Pythagoras and his followers realize?
3. What is the simplest and most commonly quoted example of a Pythagorean triangle?
4. Where does one of the simplest proofs of a Pythagorean theorem come from?
5. Is it clear whether Pythagoras himself definitively proved it?
6. How many proofs of the theorem exist now?
7. What were Pythagoras's other achievements in geometry?
8. The foundations of what theory did Pythagoreans establish?
9. What several new properties of square numbers did Pythagoreans discover?

Task 18. Write a short essay on the suggested topics. Suggest some other relevant essay topics.

1. Pythagoras's contribution to geometry.
2. The most popular proofs of the Pythagorean theory.
3. Pythagoras's contribution to number theory.

Part 2
TRIGONOMETRY: MAIN CONCEPTS

Task 1. In pairs, discuss the following questions. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. Do you agree with the opinion about trigonometry as a boring and incomprehensible section of the school mathematics course?
2. Which spheres of our life is trigonometry often used in?
3. Why do we need to know trigonometry in everyday life?

Task 2. Practise reading the following words.

№	Word	Transcription	Translation
1	ratio	[ˈreɪʃɪəʊ]	соотношение
2	right-angled	[ˈraɪt æŋgəld]	прямоугольный
3	perpendicular	[ˌpɜːrpənˈdɪkjələr]	перпендикулярный
4	hypotenuse	[haɪˈpɒtənjuːz]	гипотенуза
5	sine	[saɪn]	синус
6	cosine	[ˈkəʊsaɪn]	косинус
7	tangent	[ˈtændʒənt]	тангенс
8	conangent	[kəʊˈtændʒənt]	котангенс
9	identity	[aɪˈdentɪtɪ]	тождество
10	adjacent	[əˈdʒeɪsənt]	смежный, прилежащий

Task 3. Study the following words and expressions.

№	Word / Expression	Transcription	Translation
1	be considered as	[biː kənˈsɪdəd əz]	считаться, рассматриваться
2	trigonometry is about	[ˌtrɪgəˈnɒmɪtrɪ ɪz əˈbaʊt]	тригонометрия связана с…
3	more specifically	[mɔː spəˈsɪfɪkəlɪ]	точнее говоря, говоря конкретнее
4	base	[beɪs]	основание (треугольника)
5	measure of an angle	[ˈmeʒə əv æn æŋgl]	величина угла
6	be derived	[bɪ dɪˈraɪvd]	быть полученным
7	can be observed	[kən bɪ əbˈzɜːvd]	можно наблюдать, увидеть
8	in terms of trigonometric ratios	[ɪn tɜːmz əv ˌtrɪgəˈnɒmətrɪk ˈreɪʃɪəʊz]	с точки зрения тригонометрических соотношений
9	interrelated	[ˌɪntərɪˈleɪtɪd]	взаимосвязанный
10	standard angle values	[ˈstændəd æŋgl ˈvæljuːz]	стандартные значения углов
11	refer to	[rɪˈfɜː tuː]	ссылаться (на)
12	trigonometric	[trɪgəˈnɒmətrɪk	тригонометрические

	identities	aɪˈdentɪtɪz]	тождества
13	angles involved	[ˈæŋgəlz ɪnˈvɒlvd]	указанные углы
14	throughout history	[θruːˈaʊt ˈhɪstrɪ]	на протяжении всей истории
15	celestial mechanics	[sɪˈlestɪəl mɪˈkanɪks]	небесная механика

Task 4. Read and translate Text A using the dictionary if necessary.

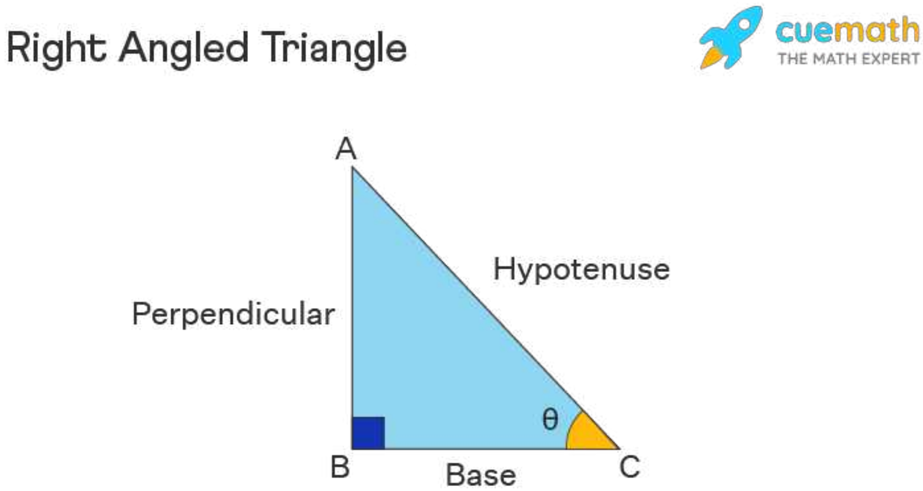
Text A

TRIGONOMETRY: MAIN CONCEPTS

Introduction to Trigonometry

Trigonometry is a branch of mathematics which is considered as a part of geometry. The word itself comes from the Greek trigōnon (which means "triangle") and metron ("measure"). As the name suggests, trigonometry deals mostly with angles and triangles; in particular, it's defining and using the relationships and ratios between angles and sides in triangles. More specifically, trigonometry is about right-angled triangles, where one of the internal angles is 90°. Trigonometry is a system that helps us to work out missing or unknown side lengths or angles in a triangle.

A right-angled triangle has a single right angle. By definition, that means that all sides cannot be the same length. A typical right-angled triangle is shown below.



Important Terms for Right-Angled Triangles

In a right-angled triangle, we have the following three sides.

Perpendicular - It is the side opposite to the angle θ.

Base - This is the adjacent side to the angle θ.

Hypotenuse - This is the side opposite to the right angle.

The side opposite θ is called the **opposite**.

The side next to θ which is not the hypotenuse is called **the base.**

Introducing Sine, Cosine and Tangent

Trigonometry especially deals with the **ratios of sides in a right triangle**, which can be used to determine the measure of an angle. These ratios are called trigonometric functions.

The three basic functions in trigonometry are **sine, cosine and tangent**. Based on these three functions the other three functions that are cotangent, secant and cosecant are derived. All the trigonometrical concepts are based on these functions.

Trigonometric Ratios

There are basic six ratios in trigonometry that help in establishing a relationship between the ratio of sides of a right triangle with the angle. If θ is the angle in a right-angled triangle, formed between the base and hypotenuse, then

- sin θ = Perpendicular/Hypotenuse
- cos θ = Base/Hypotenuse
- tan θ = Perpendicular/Base

The value of the other three functions: cot, sec, and cosec depend on tan, cos, and sin respectively as given below.

- cot θ = 1/tan θ = Base/Perpendicular
- sec θ = 1/cos θ = Hypotenuse/Base
- cosec θ = 1/sin θ = Hypotenuse/Perpendicular

Important Trigonometric Angles

Trigonometric angles are the angles in a right-angled triangle using which different trigonometric functions can be represented. Some standard angles used in trigonometry are 0º, 30º, 45º, 60º, 90º. The trigonometric values for these angles can be observed directly in a trigonometric table. Some other important angles in trigonometry are 180º, 270º, and 360º. Trigonometry angle can be expressed in terms of trigonometric ratios as,

- θ = $\sin^{-1}$ (Perpendicular/Hypotenuse)
- θ = $\cos^{-1}$ (Base/Hypotenuse)
- θ = $\tan^{-1}$ (Perpendicular/Base)

Trigonometric Table

The trigonometric table is made up of trigonometric ratios that are interrelated to each other – sine, cosine, tangent, cosecant, secant, cotangent. These ratios, in short, are written as sin, cos, tan, cosec, sec, cot, and are taken for standard angle values. You can refer to the trigonometric table chart to know more about these ratios.

Trigonometric Table

cuemath THE MATH EXPERT

θ	0^0	30^0	45^0	60^0	90^0	180^0	270^0	360^0
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined	0	Not Defined	0
cosec θ	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not Defined	-1	Not Defined
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined	-1	Not Defined	1
cot θ	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not Defined	0	Not Defined

Trigonometry Identities

In Trigonometric Identities, an equation is called an identity when it is true for all values of the variables involved. Similarly, an equation involving trigonometric ratios of an angle is called a trigonometric identity, if it is true for all values of the angles involved. In trigonometric identities, you will get to learn more about the Sum and Difference Identities.

For example, sin θ/cos θ = [Opposite/Hypotenuse] ÷ [Adjacent/Hypotenuse] = Opposite/Adjacent = tan θ

Therefore, tan θ = sin θ/cos θ is a trigonometric identity. The three important trigonometric identities are:

- $\sin^2\theta + \cos^2\theta = 1$
- $\tan^2\theta + 1 = \sec^2\theta$
- $\cot^2\theta + 1 = \text{cosec}^2\theta$

Applications of Trigonometry

Throughout history, trigonometry has been applied in areas such as architecture, celestial mechanics, surveying, etc. Its applications include in:

- Various fields like oceanography, seismology, meteorology, physical sciences, astronomy, acoustics, navigation, electronics, and many more.
- It is also helpful to find the distance of long rivers, measure the height of the mountain, etc.
- Spherical trigonometry has been used for locating solar, lunar, and stellar positions.

AFTER TEXT TASKS

Task 5. Answer the following questions.

1. What does trigonometry deal with?
2. What does trigonometry help us to do?
3. What do we call the sides of a typical right-angled triangle?
4. What are trigonometric functions used for?
5. What are some standard angles used in trigonometry?
6. What is the trigonometric table made up of?
7. What kind of equation is called an identity?
8. What are the three important trigonometric identities?
9. In which areas has trigonometry been applied throughout history?
10. What fields do applications of trigonometry include?

Task 6. Give Russian equivalents to these word combinations.

1. to come from
2. as the name suggests
3. the relationships and ratios between angles and sides in triangles
4. trigonometry is about right-angled triangles
5. to work out missing or unknown side lengths
6. the side next to θ which is not the hypotenuse
7. deals with the ratios of sides in a right triangle
8. establishing a relationship between the ratio of sides of a right triangle
9. trigonometric ratios that are interrelated to each other
10. refer to the trigonometric table chart
11. similarly
12. an equation involving trigonometric ratios of an angle
13. various fields like
14. locating solar, lunar, and stellar positions

Task 7. Find the English equivalents to the following word combinations.

1. раздел математики
2. тригонометрия имеет дело с углами и треугольниками
3. определить неизвестные длины сторон или углы в треугольнике
4. сторона, противоположная углу θ
5. сторона, прилегающая к углу θ
6. синус, косинус и тангенс
7. котангенс, секанс и косеканс
8. зависят от tan, cos и sin соответственно, как указано ниже
9. выражен в терминах тригонометрических соотношений
10. тригонометрическое тождество
11. на протяжении всей истории
12. измерить высоту горы
13. небесная механика
14. определить положение Солнца, Луны и звезд

Task 8. Match the terms with their translation.

№	Term		Translation
1	adjacent	**a**	основание
2	relationships	**b**	гипотенуза
3	the base	**c**	прямоугольный
4	cosine	**d**	смежный, прилегающий
5	hypotenuse	**e**	1) касательная 2) тангенс
6	right-angled	**f**	взаимоотношения
7	ratio	**g**	тождество
8	tangent	**h**	косинус
9	applications	**i**	отношение, соотношение
10	identity	**j**	применение

Task 9. Mark the following sentences true (T) or false (F).

1. Peppendicular is the adjacent side to the angle θ.
2. Hypotenuse is the side opposite to the right angle.
3. The base is the side opposite to θ.
4. Trigonometry especially deals with the **ratios of sides in a right triangle.**
5. There are two basic functions in trigonometry, they are **sine and cosine.**
6. There are six basic ratios in trigonometry that help in establishing a relationship between the ratio of sides of a right triangle.
7. Some standard angles used in trigonometry are 0º, 30º, 55º, 60º and 90º.
8. The trigonometric table is made up of trigonometric ratios that are interrelated to each other.
9. An equation is called an identity when it is true for most values of the variables involved.
10. Throughout history, trigonometry has been applied in many areas such as architecture, celestial mechanics, surveying, etc.

Task 10. Match the beginnings and the endings of the given sentences.

№	Beginnings		Endings
1	Trigonometry especially deals with the ratios of sides in a right triangle,	**a**	helps us to work out missing or unknown side lengths or angles in a triangle.
2	There are basic six ratios in trigonometry that help in	**b**	using which different trigonometric functions can be represented.
3	Trigonometry is a system that	**c**	the Greek trigōnon (which means "triangle") and metron ("measure").
4	Trigonometric angles are the angles in a right-angled triangle	**d**	which can be used to determine the measure of an angle.
5	The trigonometric table is made up of trigonometric ratios that are interrelated to each other –	**e**	establishing a relationship between the ratio of sides of a right triangle with the angle.

6	The word 'trigonometry' comes from	**f**	locating solar, lunar, and stellar positions.
7	Spherical trigonometry has been used for	**g**	sine, cosine, tangent, cosecant, secant, cotangent.

Task 11. Render Text A.

Task 12. Write a short essay on the suggested topics. The volume of the essay is 200-250 words. Suggest some other relevant essay topics.

1. Important Trigonometric Angles.
2. The History of Trigonometry.
3. Applications of Trigonometry.

Task 13. Read the words. Try to remember the pronunciation.

1. Sumerian [sʊˈmɪərɪən] – шумер, шумерский
2. Hellenistic [helɪˈnɪstɪk] – эллинистический
3. Ptolemy [ˈtɒləmɪ] – Птолемей
4. treatise [ˈtriːtɪs] – трактат, научный труд
5. throughout [θruːˈaʊt] – на протяжении
6. *Surya Siddhanta* [ˈsʊrɪə sɪˈdentə] – "Сурья-сиддханта" (тексты по ведической астрономии)
7. Aryabhata [ˈarɪəˈbetə] – Ариябхата, индийский астроном и математик
8. Nasir al-Din al-Tusi [nəˈsɪr əlˈdɪn əlˈtusɪ] – Насер ад-Дин Туси, персидский математик, механик и астроном XIII века
9. *De Triangulis* [di: traɪˈæŋgjulɪs] – "О треугольниках", книга средневекового математика Региомантана
10. *De revolutionibus orbium coelestium* [də ˌrevəˈlusɪɒnɪəs ˈɒrbɪəm səˈlestɪəm] – "О вращении небесных тел", книга Коперника

Task 14. Read Text B. Translate it from English into Russian.

Text B
HISTORY OF TRIGONOMETRY

Sumerian astronomers studied angle measure, using a division of circles into 360 degrees. They, and later the Babylonians, studied the ratios of the sides of similar triangles and discovered some properties of these ratios but did not turn that into a systematic method for finding sides and angles of triangles. The ancient Nubians used a similar method.

In the 3rd century BC, Hellenistic mathematicians such as Euclid and Archimedes studied the properties of chords and inscribed angles in circles, and they proved theorems that are equivalent to modern trigonometric formulae, although they presented them geometrically rather than algebraically. In 140 BC, Hipparchus (from Nicaea, Asia Minor) gave the first tables of chords, analogous to modern tables of sine values, and used them to solve problems in trigonometry and spherical trigonometry. In the 2nd century AD, the Greco-Egyptian astronomer Ptolemy (from Alexandria, Egypt) constructed detailed trigonometric tables (Ptolemy's table of chords) in Book 1, chapter 11 of his *Almagest*. Ptolemy used chord length to define his trigonometric functions, a minor difference from the sine convention we use today. (The value we call sin (θ) can be found by looking up the chord length for twice the angle of interest (2θ) in Ptolemy's table, and then dividing that value by two.) Centuries passed before more detailed tables were produced, and Ptolemy's treatise remained in use for performing trigonometric calculations in astronomy throughout the next 1200 years in the medieval Byzantine, Islamic, and, later, Western European worlds.

The modern sine convention is first attested in the *Surya Siddhanta*, and its properties were further documented by the 5th century (AD) Indian mathematician and astronomer Aryabhata. These Greek and Indian works were translated and expanded by medieval Islamic mathematicians. By the 10^{th} century, Islamic mathematicians were using all six trigonometric functions, had tabulated their values, and were applying them to problems in spherical geometry. The Persian polymath Nasir al-Din al-Tusi has been described as the creator of trigonometry as a mathematical discipline in its own right. Nasīr al-Dīn al-Tūsī was the first to treat trigonometry as a mathematical discipline independent from astronomy, and he developed spherical trigonometry into its present form. He listed the six distinct cases of a right-angled triangle in spherical trigonometry, and in his *On the Sector Figure*, he stated the law of sines for plane and spherical triangles, discovered the law of tangents for spherical triangles, and provided proofs for both these laws. Knowledge of trigonometric functions and methods reached Western Europe via Latin translations of Ptolemy's Greek *Almagest* as well as the works of Persian and Arab astronomers such as Al Battani and Nasir al-Din al-Tusi.

One of the earliest works on trigonometry by a northern European mathematician is *De Triangulis* by the 15^{th} century German mathematician Regiomontanus, who was encouraged to write, and provided with a copy of the *Almagest*, by the Byzantine Greek scholar cardinal Basilios Bessarion with whom he lived for several years. At the same time, another translation of the *Almagest* from Greek into Latin was completed by the Cretan George of Trebizond. Trigonometry was still so little known in 16th-century northern Europe that Nicolaus Copernicus devoted two chapters of *De revolutionibus orbium coelestium* to explain its basic concepts.

Driven by the demands of navigation and the growing need for accurate maps of large geographic areas, trigonometry grew into a major branch of mathematics.

AFTER TEXT TASKS

Task 15. Answer the questions on Text B.

1. What did Sumerian astronomers study?
2. What properties did Babylonians discover?
3. What theorems did Hellenistic mathematicians prove in the 3rd century BC?

4. When did Hipparchus give the first tables of chords?
5. Who constructed detailed trigonometric tables of chords in 140 BC?
6. What did Ptolemy from Alexandria construct in the 2 century AD?
7. What was the Indian mathematician and astronomer Aryabhata achievement in the 5^{th} century AD?
8. What is The Persian polymath Nasir al-Din al-Tusi famous for?
9. How did knowledge of trigonometric functions and methods reach Western Europe?
10. Who was the author of the work called *De Triangulis*?
11. Why did Nicolaus Copernicus devote two chapters of *De revolutionibus orbium coelestium* to explain the basic concepts of trigonometry?
12. What were the reasons for the rapid development of trigonometry?

Task 16. Make up a plan to the text.

Task 17. Write a short summary of the text in English.

UNIT 8. MATHEMATICAL ANALYSIS

Mathematics is the gate and key to science.
– Roger Bacon

Part 1
INTRODUCTION TO MATHEMATICAL ANALYSIS

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. What main problems does Mathematical analysis deal with?
2. What sphere of science did analysis evolve from?
3. In what way can analysis be distinguished from geometry and in what way can it be applied to a topological space and to a metric space?

Task 2. Practise reading the following words.

№	Word	Transcription
1	differentiation	[dɪfərenʃɪˈeɪʃn]
2	integration	[ɪntɪˈgreɪʃn]
3	measure	[ˈmeʒə]
4	infinite series	[ˈɪnfɪnɪt ˈsɪəriːz]
5	calculus	[ˈkælkjʊləs]
6	mathematical object	[mæθɪˈmætɪkəl ˈɒbʤɛkt]
7	topological space	[tɒpəˈlɒʤɪk(ə)l speɪs]
8	method of exhaustion	[ˈmeθəd ɒv ɪgˈzɔːsʧən]
9	regular polygon	[ˈregjʊlə ˈpɒlɪgən]
10	power series	[ˈpaʊə ˈsɪəriːz]

Task 3. Study and remember the following words and expressions.

№	Word / Expression	Transcription	Translation
1	infinitesimal	[ɪnfɪnɪ'tesɪm(ə)l]	бесконечно малый
2	sine	[saɪn]	синус
3	cosine	['kəusaɪn]	косинус
4	tangent	['tænʤ(ə)nt]	тангенс
5	arctangent	[əˈstænʤənt]	арктангенс
6	derivative	[dɪˈrɪvətɪv]	производная, производная функция
7	generating function	[ˈʤenəreɪtɪŋ ˈfʌŋkʃn]	фундаментальная последовательность (Коши)
8	discontinuities	[dɪskəntɪnˈjʊɪtɪz]	нарушение последовательности; прерывность
9	a complete set	[kəmˈpliːt set]	полное множество
10	real line	[ˈrɪəl ˈlaɪn]	вещественная прямая (ось)
11	discontinuities of	[dɪskəntɪnˈjʊɪtɪz əv	разрыв непревывности

	real functions	rɪəl fʌŋkʃnz]	вещественных функций
12	space-filling curve	[speɪs-ˈfɪlɪŋ kɜːv]	заполняющая пространство кривая
13	naive set theory	[nɑːˈiːv set ˈθɪərɪ]	наи́вная тео́рия мно́жеств (раздел математики, в котором изучаются общие свойства множеств)
14	normed vector space	[nɔːmd ˈvektə speɪs]	нормированное векторное пространство
15	metric space	[ˈmetrɪk speɪs]	метрическое пространство

Task 4. Read and translate Text A using a dictionary if necessary.

Text A

INTRODUCTION TO MATHEMATICAL ANALYSIS

Mathematical analysis is a branch of mathematics that includes the theories of differentiation, integration, measure, limits, infinite series, and analytic function.

These theories are usually studied in the context of real and complex numbers and functions. Analysis evolved from calculus, which involves the elementary concepts and techniques of analysis. Analysis may be distinguished from geometry; however, it can be applied to any space of mathematical objects that has a definition of nearness (a topological space) or specific distances between objects (a metric space).

History

Archimedes used the method of exhaustion to compute the area inside a circle by finding the area of regular polygons with more and more sides. This was an early but informal example of a limit, one of the most basic concepts in mathematical analysis.

Mathematical analysis formally developed in the 17^{th} century during the Scientific Revolution, but many of its ideas can be traced back to earlier mathematicians. Early results in analysis were implicitly present in the early days of ancient Greek mathematics. For instance, an infinite geometric sum is implicit in Zeno's paradox of the dichotomy.

Later, Greek mathematicians such as Eudoxus and Archimedes made more explicit, but informal, use of the concepts of limits and convergence when they used the method of exhaustion to compute the area and volume of regions and solids. The explicit use of infinitesimals appears in Archimedes' *The Method of Mechanical Theorems*, a work rediscovered in the 20^{th} century. In Asia, the Chinese mathematician Liu Hui used the method of exhaustion in the 3rd century AD to find the area of a circle. Zu Chongzhi established a method that would later be called Cavalieri's principle to find the volume of a sphere in the 5^{th} century. The Indian mathematician Bhāskara II gave examples of the derivative and used what is now known as Rolle's theorem in the 12^{th} century.

In the 14^{th} century, Madhava of Sangamagrama developed infinite series expansions, like the power series and the Taylor series, of functions such as sine, cosine, tangent and arctangent. Alongside his development of the Taylor series of the trigonometric functions, he also estimated the magnitude of the error terms created by truncating these series and gave a rational approximation of an infinite series. His followers at the Kerala school of astronomy and mathematics further expanded his works, up to the 16th century.

The modern foundations of mathematical analysis were established in 17th century Europe. Descartes and Fermat independently developed analytic geometry, and a few decades later Newton and Leibniz independently developed infinitesimal calculus, which grew, with the stimulus of applied work that continued through the 18th century, into analysis topics such as the calculus of variations, ordinary and partial differential equations, Fourier analysis, and generating functions. During this period, calculus techniques were applied to approximate discrete problems by continuous ones.

In the 18th century, Euler introduced the notion of mathematical function. Real analysis began to emerge as an independent subject when Bernard Bolzano introduced the modern definition of continuity in 1816, but Bolzano's work did not become widely known until the 1870s. In 1821, Cauchy began to put calculus on a firm logical foundation by rejecting the principle of the generality of algebra widely used in earlier work, particularly by Euler. Instead, Cauchy formulated calculus in terms of geometric ideas and infinitesimals. Thus, his definition of continuity required an infinitesimal change in x to correspond to an infinitesimal change in y. He also introduced the concept of the Cauchy sequence, and started the formal theory of complex analysis. Poisson, Liouville, Fourier and others studied partial differential equations and harmonic analysis. The contributions of these mathematicians and others, such as Weierstrass, developed the (ε, δ) - definition of limit approach, thus founding the modern field of mathematical analysis.

In the middle of the 19th century Riemann introduced his theory of integration. The last third of the century saw the arithmetization of analysis by Weierstrass, who thought that geometric reasoning was inherently misleading, and introduced the "epsilon-delta" definition of limit. Then, mathematicians started worrying that they were assuming the existence of a continuum of real numbers without proof. Dedekind then constructed the real numbers by Dedekind cuts, in which irrational numbers are formally defined, which serve to fill the "gaps" between rational numbers, thereby creating a complete set: the continuum of real numbers, which had already been developed by Simon Stevin in terms of decimal expansions. Around that time, the attempts to refine the theorems of Riemann integration led to the study of the "size" of the set of discontinuities of real functions.

Also, "monsters" (nowhere continuous functions, continuous but nowhere differentiable functions, space-filling curves) began to be investigated. In this context, Jordan developed his theory of measure, Cantor developed what is now called naive set theory, and Baire proved the Baire category theorem. In the early 20th century, calculus was formalized using an axiomatic set theory. Lebesgue solved the problem of measure, and Hilbert introduced Hilbert spaces to solve integral equations. The idea of normed vector space was in the air, and in the 1920s Banach created functional analysis.

Important concepts. Metric spaces

In mathematics, a metric space is a set where a notion of distance (called a metric) between elements of the set is defined.

Much of analysis happens in some metric space; the most commonly used are the real line, the complex plane, Euclidean space, other vector spaces, and the integers. Examples of analysis without a metric include measure theory (which describes size rather than distance) and functional analysis (which studies topological vector spaces that need not have any sense of distance).

Formally, A metric space is an ordered pair where is a set and is a metric on, i.e., a function such that for any x, y, x ∈, the following holds:

$d(x,y) = 0$ iff $x = y$ (*identity of indiscernible*),

$d(x,y) = d(y,x)$ (*symmetry*) and

$d(x,z) \leq d(x,y) + d(y, z)$ (*triangle inequality*) .

By taking the third property and letting $z = x$, it can be shown that

$d(x,y) \geq 0$ (*non-negative*).

Sequences and limits

A **sequence** is an ordered list. Like a set, it contains members (also called *elements*, or *terms*). Unlike a set, order matters, and exactly the same elements can appear multiple times at different positions in the sequence. Most precisely, a sequence can be defined as a function whose domain is a countable totally ordered set, such as the natural numbers.

One of the most important properties of a sequence is *convergence*. Informally, a sequence converges if it has a *limit*. Continuing informally, a (singly-infinite) sequence has a limit if it approaches some point *x*, called the limit, as *n* becomes very large. That is, for an abstract sequence (*an*) (with *n* running from 1 to infinity understood) the distance between *an* and *x* approaches 0 as $n \to \infty$, denoted

$$\lim_{n\to\infty} a_n = x$$

Adopted from *Румянцева О.А. Англійська мова для математиків (інтенсивний курс для студентів математичних спеціальностей Інституту математики, економіки і механіки) = English for mathematicians (the intensive course for the students-mathematicians of The Institute of Mathematics, Economics and Mechanics) / О.А. Румянцева. ОНУ імені І.І. Мечникова. – Одеса, 2015. – 145 с.*

AFTER TEXT TASKS

Task 5. Answer the following questions.

1. Name the ancient Greek mathematicians:

– who described the method of exhaustion?

– who represented paradox of the dichotomy?

– who used the concepts of limits and convergence?

2. What outstanding discoveries in maths were made by Chinese and Indian scientists?

3. Describe the most prominent discoveries of mathematicians in times past and conclude about their influence on current conceptions in Mathematical analysis?
4. When were established the modern foundations of mathematical analysis?
5. What European mathematicians developed such branches as analytic geometry, infinitesimal calculus?
6. Give the interpretation and definition to the following notions: the calculus of variations, ordinary and partial differential equations, Fourier analysis and generating functions.
7. Name the scientists who introduced the notion of mathematical function, differential equations and harmonic analysis. Expand upon the essence of these mathematical discoveries.
8. Give the determinations (in your own words) to the following notions: sequence, limit, complete set, metric spaces.
9. Formulate the Rolle's theorem and Cavalieri's principle in modern interpretation.

Task 6. Give Russian equivalents to these word combinations.
1. metric
2. distance
3. complex plane
4. Euclidean space
5. vector spaces
6. complex plane
7. ordered pair
8. Cavalieri’s principle (method of indivisibles)
9. harmonic analysis
10. (ε, δ) - definition of limit
11. theory of integration
12. continuum of real numbers
13. Dedekind cuts
14. Zeno's paradox of the dichotomy

Task 7. Find the English equivalents to the following word combinations.
1. функция
2. тогда и только тогда
3. аксиома треугольника, неравенство треугольника
4. вполне упорядоченный
5. исчисляемый
6. теория мер (в математическом анализе мера Жордана используется для построения интеграла Римана)
7. представление [многоразрядного числа или дроби] в десятичной форме
8. Интегра́л Ри́мана
9. лимит, предел
10. всюду разрывная функция
11. функция Вейерштрасса

12. вариационное исчисление
13. гармонический анализ, Фурье-анализ
14. порождающая функция, производящая функция
15. теория комплексного анализа
16. ряды Те́йлора, разложение функции в бесконечную сумму степенных функций
17. разложение бесконечных рядов
18. степенной ряд
19. Теорема Ро́лля (теорема о нуле производной)
20. бесконечно малая величина
21. анализ бесконечно малых величин

Task 8. Match the terms with their definitions.

№	**Term**		**Definition**
1	sine	**a**	бесконечно малая величина
2	calculus of variations	**b**	косинус
3	cosine	**c**	производная, производная функция
4	generating function	**d**	тангенс
5	tangent	**e**	порождающая функция, производящая функция
6	discontinuities	**f**	арктангенс
7	arctangent	**g**	вариационное исчисление
8	a complete set	**h**	нарушение последовательности; прерывность
9	derivative	**i**	синус
10	infinitesimal	**j**	полное множество

Task 9. Mark true (T) or false (F) sentences.

1. Cauchy formulated calculus in terms of geometric ideas and infinitesimals.
2. Riemann`s definition of continuity required an infinitesimal change in *x* to correspond to an infinitesimal change in *y*.
3. Cauchy also introduced the concept of the Cauchy sequence, and started the formal theory of complex analysis.
4. Poisson, Liouville, Fourier and others studied partial differential equations and harmonic analysis.
5. The contributions of Poisson, Liouville, Fourier and Weierstrass, developed the (ε, δ) - definition of limit approach, thus founding the modern field of mathematical analysis.
6. In the middle of the 17th century Riemann introduced his theory of integration.
7. At the beginning of the century saw the arithmetization of analysis by Weierstrass, who thought that geometric reasoning was inherently misleading, and introduced the "epsilon-delta" definition of limit.
8. Then, mathematicians started worrying that they were assuming the existence of a continuum of real numbers without proof.

9. Dedekind then constructed the real numbers by Dedekind cuts, in which irrational numbers are formally defined.
10. Jordan developed what is now called naive set theory.

Task 10. Insert the necessary word from the chart into the gaps.

Discrete; foundations; calculus; analytic geometry; Madhava of Sangamagrama; the magnitude; infinitesimal calculus; the notion; Cavalieri's principle; the derivative; Rolle's theorem in the 12th century.

1. Zu Chongzhi established a method that would later be called (1) … to find the volume of a sphere in the 5th century.
2. The Indian mathematician Bhāskara II gave examples of (2) … and used what is now known as (3) … in the 12th century.
3. In the 14th century, (4) … developed infinite series expansions, like the power series and the Taylor series, of functions such as sine, cosine, tangent and arctangent.
4. Madhava of Sangamagrama also estimated (5) … of the error terms created by truncating these series and gave a rational approximation of an infinite series.
5. The modern (6) … of mathematical analysis were established in 17th century Europe.
6. Descartes and Fermat independently developed (7) … .
7. Newton and Leibniz independently developed (8) … .
8. During this period, calculus techniques were applied to approximate (9) … problems by continuous ones.
9. In the 18th century, Euler introduced (10) … of mathematical function.
10. In 1821, Cauchy began to put (11) … on a firm logical foundation by rejecting the principle of the generality of algebra widely used in earlier work, particularly by Euler.

Task 11. Match the beginnings and the endings of the given sentences.

Beginnings

1. Mathematical analysis is a branch of mathematics that includes
2. These theories are usually studied in the context of
3. Analysis evolved from calculus, which involves
4. Analysis can be applied to any space of mathematical objects that has
5. Archimedes used the method of exhaustion to compute
6. Mathematical analysis formally developed in
7. Early results in analysis were implicitly present in the early days
8. An infinite geometric sum is implicit in
9. Later, Greek mathematicians such as Eudoxus and Archimedes made more explicit, but informal, use of
10. The explicit use of infinitesimals appears in

Endings

a. the 17th century during the Scientific Revolution.
b. Archimedes' *The Method of Mechanical Theorems*.

c. real and complex numbers and functions.
d. the elementary concepts and techniques of analysis.
e. the area inside a circle by finding the area of regular polygons with more and more sides.
f. of ancient Greek mathematics.
g. Zeno's paradox of the dichotomy.
h. the theories of differentiation, integration, measure, limits, infinite series, and analytic function.
i. a definition of nearness (a topological space) or specific distances between objects (a metric space).
j. the concepts of limits and convergence.

Task 12. Retell Text A.

Task 13. In pairs, take turns to interview your partner about the Scientific Revolution in the 17th century. What questions do you think are the most relevant?

Task 14. Find information and tell about Cavalieri's principle.

Task 15. Prepare a report on one of the famous mathematicians. The volume of the report is 150-200 words. Be ready to tell about the scientist.
Zeno of Elea, Liu Hui, Newton, Leibniz, Descartes, Fermat, Siméon Denis Poisson, Joseph Liouville, Jean Baptiste Joseph Fourier, Julius Wilhelm Richard, Simon Stevin, Karl Theodor Wilhelm Weierstrass, Georg Ferdinand, Ludwig Philipp Cantor, René-Louis Baire, Henri Léon Lebesgue, David Hilbert, Stefan Banach.

Task 16. Write a short essay on the suggested topics. The volume of the essay is 200-250 words. Suggest some other relevant essay topics.
1. History of Mathematical analysis.
2. Mathematical analysis as a branch of mathematics.
3. The modern foundations of mathematical analysis.

Task 17. Read the words. Try to remember the pronunciation.
1. Albert Einstein [ˈælbət ˈaınstaın] – Альберт Эйнштейн
2. Zurich [΄zu(ə)rik] – Цюрих
3. Prague [prɑːg] – Прага
4. the Royal Society [ðiː ˈrɔıəl səˈsaıətı] –Академия наук
5. Princeton [ˈprınstən] – Принстон

Task 18. Read Text B. Translate it from English into Russian.

Text B

ALBERT EINSTEIN

Albert Einstein is known as the greatest mathematical physicist. His relativity theory was one of the five or six great discoveries comparable to those of Galilei and Newton. Albert Einstein was born in southern Germany in 1879. As a boy, Albert was unsociable, slow and very honest. His unusual talent for mathematics and physics began to show very early. He was very good at mathematics, and at the age of twelve, he worked out his own methods for solving equations.

In 1896, Albert Einstein was admitted to the Zurich Polytechnic as a student in mathematics and physics. He soon realized that he was a physicist rather than a mathematician. At the age of 21, after four years of study at the university, which he graduated brilliantly, he began to work as a clerk at an office. And in 1905, he made some revolutionary discoveries in science. He published three papers. In his first paper, he explained the photoelectric effect with the help of M. Plank’s quantum theory.

His second paper was a mathematical development of the theory of Brownian motion.

His third paper was entitled “Special Theory of Relativity”. It must be mentioned that a great contribution to the theory of relativity had been made earlier by the great mathematicians Lorenz and Poincare. Einstein’s work was published in a physical journal. It stated that energy equals mass multiplied by the square of the speed of light. This theory is expressed by the equation: E = mc2.

Scientists all over the world met this work with interest and surprise. But only very few physicists realized the importance of his theory at that time. The word *relativity* refers to the fact that all motion is purely relative; in a ceaselessly moving universe, no point can be fixed in place and time from which events can be measured absolutely.

Another of Einstein’s great discoveries was *unified field theory*. It was the result of 35 years of intensive research work. He expressed it in four equations where he combined the physical laws that control forces of light and energy with the mysterious force of gravitation.

After his discoveries, Albert Einstein became famous. Soon he was appointed Professor of Physics at Zurich Polytechnic. Then he got the professorship at Prague, where he remained until 1913.

Albert Einstein gave all his life to science. He was an extremely talented man and a great thinker. He was always looking at the world around him with his eyes wide open, and he was always asking: “Why? Why is that so?”

Einstein was a very simple, open man. His greatest quality was modesty. He was always highly critical of his own work. Einstein improved the old law of gravitation to satisfy more of the facts. In 1921, he received the Nobel Prize for physics and was elected member of the Royal Society.

When the Nazis came to power in Germany in the 1930s, Einstein, who hated them, went to England, living in semi-secrecy and appearing from time to time at public protest meetings. In 1933, he went to America where he took up the post of Professor of Theoretical Physics at the Institute of Advanced Studies at Princeton. Albert Einstein died in 1955 at the age of 76. His ideas made a revolution in natural sciences of the 20th century, and his contribution to science is so great that his name is now familiar to all educated people on the planet.

Adopted from *Пушкина Е.Н. English for Mathematicians and Information Technologies Learners = Английский для студентов, изучающих математику и информационные технологии: учебно-методическое пособие [Электронный ресурс] / Е.Н. Пушкина. – Нижний Новгород, ННГУ, 2019. – 88 с.*

AFTER TEXT TASK

Task 19. Answer the questions on Text B.

1. Is Albert Einstein known mostly as a mathematician or as a physicist?
2. Whose discoveries was his relativity theory comparable to?
3. What country was he born in?
4. What qualities did he reveal in his childhood?
5. How old was Albert when he worked out his own methods for solving equations?
6. Where did he study when he realized the he preferred physics to mathematics?
7. Where did he work as professor when he became famous?
8. What kind of man was Einstein?
9. When was he awarded the Nobel Prize for physics?
10. Why did Einstein emigrate to England?
11. Where did he work in America?
12. Why is Einstein one of the best known scientists of the world?

Part 2
MAIN BRANCHES OF MATHEMATICAL ANALYSIS

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. Do you know the adjective of the noun "algebra"?
2. Can you name any divisions of algebra?
3. What is your favourite field in modern math?
4. Why do you like studying math?
5. What basic problems do the following fields of algebra – linear algebra, Lie group, Boolean algebra, homological algebra, vector algebra, matrix algebra – deal with?

Task 2. Practise reading the following word.

№	Word	Transcription
1	analysis	[əˈnælɪsɪs]
2	real variable	[rɪəl ˈve(ə)rɪəb(ə)l]
3	sequence	[ˈsiːkwəns]
4	convergence	[kənˈvɜːʤəns]

5	calculus	[ˈkælkjʊləs]
6	continuity	[kɒntɪˈnjuːɪtɪ]
7	investigate	[ɪnˈvestɪgeɪt]
8	hydrodynamics	[haɪdrəʊd(a)ɪˈnæmɪks]
9	thermodynamics	[θɜːməʊdaɪˈnæmɪks]
10	equation	[ɪˈkweɪʒ(ə)n]

Task 3. Study and remember the following words and expressions.

№	Word / Expression	Transcription	Translation
1	derivative	[dɪˈrɪvətɪv]	производное
2	particularly useful	[pəˈtɪkjʊləlɪ ˈjuːsf(ə)l]	особенно полезно
3	deterministic relation	[dɪtɜːmɪˈnɪstɪk rɪˈleɪʃn]	детерминированное отношение
4	continuously varying quantities	[kənˈtɪnjʊəslɪ ˈve(ə)rɪɪŋ ˈkwɒntətɪz]	непрерывно изменяющиеся величины
5	velocity	[vɪˈlɒsɪtɪ]	скорость
6	measure	[ˈmeʒə]	мера, измерять
7	meromorphic functions	[ˈmərəmɔːfɪk fʌŋkʃn]z	мероморфные функции
8	quantum field theory	[ˈkwɒntəm fiːld ˈθɪərɪ]	квантовая теория поля
9	applied mathematics	[əˈplaɪd mæθɪˈmætɪks]	прикладная математика
10	algebraic geometry	[ælʤɪˈbreɪɪk ʤɪˈɔmɪtrɪ]	алгебраическая геометрия

Task 4. Read and translate Text A using a dictionary if necessary.

Text A
MAIN BRANCHES OF MATHEMATICAL ANALYSIS

Real analysis. Real analysis (traditionally, the theory of functions of a real variable) is a branch of mathematical analysis dealing with the real numbers and real-valued functions of a real variable. In particular, it deals with the analytic properties of real functions and sequences, including convergence and limits of sequences of real numbers, the calculus of the real numbers, and continuity, smoothness and related properties of real-valued functions.

Complex analysis, traditionally known as the **theory of functions of a complex variable**, is the branch of mathematical analysis that investigates functions of complex numbers. It is useful in many branches of mathematics, including algebraic geometry, number theory, applied mathematics; as well as in physics, including hydrodynamics, thermodynamics, mechanical engineering, electrical engineering, and particularly, quantum field theory. Complex analysis is particularly concerned with the analytic functions of complex variables (or, more generally, meromorphic functions). Because the separate real and imaginary parts of any analytic function must satisfy Laplace's equation, complex analysis is widely applicable to two-dimensional problems in physics.

Functional analysis. Functional analysis is a branch of mathematical analysis, the core of which is formed by the study of vector spaces endowed with some kind of limit-related structure (e.g. inner product, norm, topology, etc.) and the linear operators acting upon these spaces and respecting these structures in a suitable sense. The historical roots of functional analysis lie in the study of spaces of functions and the formulation of properties of transformations of functions such as the Fourier transform as transformations defining continuous, unitary etc. operators between function spaces. This point of view turned out to be particularly useful for the study of differential and integral equations.

Differential equations. A differential equation is a mathematical equation for an unknown function of one or several variables that relates the values of the function itself and its derivatives of various orders. Differential equations play a prominent role in engineering, physics, economics, biology, and other disciplines.

Differential equations arise in many areas of science and technology, specifically whenever a deterministic relation involving some continuously varying quantities (modeled by functions) and their rates of change in space and/or time (expressed as derivatives) is known or postulated. This is illustrated in classical mechanics, where the motion of a body is described by its position and velocity as the time value varies. Newton's laws allow one (given the position, velocity, acceleration and various forces acting on the body) to express these variables dynamically as a differential equation for the unknown position of the body as a function of time. In some cases, this differential equation (called an equation of motion) may be solved.

Measure theory. A measure on a set is a systematic way to assign a number to each suitable subset of that set, intuitively interpreted as its size. In this sense, a measure is a generalization of the concepts of length, area, and volume. A particularly important example is the Lebesgue measure on a Euclidean space, which assigns the conventional length, area, and volume of Euclidean geometry to suitable subsets of the dimensional Euclidean space. For instance, the Lebesgue measure of the interval in the real numbers is its length in the everyday sense of the word – specifically, 1.

Technically, a measure is a function that assigns a non-negative real number or $+\infty$ to (certain) subsets of a set. It must assign 0 to the empty set and be (countably) additive: the measure of a 'large' subset that can be decomposed into a finite (or countable) number of 'smaller' disjoint subsets, is the sum of the measures of the "smaller" subsets. In general, if one wants to associate a *consistent* size to *each* subset of a given set while satisfying the other axioms of a measure, one only finds trivial examples like the counting measure. This problem was resolved by defining measure only on a sub-collection of all subsets; the so-called *measurable* subsets, which are required to form a -algebra. This means that countable unions, countable intersections and complements of measurable subsets are measurable. Non-measurable sets in a Euclidean space, on which the Lebesgue measure cannot be defined consistently, are necessarily complicated in the sense of being badly mixed up with their complement. Indeed, their existence is a non-trivial consequence of the axiom of choice.

Numerical analysis. Numerical analysis is the study of algorithms that use numerical approximation (as opposed to general symbolic manipulations) for the problems of mathematical analysis (as distinguished from discrete mathematics).

Modern numerical analysis does not seek exact answers, because exact answers are often impossible to obtain in practice. Instead, much of numerical analysis is concerned with obtaining approximate solutions while maintaining reasonable bounds on errors. Numerical analysis naturally finds applications in all fields of engineering and the physical sciences, but in the 21st century, the life sciences and even the arts have adopted elements of scientific computations. Ordinary differential equations appear in celestial mechanics (planets, stars and galaxies); numerical linear algebra is important for data analysis; stochastic differential equations and Markov chains are essential in simulating living cells for medicine and biology.

Other topics in mathematical analysis:

- Calculus of variations deals with extremizing functionals, as opposed to ordinary calculus which deals with functions.
- Harmonic analysis deals with Fourier series and their abstractions.
- Geometric analysis involves the use of geometrical methods in the study of partial differential equations and the application of the theory of partial differential equations to geometry.
- Clifford analysis, the study of Clifford valued functions that are annihilated by Dirac or Dirac-like operators, termed in general as monogenic or Clifford analytic functions.
- *p*-adic analysis, the study of analysis within the context of *p*-adic numbers, which differs in some interesting and surprising ways from its real and complex counterparts.
- Non-standard analysis, which investigates the hyperreal numbers and their functions and gives a rigorous treatment of infinitesimals and infinitely large numbers.
- Computable analysis, the study of which parts of analysis can be carried out in a computable manner.
- Stochastic calculus – analytical notions developed for stochastic processes.
- Set-valued analysis – applies ideas from analysis and topology to set-valued functions.
- Convex analysis, the study of convex sets and functions.

Techniques from analysis are also found in other areas such as physical sciences. The vast majority of classical mechanics, relativity, and quantum mechanics is based on applied analysis, and differential equations in particular. Examples of important differential equations include Newton's second law and the Einstein field equations. Functional analysis is also a major factor in quantum mechanics.

Adopted from *Румянцева О.А. Англійська мова для математиків (інтенсивний курс для студентів математичних спеціальностей Інституту математики, економіки і механіки) = English for mathematicians (the intensive course for the students-mathematicians of The Institute of Mathematics, Economics and Mechanics) / О.А. Румянцева. ОНУ імені І.І. Мечникова. – Одеса, 2015. – 145 с.*

AFTER TEXT TASKS

Task 5. Answer the questions.

1. What mathematical notions does the Real analysis deal with?
2. What types of functions is the Complex analysis concerned with?
3. What are the historical roots of functional analysis?
5. What kind of disciplines do the differential equations play a prominent role in?
6. Referring to the measure theory how can the measure of a 'large' subset be decomposed into?
7. What fields does the Numerical analysis find its applications in?
8. Enumerate the basic forms of Mathematical Analyses and expand on their principles.

Task 6. Look through the text to search for unfamiliar words and try to understand their general meanings.

Task 7. Write down the transcription and definitions of unfamiliar words, practise reading the words and try to remember them.

Task 8. Give Russian equivalents to these word combinations.

1. real analysis
2. theory of functions
3. real variable
4. branch of mathematical analysis
5. real-valued functions
6. convergence and limits
7. sequences of real numbers
8. the calculus of the real numbers
9. real-valued functions
10. complex numbers

Task 9. Find the English equivalents to the following word combinations.

1. дифференциальные уравнения
2. квантовая механика
3. стохастическое исчисление
4. гиперреальные числа
5. экстремальные функционалы
6. небесная механика
7. получение приближенных решений
8. поддержание разумных границ
9. аксиомы меры
10. бесконечно большие числа

Task 10. Match the beginnings and the endings of the given sentences.

Beginnings

1. The theory of functions of a real variable is a branch of mathematical analysis dealing with
2. The theory of functions deals with
3. The theory of functions of a complex variable, is the branch of mathematical analysis that investigates
4. The theory of functions is useful in many branches of mathematics, including
5. Complex analysis is particularly concerned with
6. Functional analysis is a branch of mathematical analysis, the core of which is formed
7. The historical roots of functional analysis lie in the study of
8. A differential equation is a mathematical equation for
9. Differential equations play a prominent role in
10. Differential equations arise in
11. A measure on a set is

Endings

a. the analytic functions of complex variables.
b. many areas of science and technology.
c. a systematic way to assign a number to each suitable subset of that set, intuitively interpreted as its size.
d. engineering, physics, economics, biology, and other disciplines.
e. algebraic geometry, number theory, applied mathematics and others.
f. spaces of functions and the formulation of properties of transformations of functions.
g. functions of complex numbers.
h. an unknown function of one or several variables that relates the values of the function itself and its derivatives of various orders.
i. the real numbers and real-valued functions of a real variable.
j. the analytic properties of real functions and sequences.
k. by the study of vector spaces endowed with some kind of limit-related structure and the linear operators acting upon these spaces and respecting these structures in a suitable sense.

Task 11. Write out key words from the text.

Task 12. Use the key words of the text to make up the outline of the text.

Task 13. Write out the main idea of the text. Be ready to speak about it.

Task 14. Give the summary of Text A.

Task 15. In pairs, take turns to interview your partner about branches of Mathematical analysis. What questions do you think are the most relevant?

Task 16. Write a short essay on the suggested topics. Suggest some other relevant essay topics.

1. The future of Mathematical Analysis.
2. The most significant inventions in Mathematical Analysis.
3. The main features of Mathematical Analysis.

Task 17. Read the words. Try to remember the pronunciation.

1. Galileo Galilei [gæləˈliːəʊ gəˈlɪlɪ] – Галилео Галилей
2. Pisa [ˈpiːzə] – Пиза
3. Florentine [ˈflɒrəntaɪn] – флорентийский
4. Padua [ˈpædjʊə] – Падуя
5. Venus [ˈviːnəs] – Венера
6. Saturn [ˈsætən] – Сатурн
7. Jupiter [ˈʤuːpɪtə] – Юпитер
8. satellite [ˈsæt(ɪ)laɪt] – спутник
9. inquisition [ɪnkwɪˈzɪʃn] – инквизиция
10. condemn [kənˈdem] – осуждать
11. telescope [ˈtelɪskəʊp] – телескоп
12. equivalence [ɪˈkwɪvələns] – тождество, эквивалентность
13. dynamics [daɪˈnæmɪks] – динамика
14. Leyden [laɪdn] – Лейден

Task 18. Read Text B. Translate it from Russian into English.

Text B

ГАЛИЛЕЙ

Галилео Галилей был выдающимся итальянским астрономом, который внес свой вклад в математику в начале 17 века. Галилей родился в Пизе в 1564 году. Он был сыном обедневшего флорентийского дворянина. Галилей начинал как студент-медик, но позже занялся наукой и математикой, в которых обладал замечательным талантом.

Когда Галилею было 25 лет, его назначили профессором математики в Пизе, и в то же время он продолжал проводить эксперименты. Но социальная атмосфера в Пизе не была дружественной, и в 1592 году Галилей покинул этот город и стал профессором математики в Падуе. Здесь в течение почти 18 лет он продолжал свои эксперименты и преподавание и стал очень популярным.

Примерно в 1607 году Галилей услышал об изобретении телескопа и решил сделать несколько собственных инструментов. Вскоре он изготовил телескоп с увеличительной силой более 30 диаметров. С помощью своего телескопа он наблюдал солнечные пятна, горы на Луне, фазы Венеры, Кольца Сатурна и четыре ярких спутника Юпитера. Эти открытия вызвали противодействие Церкви, и в 1633 году Галилея вызвали предстать перед инквизицией и заставили отречься и публично заявить, что Земля не движется. Но борьба еще не закончилась. В 1634 году Галилей закончил еще одну книгу, в которой вновь были озвучены идеи, осужденные Церковью. Несколько лет спустя он ослеп. Он умер в 1642 году. Галилею мы обязаны идеей гармонии между экспериментом и теорией. Он основал механику свободно падающих тел и заложил основы динамики в целом. Он изобрел первый современный тип микроскопа. Галилей сделал очень интересные заявления, показывающие, что он понял идею эквивалентности бесконечных классов, фундаментальный момент в теории множеств Кантора в 19 веке, который повлиял на развитие современного анализа. Эти утверждения и многие идеи Галилея в области динамики были опубликованы в Лейдене в 1638 году.

Adopted from *Румянцева О.А. Англійська мова для математиків (інтенсивний курс для студентів математичних спеціальностей Інституту математики, економіки і механіки) = English for mathematicians (the intensive course for the students-mathematicians of The Institute of Mathematics, Economics and Mechanics) / О.А. Румянцева. ОНУ імені І.І. Мечникова. – Одеса, 2015. – 145 с.*

UNIT 9. SETS AND SET THEORY

"No one shall expel us from the paradise which Cantor has created for us".
[Expressing the importance of Georg Cantor's set theory in the development of mathematics.]
— David Hilbert

Part 1
SET THEORY AND DESCRIBING SETS

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.
1. Can you give some examples of sets in real life?
2. Do you know anything about kinds of sets?
3. Can you tell who and when created the set theory?

Task 2. Practise reading the following words.

№	Word	Transcription	Translation
1	George Cantor	[dʒɔːdʒ ˈkæntɔː]	Георг Кантор
2	well-defined	[ˈwel dɪˈfaɪnd]	четко определенный
3	therefore	[ˈðeəfɔː]	поэтому
4	curly braces	[ˈkɜːlɪ breɪsɪs]	фигурные скобки
5	uppercase letters	[ˈʌpəkeɪs ˈletəz]	прописные буквы
6	lowercase letters	[ˈləʊəkeɪs ˈletəz]	строчные буквы
7	equality sign	[ɪˈkwɒlɪtɪ saɪn]	знак равенства
8	intersection	[ɪntəˈsekʃ(ə)n]	пересечение (множеств)
9	encounter	[ɪnˈkaʊntə]	сталкиваться
10	ellipsis sign	[ɪˈlɪpsɪs saɪn]	многоточие

Task 3. Study and remember the following words and expressions.

№	Word / Expression	Transcription	Translation
1	concept	[ˈkɒnsept]	понятие, концепция
2	mental list of places	[ˈmentl lɪst əv ˈpleɪsɪz]	воображаемый список мест
3	throughout all branches	[θruːˈaʊt ɔːl ˈbrɑːntʃɪz]	во всех разделах
4	relate to	[rɪˈleɪt tə]	относиться, быть связанным
5	by convention	[baɪ kənˈvenʃn]	по соглашению
6	set membership	[set ˈmembəʃɪp]	принадлежность к множеству
7	to establish the pattern	[ɪˈstæblɪʃ ðə ˈpætərn]	создать образец, модель
8	composite number	[ˈkɒmpəzɪt ˈnʌmbə]	составное число

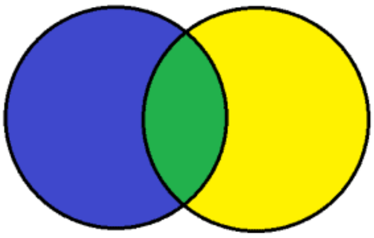
9	exclusive	[ɪkˈskluːsɪv]	исключительный
10	prime numbers	[praɪm ˈnʌmbəz]	простые числа

Task 4. Read and translate Text A using a dictionary if necessary.

Text A

SET THEORY AND DESCRIBING SETS

While the concept of “set” may seem unfamiliar, most people use sets every day without calling it **set theory**. For instance, most people probably have a mental list of places they go to meet with friends and places they go to study or get work done. Perhaps the park is only for friends and the office is only for work while a café works for both (see the picture below).



Set theory was created by George Cantor between 1874 and 1884 and has become a fundamental part of modern mathematics and its basic concepts are used throughout all the various branches of mathematics.

These are the basic ideas behind set theory. This section aims to explore these concepts as they relate to mathematical structures.

In mathematics, we deal with different collections of numbers, symbols, or even equations. We give these kinds of collections a special name in mathematics; we call them **sets**.

Let’s start by defining a set. A set is a collection of well-defined objects. We refer to these objects as **members** or **elements** of the set. For example, the set of natural numbers contains all the natural numbers. Therefore, each natural number is an element or member of that set.

Writing a set in math is pretty simple. We just:

• list the elements in the set,
• separate each element in the set using a comma,
• enclose the elements in the set using curly braces, {}.

For example, the numbers 5,6 and 7 are members of the set {5,6,7}

By convention, we should use an uppercase letter to denote a set and lowercase letters to denote a set’s elements. Also, we should always put an equality sign after the uppercase letter just before writing the elements of the set.

Let’s say we want to write down set A with the elements a, b, and c. So, we will write it as follows: A={a,b,c}

We can also write sets within a set. For example, sets D and E below.

D={p,q,{p,q,r}}

E={1,2,{3,5},6}

It shows that set D contains the set {p,q,r}, and set E contains the set {3,5}.

Set Membership. We use the symbol ∈ to show that an object is a member of a set. The symbol ∈ is read as 'is an element of' or 'is a member of.' For example, 1 is an element of set B above, so we write: $1 \in B$.

We use the symbol ∉ to show that an object is not a member of a set. The symbol is read as 'is not an element of' or 'is not a member of.' For example, 7 is not an element of set B above, so we write $7 \notin B$.

$$\mathbf{B} = \{1, 2, 3, 4, 5\}$$

$$1 \in \mathbf{B}, 5 \in \mathbf{B}, 0 \notin \mathbf{B}, 6 \notin \mathbf{B}$$

In some cases, we will encounter very large sets or even infinite sets in mathematics. This makes it impossible to list all the elements in the set. In such cases, we:

• write down a few elements of the set to establish the pattern, say, 4 or 5 elements; for example, we write the set N of all the natural numbers as: **N** = {1,2,3,4,…}

• put an ellipsis sign or three dots to show that the set has elements that continue in the same pattern; for example, we write the set A of all the odd numbers between 30 and 70 as: **A** = {31,33,35,…,67,69}

Sets can be compared through operations, just like numbers can be compared through operations. Of course, when discussing sets, the operations do not follow the lines of traditional addition, subtraction, multiplication, and division.

Downloaded from *Story of Mathematics. URL: https://www.storyofmathematics.com/describing-sets*

AFTER TEXT TASKS

Task 5. Answer the following questions.

1. Who created set theory? When was it?
2. In which fields are the basic concepts of set theory used?
3. What kinds of sets do we deal with in mathematics?
4. What is the definition of a set?
5. How can we write a set in math?
6. What kind of letters should use an uppercase letter to denote a set's elements?
7. What sign should we use just before writing the elements of the set?
8. What symbol do we use to show that an object is a member of a set?
9. What symbol do we use to show that the set is infinite?
10. What kinds of set operations are mentioned in the Text?

Task 6. Look through the text to search for unfamiliar words and try to understand their general meanings.

Task 7. Write down the transcription and definitions of unfamiliar words, practise reading the words and try to remember them.

Task 8. Give Russian equivalents to these word combinations.

1. seem unfamiliar
2. to study or get work done
3. throughout all the branches
4. to aim to explore
5. to refer to these objects
6. by convention
7. to denote a set
8. to write as follows
9. to be compared through operations
10. the set of prime or odd numbers

Task 9. Find the English equivalents to the following word combinations.

1. понятие множества
2. важнейшая часть современной математики
3. относиться к математическим структурам
4. различные разделы математики
5. набор четко определенных объектов
6. включить элементы в множество, используя фигурные скобки
7. принадлежность к множеству
8. бесконечное множество
9. знак многоточия
10. установить закономерность

Task 10. Match the terms with their definitions.

№	Term		Definition
1	set	**a**	the smallest set which contains all the elements of both the sets
2	element of a set	**b**	three little dots showing that something has been left out
3	set theory	**c**	any one of the distinct objects that belong to that set
4	finite set	**d**	a whole number that is not able to be divided by two into two equal whole numbers
5	ellipsis sign	**e**	collection of well-defined elements
6	odd number	**f**	set that has a fixed number of elements
7	intersection of sets	**g**	the branch of mathematics which deals with the formal properties of sets
8	union of sets	**h**	the largest set which contains all the elements that are common to both the sets

Task 11. Match the beginnings and the endings of the sentences.

№	Beginnings		Endings
1	Set theory was created	**a**	it is impossible to list all the elements in the set.
2	In mathematics, we deal with	**b**	and lowercase letters to denote a set's elements.
3	For example, the set of natural numbers contains	**c**	that an object is not a member of a set.
4	We use an uppercase letter to denote a set	**d**	and unions, which are the combination of two sets.
5	We use the symbol ∉ to show	**e**	by George Cantor between 1874 and 1884.
6	When we have an infinite set,	**f**	all natural numbers.
7	Sets can be compared through operations,	**g**	different collections of numbers, symbols, or even equations.
8	Set operations focus on intersections, which are the overlaps of sets,	**h**	just like numbers can be compared through operations.

Task 12. Insert the necessary word from the chart into the gaps. One word can be used more that once.

Sets, fundamental, deal, member, operations (2), sets theory, concepts, lowercase, equality, well-defined, uppercase, members

1. Most people use (1) … every day without calling it (2) … … .
2. Set theory has become a (3) … part of modern mathematics.
3. Its basic (4) … are used throughout all the various branches of mathematics.
4. In mathematics, we (5) … with different collections of numbers and symbols.
5. A set is a collection of (6) … objects.
6. By convention, we should use an (7) … letter to denote a set and (8) … letters to denote a set's elements.
7. We should always put an (9) … sign after the uppercase letter just before writing the elements of the set.
8. We use the symbol ∈ to show that an object is a (10) … of a set.
9. Sets can be compared through (11) …, just like numbers can be compared through (12) … .
10. For example, the numbers 5,6 and 7 are (13) … of the set {5,6,7}.

Task 13. Mark the sentences true (T) or false (F).

1. Most people use sets every day calling it set theory.
2. For most people the park is only for friends and the office is only for work while a café works for both.
3. Set theory was created by George Cantor between 1874 and 1880.

4. The concepts of set theory are used throughout all the various branches of mathematics.
5. In chemistry, we deal with different collections of numbers, symbols, or even equations.
6. A set is a collection of well-defined objects.
7. The set of natural numbers contains all negative numbers.
8. Writing a set in math is pretty simple.
9. We always put an equality sign after writing the elements of the set.
10. We use the symbol ∉ to show that an object is a member of a set.

Task 14. Look through the text and make up a plan for the text.

Task 15. Render Text A according to the plan using mathematical terms.

Task 16. Translate from Russian into English.
1. Теория множеств была создана Георгом Кантором между 1874 и 1884 годами и стала фундаментальной частью современной математики.
2. Основные концепции теории множеств используются в различных областях математики.
3. В математике мы имеем дело с различными совокупностями чисел, символов и даже уравнений.
4. Мы называем такие совокупности множествами.
5. Множество – это совокупность четко определенных объектов.
6. Чтобы описать множество, нужно перечислить каждый элемент через запятую, а затем заключить элементы в фигурные скобки.
7. По соглашению, мы должны использовать прописные буквы для обозначения множества и строчные буквы для обозначения элементов множества.
8. Мы используем символ ∈, чтобы показать, что объект является элементом множества.
9. В математике встречаются большие или даже бесконечные множества, что делает невозможным перечисление всех элементов.
10. В таких случаях мы записываем несколько элементов набора, чтобы установить закономерность, затем ставим знак многоточия.
11. Множества, как и числа, допускают операции над ними.
12. Пересечения и объединения множеств являются операциями над множествами.

Task 17. Read the words and try to remember the pronunciation.
1. set theory [ˈset ˈθɪərɪ] – теория множеств
2. collection [kəˈlekʃn] – совокупность
3. to group [gruːp] – сгруппировать
4. braces, curly braces [ˈbreɪsɪz, ˈkɜːlɪ ˈbreɪsɪz] – фигурные скобки
5. concept of a set [ˈkɒnsept əv ə ˈset] – понятие множества
6. empty set [ˈemptɪ set] – пустое множество
7. infinite set [ˈɪnfɪnɪt set] – бесконечное множество

8. even two-digit numbers [ˈiːvn tuː ˈdɪdʒɪt ˈnʌmbəz] – четные двузначные числа
9. ellipsis [ɪˈlɪpsɪs] – многоточие
10. set of natural numbers [set əv ˈnætʃrəl ˈnʌmbəz] – множество натуральных чисел

Task 18. Read Text B. Translate it from Russian into English.

Text B

МНОЖЕСТВА И ПОДМНОЖЕСТВА

Множествами занимается специальный раздел математики теория множеств. Множество – одно из главных и фундаментальных понятий. Давайте попробуем понять, что же такое множество? Множество – это совокупность различных элементов, их можно посчитать, сгруппировать. Примерами множеств могут служить буквы алфавита – множество, состоящее из 33 элементов. Множество яблок на дереве – количество яблок на дереве, конечно и его можно посчитать и пронумеровать. Примеров множеств можно придумать очень много.

В математике множество обозначается в фигурных скобках {,}. Например, множество первых пяти букв английского алфавита обозначают вот так: {A,B,C,D,E}. Если записать это множество в другом порядке, оно не изменится.

Математика настолько интересный предмет, что у нас есть понятие пустого множества и бесконечного множества. Пустое множество – множество, в котором нет ни одного элемента, его обозначают без скобок и используют значок Ø. Бесконечное множество, наверняка понятно из названия – множество, в котором бесконечное количество элементов, например, множество всех чисел.

Множества можно также описать различными словами, например, {10, 12, 16, 18, ..., 96 ,98} – это множество четных двузначных чисел. Многоточие используется, когда элементов очень много и все их записать сложно, но при этом запись множества должна быть понятной, и чтобы по ней можно было определить, что это за множество.

Существуют специальные обозначения множеств. Например, символ N служит для обозначения множества натуральных чисел. Для обозначения принадлежности элемента множеству используется специальный знак ϵ. Запись 2ϵ{2,4,6,8...}2ϵ{2,4,6,8...} читается так: "Два принадлежит множеству четных чисел".

Downloaded from *Множества и подмножества*. *URL: https://mathematics-tests.com/9-klass-uroki-prezentatsii-obzor/mnozhestva-podmnozhestva*

Part 2
THE BASIC SET OPERATIONS AND THEIR PROPERTIES

Task 1. In pairs, discuss the following questions. Try to prove your idea. Express your attitude to the received response, your agreement or disagreement with the partner's opinion.

1. Do you know any operations on sets, apart from union and intersections of sets?
2. Have you heard about the Venn diagram? What does it depict?
3. As soon as we know that the properties of set operations are similar to the properties of fundamental operations on numbers, then what properties must set operations have, according to the logic?

Task 2. Practise reading the following words.

№	Word	Transcription	Translation
1	diagram	[dæıəgræm]	диаграмма
2	finite	[ˈfaınaıt]	конечный, имеющий предел
3	imply	[ımˈplaı]	подразумевать
4	denote	[dıˈnəʊt]	обозначать, означать
5	commutative	[kəˈmjuːtətıv]	коммутативный, перестановочный
6	complement	[ˈkɒmplıment]	дополнять, дополнение
7	explore	[ıkˈsplɔː]	изучать, выяснять
8	associative	[əˈsəʊʃıətıv]	ассоциативный
9	intersection	[ˌıntərˈsekʃn]	пересечение
10	disjoint	[dısˈdʒɔınt]	непересекающийся

Task 3. Study and remember the following expressions.

№	Expression	Transcription	Translation
1	union of sets	[ˈjuːnjən əv ˈsets]	объединение множеств
2	intersection of sets	[ˌıntərˈsekʃn əv ˈsets]	пересечение множеств
3	complement of a set	[ˈkɒmplıment əv ə ˈset]	дополнение множества
4	difference between sets	[ˈdıfrəns bıˈtwiːn ˈsets]	разность между множествами
5	set operations	[ˈset ˌɒpəˈreıʃənz]	операции над множествами
6	the set of distinct elements	[ðə ˈset əv dıˈstıŋkt ˈelımənts]	множество различных элементов
7	let us consider	[ˈlet ʌs kənˈsıdə]	давайте рассмотрим
8	the set of common elements	[ðə ˈset əv ˈkɒmən ˈelımənts]	множество общих элементов
9	the intersection of sets	[ðə ˌıntərˈsekʃn əv ˈsets]	пересечение множеств
10	the concept of the	[ðə ˈkɒnsept əv ðə	понятие разности между

	difference between numbers	ˈdɪfrəns bɪˈtwiːn ˈnʌmbəz]	числами
11	is defined as	[ɪz dɪˈfaɪnd æz]	определяется как
12	let us explore the properties	[let əs ɪkˈsplɔː ðə ˈprɒpətɪz]	изучим свойства
13	Commutative Law	[kəˈmjuːtətɪv lɔː]	правило (закон) коммуникативности
14	Associative Law	[əˈsəʊʃɪətɪv lɔː]	правило (закон) ассоциативности
15	the set operation formula	[ðə set ɒpəˈreɪʃ(ə)n ˈfɔːmjʊlə]	формулы операций над множествами

Task 4. Read and translate Text A using a dictionary if necessary.

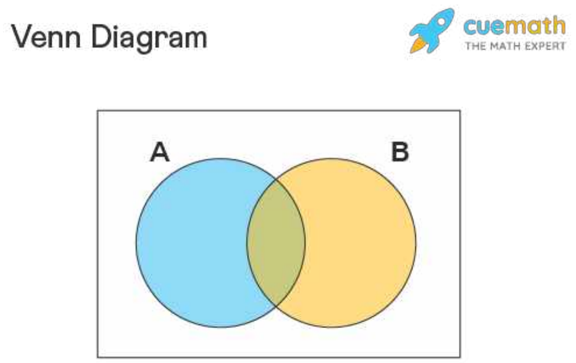
Text A

THE BASIC SET OPERATIONS AND THEIR PROPERTIES

There are four main kinds of set operations which are:

1. Union of sets
2. Intersection of sets
3. Complement of a set
4. Difference between sets/Relative Complement

There is a Venn diagram that shows the possible relationship between different finite sets. It looks as shown below.



Let us consider the set operations.

Union of Sets

For two given sets A and B, A∪B (read as: *A union B*) is the set of distinct elements that belong to set A and B or both. The number of elements in A ∪ B is given by n(A∪B) = n(A) + n(B) − n(A∩B), where n(X) is the number of elements in set X. To understand this set operation of the union of sets better, let us consider an example: If A = {1, 2, 3, 4} and B = {4, 5, 6, 7}, then the union of A and B is given by A ∪ B = {1, 2, 3, 4, 5, 6, 7}.

Intersection of Sets

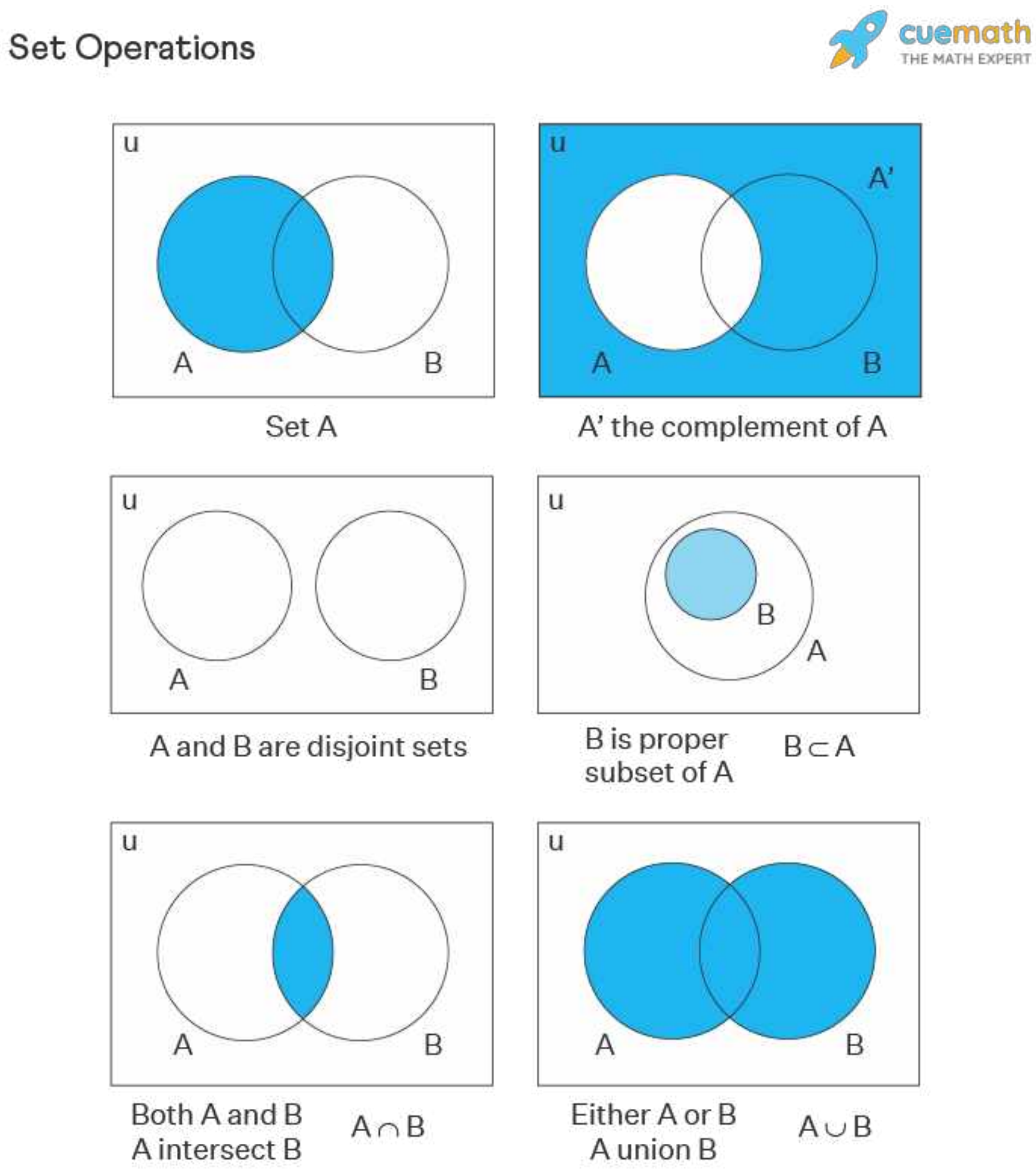
For two given sets A and B, A∩B (*read as: A intersection B*) is the set of common elements that belong to set A and B (See the picture on the next page). The number of elements in $A \cap B$ is given by $n(A \cap B) = n(A)+n(B)-n(A \cup B)$, where $n(X)$ is the number of elements in set X. To understand this set operation of the intersection of sets better, let us consider an example: If A = {1, 2, 3, 4} and B = {3, 4, 5, 7}, then the intersection of A and B is given by A ∩ B = {3, 4}.

Set Difference

The set operation difference between sets implies subtracting the elements from a set which is similar to the concept of the difference between numbers (See the picture on the next page). The difference between sets A and B denoted as A − B lists all the elements that are in set A but not in set B. To understand this set operation of set difference better, let us consider an example: If A = {1, 2, 3, 4} and B = {3, 4, 5, 7}, then the difference between sets A and B is given by A - B = {1, 2}.

Complement of Sets

The complement of a set A denoted as A′ or A^c (read as A complement) is defined as the set of all the elements in the given universal set(U) that are not present in set A (See the picture on the next page). To understand this set operation of complement of sets better, let us consider an example: If U = {1, 2, 3, 4, 5, 6, 7, 8, 9} and A = {1, 2, 3, 4}, then the complement of set A is given by A' = {5, 6, 7, 8, 9}.



Now, let us explore the properties of the set operations that we have discussed.

Properties of Set Operations

The properties of set operations are similar to the properties of fundamental operations on numbers. The important properties on set operations are stated below:

Commutative Law. For any two given sets A and B, the commutative property is defined as: $A \cup B = B \cup A$

This means that the set operation union of two sets is commutative.

Associative Law. For any three given sets A, B and C the associative property is defined as: $(A \cup B) \cup C = A \cup (B \cup C)$

This means the set operation union of sets is associative.

De-Morgan's Law. The law states that for any two sets A and B, we have $(A \cup B)' = A' \cap B'$ and $(A \cap B)' = A' \cup B'$

$A \cup A = A$

$A \cap A = A$

$A \cap \emptyset = \emptyset$

$A \cup \emptyset = A$
$A \cap B \subseteq A$
$A \subseteq A \cup B$

Important Notes on Set Operations

Set operation formula for union of sets is $n(A\cup B) = n(A) + n(B) - n(A\cap B)$ and set operation formula for intersection of sets is $n(A\cap B) = n(A)+n(B)-n(A\cup B)$.
The union of any set with the universal set gives the universal set and the intersection of any set A with the universal set gives the set A.
Union, intersection, difference, and complement are the various operations on sets.
The complement of a universal set is an empty set $U' = \phi$. The complement of an empty set is a universal set $\phi' = U$.

Solving the task with sets

In a school, every student plays either football or soccer or both. It was found that 200 students played football, 150 students played soccer and 100 students played both. Find how many students were there in the school using the set operation formula.
Solution: Let us represent the number of students who played football as n(F) and the number of students who played soccer as n(S). We have $n(F) = 200$, $n(S) = 150$ and $n(F \cap S) = 100$. We know that,
$n(F\cup S) = n(F) + n(S) - n(F\cap S)$
Therefore, $n(F\cup S)=(200+150)-100$
$n(F\cup S) = 350 - 100 = 250$
(*Answer: The total number of students in school is 250*)

AFTER TEXT TASKS

Task 5. Answer the following questions.

1. What are the four main kinds of set operations?
2. What does the Venn diagram show?
3. How do we define the union of sets A and B?
4. What is the definition of the intersection of sets A and B?
5. What is the operation of difference between sets similar to?
6. What does the difference between sets A and B list?

7. How do we define the complement of set A?
8. What is the commutative property for any two given sets?
9. What is the associative property for any three given sets?
10. What does De-Morgan's Law state?
11. What is the set operation formula for union of sets?
12. What is the set operation formula for intersection of sets?

Task 6. Look through the text to search for unfamiliar words and try to understand their general meanings.

Task 7. Write down the transcription and definitions of unfamiliar words, practise reading the words and try to remember them.

Task 8. Give Russian equivalents to these word combinations.
1. relationship between different finite sets
2. let us consider an example
3. the set of common elements
4. implies subtracting the elements
5. the concept of the difference between numbers
6. the complement of a set A denoted as A′
7. the properties of the set operations
8. the union of any set with the universal set
9. an empty set
10. set operation formula

Task 9. Find the English equivalents to the following word combinations.
1. четыре основных операции над множествами
2. возможные отношения между различными множествами
3. количество элементов определяется выражениями
4. давайте рассмотрим пример
5. множество общих элементов
6. чтобы лучше понять эту операцию
7. разность между множествами A и B
8. исследуем свойства операций над множествами
9. для любых заданных множеств
10. Закон Де-Моргана гласит, что
11. важные замечания
12. решение задач с помощью множеств

Task 10. Match the terms with their definitions.

№	Term		Definition
1	The union of two sets A and B is	**a**	a set containing all the elements that are in set A but not in set B.
2	The intersection of two	**b**	are the elements not in A.

	sets is		
3	The difference between sets A and B is	**c**	A set containing all elements that are in A or in B (possibly both).
4	The complement of a set A, often denoted by A^c (or A′),	**d**	the diagram that shows the possible relationship between different finite sets.
5	Venn diagram is	**e**	is the **set** of elements that are common to each of the two **sets**.

Task 11. Match the beginnings and the endings of the sentences.

№	**Beginnings**		**Endings**
1	For two given sets A and B, A∪B (read as: *A union B*)	**a**	lists all the elements that are in set A but not in set B.
2	For two given sets A and B, A∩B (*read as: A intersection B*)	**b**	is the set of distinct elements that belong to set A and B or both.
3	The difference between sets A and B denoted as A − B	**c**	(A ∪ B) ∪ C = A ∪ (B ∪ C).
4	To understand this set operation of complement of sets better,	**d**	is the set of common elements that belong to set A and B.
5	The properties of set operations are similar to	**e**	let us consider an example.
6	For any three given sets A, B and C the associative property is defined as:	**f**	is defined as: A ∪ B = B ∪ A.
7	The union of any set with the universal set gives	**g**	the properties of fundamental operations on numbers.
8	For any two given sets A and B, the commutative property	**h**	the universal set and the intersection of any set A with the universal set gives the set A.

Task 12. Give the main idea of the text.

Task 13. Make up a plan to the text.

Task 14. Retell Text A.

Task 15. In pairs, take turns to interview your partner about understanding basic set operations. What questions do you think are the most relevant?

Task 16. Translate from Russian into English.

1. Существует четыре основных вида операций с множествами, а именно: объединение множеств, пересечение множеств, дополнение множества и разность между множествами.

2. Чтобы лучше понять операцию объединения множеств, рассмотрим следующий пример.
3. Для двух заданных множеств A и B, объединение A и B – это множество различных элементов, которые принадлежат множеству A и B или обоим.
4. Для двух заданных множеств A и B, пересечение A и B – это множество общих элементов, которые принадлежат множествам A и B
5. Операция разности множеств подразумевает вычитание элементов из множества, что аналогично понятию разности чисел.
6. Дополнение множества A, обозначаемое как A′ или Ac, определяется как множество всех элементов данного универсального множества (U), которые отсутствуют в множестве A.
7.Свойства операций над множествами аналогичны свойствам фундаментальных операций над числами.
8. Для любых двух заданных множеств A и B свойство коммутативности определяется как: $A \cup B = B \cup A$. Это означает, что операция объединения двух множеств является коммутативной.
9. Для любых трех заданных множеств A, B и C ассоциативное свойство определяется как: $(A \cup B) \cup C = A \cup (B \cup C)$. Это означает, что операция объединения множеств ассоциативна.
10. Объединение любого множества с универсальным множеством дает универсальное множество, а пересечение любого множества A с универсальным множеством дает множество A.

Task 17. Read the words. Try to remember the pronunciation.
1. well-defined [ˈwel dɪˈfaɪnd] – четко определенный
2. valuable [ˈvæljʊəbl] – ценный, полезный
3. sophisticated [səˈfɪstɪkeɪtɪd] – сложный
4. set of entities [ˈset əv ˈentɪtɪz] – набор сущностей
5. distinguishable [dɪˈstɪŋgwɪʃəbl] – различимый, отличимый
6. on an equal footing [ɒn ən ˈiːkwəl ˈfʊtɪŋ] – в равном положении
7. simultaneously [ˌsɪmlˈteɪnɪəslɪ] – одновременно
8. encroach [ɪnˈkrəʊtʃ] – посягать, вторгаться, покушаться
9. contradiction [kɒntrəˈdɪkʃ(ə)n] – противоречие
10. analogous [əˈnæləgəs] – аналогичный, сходный
11. stature [ˈstætʃə] – положение, статус
12. derive [dɪˈraɪv] – происходить, быть следствием

Task 18. Read Text B. Translate it from English into Russian.

Text B

THE IMPORTANCE OF SET THEORY

Set theory, branch of mathematics that deals with the properties of well-defined collections of objects, which may or may not be of a mathematical nature, such as numbers or functions. The theory is less valuable in direct application to ordinary experience than as a basis for precise and adaptable terminology for the definition of complex and sophisticated mathematical concepts.

Between the years 1874 and 1897, the German mathematician and logician Georg Cantor created a theory of abstract sets of entities and made it into a mathematical discipline. This theory grew out of his investigations of some concrete problems regarding certain types of infinite sets of real numbers. A set, wrote Cantor, is a collection of definite, distinguishable objects of perception or thought conceived as a whole. The objects are called elements or members of the set.

The theory had the revolutionary aspect of treating infinite sets as mathematical objects that are on an equal footing with those that can be constructed in a finite number of steps. Since antiquity, a majority of mathematicians had carefully avoided the introduction into their arguments of the actual infinite (i.e., of sets containing an infinity of objects conceived as existing simultaneously, at least in thought). Since this attitude persisted until almost the end of the 19^{th} century, Cantor's work was the subject of much criticism to the effect that it dealt with fictions—indeed, that it encroached on the domain of philosophers and violated the principles of religion. Once applications to analysis began to be found, however, attitudes began to change, and by the 1890s Cantor's ideas and results were gaining acceptance. By 1900, set theory was recognized as a distinct branch of mathematics.
At just that time, however, several contradictions in so-called naive set theory were discovered. In order to eliminate such problems, an axiomatic basis was developed for the theory of sets analogous to that developed for elementary geometry. The degree of success that has been achieved in this development, as well as the present stature of set theory, has been well expressed in the Nicolas Bourbaki Éléments de mathématique (begun 1939; "Elements of Mathematics"): "Nowadays it is known to be possible, logically speaking, to derive practically the whole of known mathematics from a single source, The Theory of Sets."

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AFTER TEXT TASK

Task 19. Answer the questions on Text B.
1. What does set theory deal with?
2. When did George Cantor create his set theory?
3. What is the definition of a set, according to Cantor?
4. What was the revolutionary aspect of the set theory?
5. Why was Cantor's work the subject of much criticism?
6. When did Cantor's ideas and results gain acceptance?
7. When was set theory recognized as a distinct branch of mathematics?
8. When was axiomatic basis developed for the theory of sets?
9. What book does this citing come from? "Nowadays it is known to be possible, logically speaking, to derive practically the whole of known mathematics from a single source, The Theory of Set.

APPENDIX 1

BASIC ARITHMETIC EXPRESSIONS, FORMULAS, EQUATIONS AND RULES FOR READING THEM IN ENGLISH

Common fractions

Числитель выражается количественным числительным (например, one, four, twentyfive и т.д.), а знаменатель — порядковым числительным (например, first, fourth, twenty fifth и т.д.). Если числитель больше единицы, то знаменатель принимает окончание множественного числа s. Если дробь является буквенным выражением, то обычно используется конструкция "числитель" over "знаменатель".

$\frac{1}{2}$	one half
$\frac{1}{3}$	one third
$\frac{2}{3}$	two thirds
$\frac{26}{38}$	twenty six thirty eights
$2\frac{1}{2}$	two and a half
$3\frac{4}{5}$	three and four fifth
$\frac{m}{n}$	m over n

Decimal fractions

В десятичных дробях целое число отделяется от дроби точкой, называемой “point”. Каждая цифра читается отдельно. Ноль читается либо как “o [ou]”, либо как “zero”. Ноль целых может совсем не читаться.

0.2	1. o point two 2. zero point two 3. point two
0.06	1. o point o six 2. zero point zero six 3. point o six 4. point zero six
1.25	1. one point twenty five 2. one point two five

Indexes, powers and roots

Верхние и нижние индексы в английском называются соответственно “subscript” и “superscript”. Например, произносится как “x sub n”, произносится как “z super k”. Часто, когда не возникает путаницы с верхними и нижними индексами, (например, если верхние индексы не используются) слова “sub” и “super” опускаются, и тогде произносится как “a two”.

a_m	a sub m (a m)
a_0	1. a zero 2. a naught
b^n	b super n (b n)
c_{mn}^{jk}	c sub m n super j k

“Возвести в степень” по-английски — “raise to the power”. Например , произносится как “x raised to the fifth power”. Укороченными вариантами являются “x to the fifth power” или просто “x to the fifth”. Для второй и третьей степени обычно используются выражения “x squared” и “x cubed”. Корень n-ой степени читается как ‘the n-th root”. Корни 2-й и 3-й степени читаются как “square root” и “cube root” соответственно.

x^2	1. x squared 2. x raised to the second power 3. x to the second power 4. x to the second 5. the square of x 6. the second power of x
y^3	y cubed
z^{-10}	1. z to the negative tenth 2. z to the minus tenth
$t^{\frac{3}{2}}$	t to the three halves
$\sqrt{a}$	the square root of a
$\sqrt[3]{7}$	the cube root of seven
$\sqrt[n+k]{7}$	the root of the power n plus k of three

Certain mathematical signs and expressions.

+	plus (the addition sign)
$a+b$	1. a plus b; 2. sum of a and b;
−	minus (the subtraction sign)
$a-b$	1. a minus b; 2. difference of a and b
$-x$	1. minus x; 2. negative x
$a \pm b$	a plus minus b
$\cdot$	times (the multiplication sign)

$a \cdot b$ или ab	1. a times b; 2. a b 3. product of a and b
: или /	the division sign
$a : b$ или a/b	1. a divided by b; 2. a by b 3. ratio of a and b; 4. a over b
$d \equiv h$ или $h \mid d$	1. d is divisible by h 2. h is a divisor of d
=	the equality sign
$a = b$	1. a equals b; 2. a is equal to b
$a \neq b$	a is not equal to b
$a \approx b$	a is approximately equal to b
$\equiv$	the identity sign
$a \equiv b$	a is identically equal to b

Inequalities, absolute values, factorials

$a < b$	a is greater than b
$a > b$	a is less than b
$a \leq b$	a is greater than or equal to b
$a \geq b$	a is less than or equal to b
$\lvert x \rvert$	1. absolute value of x; 2. magnitude of x
$n!$	n factorial

Brackets

( )	1. parentheses (ед. число parenthesis) 2. round brackets
[ ]	1. brackets 2. square brackets
{ }	braces

Functions

$y = f(x)$	1. y equals f of x 2. y is a function of x
$x = f^{-1}(y)$	1. x equals f inverse of y

Limits

$\lim\limits_{n \to \infty} x_n = A$	1. limit of x n as n tends to infinity is equal to A 2. x n converges to A as n tends to infinity
$\lim\limits_{x \to a} f(x)$	limit of f of x as x approaches a
$\lim\limits_{x \to a-0} f(x)$	limit of f of x as x approaches a from the left
$\lim\limits_{x \to a+0} f(x)$	limit of f of x as x approaches a from the right

Derivatives

f'	1. f prime; 2. derivative of f
f''	1. f double prime; 2. second derivative of f
f'''	1. f triple prime; 2. third derivative of f
$f^{(n)}$	1. n-th derivative of f 2. derivative of f of the order n
f'_x	1. f prime x 2. derivative of f with respect to x
Δx	1. delta x; 2. increment of x
dx	1. d x; 2. differential ot x

Sums

$\sum_{n=1}^{m} x_n$	sum of x n as n goes from 1 to m
$\sum_{\alpha \in A} x_\alpha$	sum of x alpha as alpha runs over A

Integrals

$\int$	integral
$\iint$	double integral
$\iiint$	triple integral
$\int f(x)\,dx$	1. indefinite integral of f of x d x 2. general antiderivative of f of x
$\int_a^b$	integral from a to b
$\int_a^\infty$	(improper) integral from a to infinity
$\int_\Gamma \omega$	intergral of omega over gamma

READING SOME MATHEMATICAL EXPRESSIONS

1. $x > y$ «*x* is greater than *y*»
2. $x < y$ «*x* is less than *y*»
3. $x = 0$ «*x* is equal to *zero*»
4. $x \leq y$ «*x* is equal or less than *y*»
5. $x < y < z$ «*y* is greater than *x* but less than *z*»
6. xv «*x* times or *x* multiplied by *y*»
7. $a + b$ «*a* plus *b*»
8. $7 + 5 = 12$ «*seven* plus *five* equals *twelve*; *seven* plus *five* is equal to *twelve*; *seven* and *five* is (are) *twelve*; *seven* added to *five* makes *twelve*»
9. $a - b$ «*a* minus *b*»
10. $7 - 5 = 2$ «*seven* minus *five* equals *two*; *five* from *seven* leaves *two*; difference between *five* and *seven* is *two*; *seven* minus *five* is equal to *two*»
11. $a \times b$ «*a* multiplied by *b*»
12. $5 \times 2 = 10$ «*five* multiplied by *two* is equal to *ten*; *five* multiplied by *two* equals *ten*; *five* times two is *ten*»
13. $a : b$ «*a* divided by *b*»
14. a/b «*a* over *b*, or *a* divided by *b*»
15. $10 : 2 = 5$ «*ten* divided by *two* is equal to *five; ten* divided by *two* equals *five*»
16. $a = b$ «*a* equals *b*, or *a* is equal to *b*»
17. $b \neq 0$ «*b* is not equal to *0*»
18. $m : ab$ «*m* divided by *a* multiplied by *b*»
19. $\sqrt{ax}$ «The square root of *ax*»
20. ½ «one second»

21. $\frac{1}{4}$ «one quarter»
22. $-7/5$ «minus seven fifth»
23. a^4 «*a* fourth, *a* fourth power or *a* exponent 4»
24. a^n «*a* nth, *a* nth power, or *a* exponent *n*»
25. e^{π} «*e* to the power *π* »
26. $\sqrt[n]{b}$ «The *n*th root of *b*»
27. $\sqrt[3]{8}$ «The *cube* root of *eight* is *two*»
28. $Log_{10} 3$ «Logarithm of *three* to the base of *ten*»
29. $2 : 50 = 4 : x$ «*two* is to *fifty* as *four* is to *x*»
30. $4!$ «factorial 4»
31. $(a + b)^2 = a^2 + 2ab + b^2$ «The square of the sum of two numbers is equal to the square of the first number, plus twice the product of the first and second, plus the square of the second»
32. $(a-b)^2 = a^2 - 2ab + b^2$ «The square of the difference of two numbers is equal to the square of the first number minus twice the product of the first and second, plus the square of the second»
33. Δx «Increment of *x*»
34. $\Delta x \rightarrow 0$ «delta *x* tends to *zero*»
35. $\sum$ «Summation of ...»
36. dx «Differential of *x*»
37. dy/dx «Derivative of *y* with respect to *x*»
38. d^2y/dx^2 «Second derivative of *y* with respect to *x*»
39. d^ny/dx^n «*n*th derivative of *y* with respect to *x*»
40. dy/dx «Partial derivative of *y* with respect to *x*»
41. d^ny/dx^n «*n*th partial derivative of *y* with respect to *x*»
42. $\int$ «Integral of ...»
43. $\int_b^a$ «Integral between the limits *a* and *b*»
44. $\sqrt[5]{d^n}$ «The fifth root of *d* to the *n*th power»
45. $\sqrt{a+b}/ a - b$ «The square root of *a* plus *b* over *a* minus *b*»
46. $a^3 = logcd$ «*a* cubed is equal to the logarithm of *d* to the base *c*»
47. $\int_\tau^t f[S, \varphi(S)]\ ds$ «The integral of *f* of *S* and *φ* of *S*, with respect to *S* from *τ* to *t*»
48. $X_{a-b} = e^{tl}$ «*X* sub *a* minus *b* is equal to *e* to the power *t* times *l*»
49. $f(z) = K_{ab}$ «*f* of z is equal to *K* sub *ab*»

50. $\frac{d^2u}{dt^2} = 0$ «The second partial (derivative) of и with respect to t is equal to zero»

APPENDIX 2

HOW TO WRITE A SUMMARY

What is a summary?

A summary – a short version of a larger reading. To write a summary means to use your own words to express briefly the main idea and relevant details of the piece you have read. The purpose in writing the summary is to give the basic ideas of the original reading. The size of the summary is usually onethird of the original article.

Before writing a summary:

For a text, read, mark, and annotate the original. (For a lecture, work with the notes you took.)

– highlight the topic sentence
– highlight key points/ key words/ phrases
– highlight the concluding sentence
– outline each paragraph in the margin

Take notes on the following:

the source (author-- first/last name, title, date of publication, volume number, place of publication, publisher, URL, etc.)
the main idea of the original (paraphrased)
the major supporting points (in outline form)
major supporting explanations (e.g. reasons/causes or effects).

Remember:

Do not rewrite the original piece.
Keep your summary short.
Use your own wording.
Refer to the central and main ideas of the original piece.
Read with who, what, when, where, why and how questions in mind.

How Should I Organize a Summary?

Like traditional essays, summaries have an introduction, a body, and a conclusion. What these components look like will vary some based on the purpose of the summary you're writing. The introduction, body, and conclusion of work focused specifically around summarizing something is going to be a little different than in work where summary is not the primary goal.

Introducing a Summary

You will almost always begin a summary with an introduction to the author, article, and publication so the reader knows what we are about to read.

The introduction should accomplish a few things:

- Introduce the name of the author whose work you are summarizing.
- Introduce the title of the text being summarized.
- Introduce where this text was presented.
- State the main ideas of the text you are summarizing—just the big-picture components.
- Give context when necessary. Is this text responding to a current event? That might be important to know.

Presenting the "Meat" (or Body) of a Summary

Depending on the kind of text you are summarizing, you may want to note how the main ideas are supported (although, again, be careful to avoid making your own opinion about those supporting sources known).

When you are summarizing with an end goal that is broader than just summary, the body of your summary will still present the idea from the original text that is relevant to the point you are making (condensed and in your own words).

Concluding a Summary

Now that we've gotten a little more information about the main ideas of this piece, are there any connections or loose ends to tie up that will help your reader fully understand the points being made in this text. This is the place to put those.

Discuss the summary you've just presented. How does it support, illustrate, or give new information about the point you are making in your writing? Connect it to your own main point for that paragraph so readers understand clearly why it deserves the space it takes up in your work.

Useful phrases for writing a summary

In "… (Title, source and date of piece)", **the author shows that** … (central idea of the piece). **The author supports the main idea by** using …. and showing that….

The text (story, article, poem, excerpt…) is about…

deals with…

presents…

describes…

In the text (story, article, poem, excerpt…) the reader gets to know…

the reader is confronted with…

the reader is told about…

The author (the narrator) says, states, points out that…

claims, believes, thinks that…

describes, explains, makes clear that…

uses example to confirm, prove that…

agrees/disagrees with the view /thesis…

contradicts the view…

criticises, analyses, comments on…

tries to express…

argues that…

suggests that…

compares X to Y…

emphasises his thesis by saying that…

doubts that…

tries to convince the readers that…

concludes that…

About the structure of the text:

The text consists of/ may be divided into…

In the first paragraph/ exposition the author introduces…

In the second paragraph of the text / paragraph the author introduces…
Another example can be found in…
As a result…
The climax/ turning point is reached when…
To sum up / to conclude…
In the conclusion/ starting from line…, the author sums up the main idea/ thesis…
In his last remark/ with his last remark / statement the author concludes that…

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APPENDIX 3

ENGLISH-RUSSIAN DICTIONARY OF MATHEMATICAL TERMS

A

abscissa	- абсцисса
absolute	- абсолютный
absolute extremum	- абсолютный экстремум
absolute value	- абсолютная величина, модуль
absolute value of a complex number	- абсолютная величина комплексного числа
accuracy	- точность
acnode	- изолированная точка
acute angle	- острый угол
acute triangle	- остроугольный треугольник
add	- прибавлять, суммировать
addend	- слагаемое
addition	- суммирование
addition sign	- знак сложения
adjacent angle	- соседний (прилежащий, смежный) угол
adjacent side	- прилежащая сторона
adjacent supplementary angles	- смежные углы
adjoint	- сопряженный
admissible	- допустимый
admissible solution	- допустимое решение
affine coordinates	- аффинные координаты
algebra	- алгебра
algebraic equation	- алгебраическое уравнение
algebraic expression	- алгебраическое выражение
algorithm	- алгоритм

algorithm for division	-	алгоритм деления
alternance	-	чередование
alteration	-	изменение
alternate angles	-	накрест лежащие углы
alternate exterior angles	-	внешние накрест лежащие углы
alternate interior angles	-	внутренние накрест лежащие углы
altitude of a triangle	-	высота треугольника
amplitude	-	амплитуда
analogous	-	аналогичный
analogical	-	аналогичный
analogy	-	аналогия
analysis	-	анализ
analyze	-	анализировать
angle	-	угол
anticosine	-	арккосинус
antisine	-	аксинус
applicate	-	аппликата
approximate solution	-	приближенное решение
arbitrary	-	произвольный
arc	-	дуга
arccosine	-	арккосинус
arc-length	-	длина дуги
arcsine	-	арксинус
arctangent	-	арктангенс
area	-	площадь
argument	-	аргумент
argument of a function	-	аргумент функции

arithmetic mean	- среднее арифметическое
arithmetic progression	- арифметическая прогрессия
associative law	- сочетательный (ассоциативный) закон
associative property	- сочетательное (ассоциативное) свойство
assumption	- предположение
asymmetric(al)	- асимметричный
asymmetry	- асимметрия
asymptote	- асимптота
asymptotes of a hyperbola	- асимптоты гиперболы
average	- среднее значение
average value	- усреднение, среднее значение
axiom	- аксиома

B

back-substitution	- обратная подстановка
bar	- дробная черта, черта
base	- база, базис
base angle {of a triangle}	- угол при основании треугольника
base vector	- базисный вектор
basis	- база, базис
billion	- биллион
binary	- бинарный
binomial	- двучлен (бином)
binomial coefficient	- биноминальный коэффициент
binomial expansion	- разложение
binomial formula	- формула бинома
biquadratic equation	- биквадратное уравнение

bisector - биссектриса (более частотный термин)

bisectrix - биссектриса

bounded interval - ограниченный интервал

braces - фигурные скобки

brackets - квадратные скобки

branch - ветвь

bridging - перенос

C

calculus - математический анализ, исчисление

cancel - сокращать

canonical - канонический

Cartesian coordinate system - декартова система координат

Cartesian coordinates - декартовы координаты

central angle - центральный угол

central conic - центральное коническое сечение

central symmetry - центральная симметрия

centre - центр

centre of the escribed circle - центр вневписанной окружности

center - центр

change of variable - замена переменной

chord - хорда

circle - окружность, круг

circumcenter - центр описанной окружности

circumscribed figure - описанная фигура

closed - замкнутый

closed interval - замкнутый интервал

coefficient	- коэффициент
coincident	- совпадающий
collinearity	- коллинеарность
combination	- комбинация
combine similar terms	- приведение подобных членов
common denominator	- общий знаменатель
common difference	- разность арифметической прогрессии
common divisor	- общий делитель
common factor	- общий делитель
common fraction	- арифметическая (простая) дробь
common logs	- десятичный логарифм
common multiple	- общее кратное
common ratio	- частное геометрической прогрессии
commutative law	- переместительный (коммутативный) закон
comparison	- сравнение
complementary angle	- дополнительный угол {до 90 º}
complete induction	- полная индукция
complete solution	- полное решение
complex number	- комплексное число
computable	- вычислимый
computation	- вычисление
concave	- вогнутый
concave curve	- вогнутая кривая
concave function	- вогнутая функция
concentric circles	- концентрические окружности
condition	- условие

cone	-	конус
congruence	-	конгруэнтность
congruent angles	-	равные углы
congruent figures	-	равные фигуры
congruent polygons	-	равные многоугольники
congruent segments	-	равные отрезки
conic	-	коническое сечение
conic section	-	коническое сечение
conjugate	-	сопряженный
conjugate angle	-	дополнительный угол до 360^o
conjugate roots	-	сопряженные корни
consecutive integers	-	последовательные целые числа
constant	-	константа
continuity	-	непрерывность
continuous function	-	непрерывная функция
convex	-	выпуклый
convex curve	-	выпуклая кривая
convex polygon	-	выпуклый многоугольник
convexity	-	выпуклость
coordinate	-	координата
coordinate axis	-	координатная ось
coordinate system	-	система координат
coplanar	-	компланарный
coplanar vector	-	компланарный вектор
coprime numbers	-	взаимно простые числа
corollary	-	следствие
corresponding angles	-	соответственные углы

count	-	подсчитать, считать
criterion	-	критерий
criterion for divisibility	-	признак делимости
cross-product	-	векторное (внешнее) произведение
cube	-	куб
cubic	-	кубическая кривая
cubic curve	-	кубическая кривая
curve	-	кривая
cut	-	сечение
cylinder	-	цилиндр

D

data	-	данные
decimal	-	десятичный
decimal fraction	-	десятичная дробь
decimal number	-	десятичное число
decision	-	решение
decomposition	-	разложение
decomposition of a fraction	-	разложение дроби
decrease	-	убывать
decreasing function	-	убывающая функция
deduction	-	дедукция
define	-	определять
definition	-	определение
degenerate	-	вырожденный
degenerate conic	-	вырожденное коническое сечение
degree	-	степень
degree of a polynomial	-	степень многочлена

denominator	- знаменатель
dependent	- зависимый
derivative	- производная
derivative at a point	- производная в точке
determinant	- определитель
determine	- определять
deviation	- отклонение
diagonal	- диагональ
diagonal element	- диагональный элемент
diagonal matrix	- диагональная матрица
diameter	- диаметр
diametrically opposite point	- диаметрально противоположная точка
difference	- разность
differentiability	- дифференцируемость
differentiable function	- дифференцируемая функция
differential of area	- элемент площади
digit	- цифра
dihedral angle	- двугранный угол
dilatation	- растяжение
dimension	- размерность
direction	- направление
direction cosine	- направляющий косинус
directly proportional	- прямо пропорциональный
discontinuous	- разрывный
discontinuous function	- разрывная функция
discriminant	- дискриминант
disposition	- расположение

distance	- расстояние
distinct	- различный
distributive law	- распределительный (дистибутивный) закон
dividend	- делимое
divisible {by}	- делимый
division	- деление
division algorithm	- алгоритм деления
divisor	- делитель
domain	- область; источник
domain of definition	- область определения
dot	- точка
dot product	- скалярное произведение
dotted line	- пунктирная линия
double root	- двойной корень
dual	- двойственный (дуальный)
duality principle	- принцип двойственности

E

edge	- ребро
element	- элемент
element of area	- элемент площади
elimination	- исключение
elimination by substitution	- исключение посредством подстановки
elimination method	- метод исключения
ellipse	- эллипс
empty set	- пустое множество
equation	- уравнение

equation of a straight line	-	уравнение прямой
equilateral	-	равносторонний
equilateral polygon	-	правильный многоугольник
equilateral triangle	-	равносторонний треугольник
equivalent	-	равносильный (эквивалентный)
equivalent figure	-	конгруэнтная фигура
error	-	ошибка
enscribed	-	описанный, вневписанный
essential	-	существенный
estimation	-	оценка
Euclidean algorithm	-	алгоритм Евклида
Euclidean geometry	-	евклидова геометрия
Euclidean space	-	евклидово пространство
evaluation	-	вычисление
evaluation of determinant	-	вычисление определителя
even	-	четный
even function	-	четная функция
even number	-	четное число
everywhere defined	-	всюду определенный
exact	-	точный
exact division	-	деление без остатка
exact solution	-	точное решение
example	-	пример
excentre	-	центр вневписанной окружности
exclusion	-	исключение
existential quantifier	-	квантор существования
expansion	-	разложение

expansion of a determinant	-	разложение определителя
explementary angle	-	дополнительный угол до 360º
exponent	-	показатель, экспонент
exponential	-	показательный, экспоненциальныйы
exponential equation	-	показательное уравнение
expression	-	выражение
exterior angle {of a triangle}	-	внешний угол {треугольника}
extremal	-	экстремальный

F

factor	-	множитель
factor theorem	-	теорема Безу
factoring	-	разложение
factorization	-	разложение на множители
family	-	семейство
field	-	поле
first derivative	-	первая производная
first-order equation	-	уравнение первого порядка
flow chart	-	блок-схема
flux	-	поток
focal point	-	фокальная точка
focus	-	фокус
foot {of a perpendicular}	-	основание {перпендикуляра}
formula	-	формула
fraction	-	дробь
function	-	функция
function of a complex variable	-	функция комплексной переменной
function of a single variable	-	функция одной переменной
function of several variables	-	функция несколько независимых

		переменных
fundamental	-	основной
fundamental theorem of arithmetic	-	основная теорема арифметики

G

general	-	общий
general form	-	общий вид
general solution	-	общее решение
general term	-	общий член
geometric average	-	среднее геометрическое
geometric locus	-	геометрическое место точки
geometric mean	-	среднее геометрическое
geometric progression	-	геометрическая прогрессия
geometry	-	геометрия
grade	-	степень
greatest common divisor	-	наибольший общий делитель
greatest common factor	-	наименьшее общее кратное

H

half-angle formulas	-	формулы половинного угла
halve	-	делить пополам
height	-	высота
hemisphere	-	полусефра, полушар
hexagon	-	шестиугольник
hexaeder	-	гексаэдр, шестигранник
hexahedron	-	гексаэдр, шестигранник
hill climbing	-	поиск экстремума

homogeneous	-	однородный
homogeneous equation	-	однородное уравнение
homogeneous system	-	однородная система
horizontal	-	горизонтальный
horizontal axis	-	горизонтальная ось
Horner's scheme	-	схема Горнера
hyperbola	-	гипербола
hyperbolic	-	гиперболический
hypotenuse	-	гипотенуза
hypothesis	-	гипотеза

I

identity	-	тождество
if and only if	-	тогда и только тогда
image	-	образ
implication	-	импликация
improper fraction	-	неправильная дробь
incenter	-	центр вписанной окружности
incircle	-	вписанная окружность
include	-	включать
inclusion	-	включение, вложение
inconsistent	-	несовместимый, противоречивый
incorrect	-	ошибочный, неточный
increase	-	расти
increment	-	приращение
indefinite	-	неопределенный
independent	-	независимый
independent variable	-	независимая переменная

indeterminancy	-	неопределенность
induction	-	индукция
inequality	-	неравенство
infinite	-	бесконечный
infinite decimal fraction	-	бесконечная десятичная дробь
inflexion	-	перегиб
inhomogeneous	-	неоднородный
initial	-	начальный
initial condition	-	начальное условие
initial value	-	начальное значение
inscribe	-	вписать
inscribed angle	-	вписанный угол
inscribed circle	-	вписанная окружность
inscribed polygon	-	вписанный многоугольник
integer number	-	целое число
integral	-	интеграл
integral curve	-	интегральная кривая
integrand	-	подинтегральное выражение
integration by parts	-	интегрирование по частям
integration constant	-	постоянная интегрирования
inter-stage function	-	ступенчатая функция
intercept	-	отрезок; отрезок отсекаемый с оси
intercept theorem	-	теорема Фалеса
interdependency	-	взаимозависимость
interior angle	-	внутренний угол
intersection	-	пересечение
interval	-	интервал

inverse - обратно

inversely proportional - обратно пропорциональный

irrational number - иррациональное число

irreductibility - неприводимость

isosceles triangle - равнобедренный треугольник

J

jump - скачок

jump function - ступенчатая функция

K

kilogram(me) - килограмм

kilometre - километр

known - известный

L

law - закон

law of composition - закон композиции

law of sines - теорема синусов

law of the excluded middle - закон исключенного третьего

least common denominator - наименьший общий делитель

least common multiple - наименьшее общее кратное

leg - боковая сторона

Leibniz rule - формула Лейбница

lemma - лемма

length - длина

like denominators - одинаковые знаменатели

like signs - одинаковые знаки

limit - предел

limit value	-	предельное значение
limits of integration	-	пределы интегрирования
line	-	прямая
line segment	-	отрезок
linear	-	линейный
linear equation	-	линейное уравнение
linear function	-	линейная функция
linear independency	-	линейная независимость
linearity	-	линейность
local	-	локальный
logarithm	-	логарифм
lower limit	-	нижний предел
lowest common denominator	-	наименьший общий знаменатель
lowest common multiple	-	наименьшее общее кратное
lozenge	-	ромб

M

magnitude	-	величина
main diagonal	-	главная диагональ
major axis	-	главная ось
many-variable system	-	система с несколькими переменными
map	-	отображение
mapping	-	отображение
meter	-	метр
mathematical induction	-	математическая (полная) индукция
mathematics	-	математика
matrix	-	матрица

matrix of the transformation	-	матрица преобразования
maximum	-	максимум
mean	-	среднее арифметическое
mean proportional	-	среднее геометрическое
measurable	-	измеримый
measure	-	мера
median {of a triangle}	-	медиана
member {of a set}	-	элемент {множества}
minimum	-	минимум
minuend	-	уменьшаемое
minus {sign}	-	знак минус
module	-	модуль
monom	-	одночлен (моном)
monotone decreasing function	-	монотонно убывающая функция
monotone increasing function	-	монотонно возрастающая функция
monotonic function	-	монотонная функция
monotonous	-	монотонный
multiple	-	многократный; кратное
multiplex	-	многократный
multiplicand	-	множимое
multiplication	-	умножение
multiplier	-	множитель
multiply	-	множить
mutually	-	взаимно

N

natural logarithm	-	натуральный логарифм
natural number	-	натуральное число
necessary and sufficient condition	-	необходимое и достаточное условие

negative - отрицательный

negligible - пренебрегаемый

node - узел

non-decreasing - неуменьшающийся

non-degnerate - невырожденный

non-degenerate conic - невырожденное коническое сечение

non-degenerate conic section - невырожденное коническое сечение

non-linear - нелинейный

non-linear equation - нелинейное уравнение

non-orthogonal coordinate system - неортогональная координатная система

non-periodic - непериодический

non-symmetric - несимметричный

non-terminating decimal - бесконечная десятичная дробь

non-trivial solution - нетривиальное (ненулевое) решение

non-zero solution - нетривиальное (ненулевое) решение

normal - нормаль

normal to the surface - нормаль к поверхности

normal vector - вектор нормали

null-vector - нулевой вектор

null-matrix - нулевая матрица

null-set - нуль-множество, пустое множество

number - число

number line - числовая прямая

O

oblique - oblic

obtuse angle - тупой угол

obtuse triangle	-	тупоугольный треугольник
octagon	-	восьмиугольник
odd	-	нечетный
odd-function	-	нечетая функция
one-to-one	-	взаимно-однозначный
open interval	-	открытый интервал
opposite interior angles	-	внутренние накрест лежащие углы
order	-	порядок
order of derivative	-	порядок производной
order of equation	-	порядок уравнения
ordered pair	-	упорядоченная пара
ordinal number	-	порядковый номер
ordinate	-	ордината
origin {of coordinates}	-	начало {координат}
orthogonal base	-	ортогональный базис
orthogonal coordnate system	-	ортогональная система координат
orthonormal basis	-	ортонормированный базис
oval	-	овал

P

pair	-	пара
parabola	-	парабола
parabolic	-	параболический
parallel	-	параллельный
parallelepiped	-	параллелепипед
parallelogram	-	параллелограмм
parallelogram law	-	закон параллелограмма
parallelogram rule	-	закон параллелограмма

parameter	-	параметр
parametric form	-	параметрическая форма
parentheses	-	круглые скобки
partial fraction	-	элементарная дробь
partial-fraction expansion	-	разложение правильной дроби на простейшие дроби
pencil	-	пучок
pencil of lines	-	пучок прямых
pentagon	-	пятиугольник
per cent	-	процент
perimeter	-	периметр
period	-	период
periodic decimal fraction	-	периодическая десятичная дробь
periodic function	-	периодическая функция
periodic solution	-	периодическое решение
permissible solution	-	допустимое решение
perpendicular	-	перпендикуляр
pivot	-	ось вращения, центр вращения
plane	-	плоскость
plane geometry	-	планиметрия
planimetry	-	планиметрия
plus sign	-	знак плюс
point	-	точка
point of discontinuity	-	точка разрыва
point of inflexion	-	точка перегиба
polygon	-	многоугольник
polyhedron	-	многогранник

polynomial	-	многочлен
positive	-	положительный
possibility	-	возможность
power	-	показатель степени
pre-image	-	прообраз
preceding	-	предыдущий
prime factorization	-	разложение на простые множители
prime number	-	простое число
primitive	-	первообразная функция
principal	-	главный
principal axis	-	главная ось
principal diagonal	-	главная диагональ
principle of complete induction	-	метод полной индукции
prism	-	призма
product	-	произведение
progression	-	прогрессия
projection	-	проекция
proof	-	доказательство
proper factor	-	собственный делитель
proper fraction	-	дробь
property	-	свойство
proportion	-	пропорция
proposition	-	предложение
prove	-	доказывать
pyramid	-	пирамида
Pythagorean theorem	-	теорема Пифагора

Q

quadrate - квадрат

quadratic - квадратный

quadratic equation - квадратное уравнение

quadratic formula - формула корней квадратного уравнения

quantifier - квантор

quotient - частное

R

radian - радиан

radical - радикал, знак корня

radical sign - радикал, знак корня

radius - радиус

radius vector - радиус-вектор

raise to a power - возводить в степень

range - область, множество значений

rank of a matrix - ранг матрицы

ratio - частное, отношение

rational - рациональный

rational function - рациональная функция

rational number - рациональное число

ray - полуось

real - действительный, вещественный

real number - действительное (вещественное) число

reciprocal matrix - обратная матрица

rectangle - прямоугольник

rectangular coordinate system - {декартова} прямоугольная система координат

reduce - приводить, сокращать

reducible - приводимый

regular polygon - правильный многоугольник

relation - отношение

relative - относительный

relatively prime numbers - взаимно простые числа

remainder - остаток

repeated root - кратный корень

replace - подставлять, заменять

represent - представлять

rest - остаток

restriction - ограничение, рестрикция

rhomb - ромб

rhombus - ромб

right angle - прямой угол

right triangle - прямоугольный треугольник

root - корень

rotation - вращение

round - округлять

rounding error - ошибка округления

rule - правило

S

satisfy - удовлетворять

scalar - скаляр

scalene triangle - разносторонний треугольник

secant - секущая

sector of a circle - сектор круга

segment - отрезок, сегмент

semi-circle	-	полукруг
semiclosed interval	-	полузамкнутый интервал
set	-	множество
set theory	-	теория множеств
side {of an angle}	-	сторона {угла}
sign	-	знак
signum function	-	сигнум-функция
similar fractions	-	дроби с равными знаменателями
similar polygons	-	подобные многоугольники
similar terms	-	подобные члены
similar triangle	-	подобный треугольник
similarity	-	подобие
similitude	-	подобие
simple	-	простой
simple root	-	простой (однократный) корень
simplification	-	упрощение
sine curve	-	синусоида
sine rule	-	теорема синусов
single	-	один, отдельный, единственный
single root	-	простой (однократный) корень
skew lines	-	скрещивающиеся прямые
slope	-	наклон; угловой коэффициент
slope angle	-	угол наклона
slope formula	-	формула углового коэффициента
slope-intercept form of a straight line equation	-	уравнение прямой с угловым коэффициентом
solution {of a problem}	-	решение {задачи}

solution set	-	множество решений
solve	-	решать
space	-	пространство
speed	-	скорость
sphere	-	сфера, шар
square	-	квадрат; возводить в квадрат
square brackets	-	квадратные скобки
square root	-	квадратный корень
standard form	-	общий вид, стандартная форма, нормальная форма
statement	-	утверждение, высказывание
step function	-	ступенчатая функция
straight	-	прямой, правый
straight angle	-	развернутый угол
straight-line	-	прямая линия
straight-line segment	-	отрезок прямой, отрезок
stretching	-	растяжение
strict	-	строгий
strongly monotonic	-	строго монотонный
subset	-	подмножество
substitution	-	подстановка
subtraction	-	вычитание
subtrahend	-	вычитаемое
sum	-	сумма
summand	-	слагаемое
supplementary angles	-	дополнительные углы
surface	-	поверхность

surface area	-	площадь поверхности
surface element	-	элемент площади {поверхности}
surface of revolution	-	поверхность вращения
symbol	-	символ
symmetric	-	симметричный
symmetric function	-	симметричная (четная) функция
synthetic division	-	схема Горнера
system	-	система

T

tangent	-	касательная; тангенс, функция тангенс
tangent line	-	касательная
tangent plane	-	касательная плоскость
term	-	член
term of a fraction	-	числитель дроби
terminating decimal fraction	-	конечная десятичная дробь
tetragon	-	четырехугольник
tetrahedron	-	четырехгранник, тетраэдр
theorem	-	теорема
theory	-	теория
transcendental number	-	трансцендентное число
transform	-	преобразовать
transform of coordinates	-	преобразование координат
transitivity	-	транзитивность
translation	-	трансляция
trapezium	-	трапеция
trapezoid	-	трапеция
triangle	-	треугольник

triangular	-	треугольный
trigonometric	-	тригонометрический
trigonometric function	-	тригонометрическая функция
trigonometry	-	тригонометрия
trisection of the angle	-	трисекция угла
truth	-	истинность

U

unambiguous	-	недвусмысленный, однозначный
unbounded	-	неограниченный
uncertainty	-	недостоверность, неопределенность
undefined	-	неопределенный (недефинированный)
undetermined	-	неопределенный
unequal	-	неравный
union	-	объединение
unique solution	-	единственное решение
uniqueness	-	единственность
unit	-	единица
unit circle	-	единичная окружность
unit tangent vector	-	касательный единичный вектор
unit vector	-	единичный вектор
universal quantifier	-	квантор общности
universal set	-	универсальное множество
unknown	-	неизвестное
unlike denominators	-	неодинаковые знаменатели
unsymmetric	-	несимметричный

V

valid	-	справедливый

value - величина, стоимость

vanish - исчезать, обратиться в нуль

variable - переменная

vector - вектор

vector product - векторное (внешнее) произведение

velocity - скорость

verify - проверять

vertex - вершина

vertex angles - вертикальные углы

vertical - вертикаль; вертикальный

vertical axis - вертикальная ось

vice versa - наоборот, обратно

vinculum - дробная черта

volume - объем

W

way - путь

well-defined - вполне определенный; однозначно определенный

whole - целый

X

x-axis - ось *x*

x-intercept - отрезок на оси *x*

xy-plane - плоскость *xOy*

Y

y-axis - ось *y*

Z

z-axis	-	ось z
zero	-	нуль
zero solution	-	нулевое решение

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