

## Propagation of gravitational waves in Chern-Simons axion Einstein gravity

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In this paper we shall investigate the propagation of gravitational waves in a flat Friedman-Robertson-Walker background, in the context of a string motivated corrected Einstein gravity. Particularly, we shall consider a misalignment axion Einstein gravity in the presence of a string originating Chern-Simons coupling of the axion field to the Chern-Pontryagin density in four dimensions. We shall focus our study on the propagation of the gravitational waves, and we shall investigate whether there exists any difference in the propagation of the polarization states of the gravitational waves. As we demonstrate, the dispersion relations are different in the right-handed mode and the left-handed mode. Finally, we compare the propagation of the axion Chern-Simons Einstein theory with that of standard  $F(R)$  gravity.

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### I. INTRODUCTION

String theory is one solid theoretical framework that may describe, in a consistent way, the ultraviolet completion of classical gravity and of the Standard Model of particle physics. In some cases, several low-energy string theory effects may have their impact in the classical gravitational phenomena. One such example is offered by axionlike particles [1–11], and specifically from the misalignment axions [1]. For these axions, there exists a primordial  $U(1)$  symmetry which is unbroken quite earlier from the inflationary era, but it is broken during the inflationary era. The cosmological implications of the axions have quite appealing features and, in view of the growing research stream that is related with axion experiments and observations ([12–22] see also [23]), axion physics has become quite timely. Actually, the axion may be one of the last realistic weakly interacting massive particles (WIMP) [24], since a longstanding number of experiments focusing on large WIMP masses ended up with no result. Thus, unless

supersymmetry is discovered in the Large Hadron Collider, the axion seems to be the last realistic particle dark matter candidate.

Very closely related to axion gravity, the so-called Chern-Simons term [25–41], coupled with a function depending on the axion field, may be a realistic string theory originating correction, which may have a direct impact on low-energy gravitational phenomena and even in the inflationary era itself. The Chern-Simons term coupled to the axion has the form  $U(\phi)\tilde{R}R$ , and it is just the Chern-Pontryagin density. This string motivated term can have quite interesting effects on inflationary physics, as it was demonstrated first in Refs. [42,43]; see also Refs. [3–5] for some recent modified gravity applications. As it was shown in [42], the Chern-Simons term can affect directly the tensor modes of the primordial perturbations, and this can have a measurable effect on the tensor-to-scalar ratio. This feature was also shown to occur in the context of the Chern-Simons  $F(R)$  gravity [3]. Also in Ref. [43], it was demonstrated that the primordial gravitational waves have two polarizations that propagate in a different way. This phenomenon is similar and is related to chiral gravitational waves, which are also a very timely subject of current research [44–51].

Motivated by the above, in this work we shall investigate the propagation of gravitational waves, in the presence of

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influences the polarization of cosmic microwave background, E mode, and B mode in the early Universe [60]. It is conceivable that more complex axion scalar couplings to the Chern-Simons term may further perplex the dispersion relations for the two polarization modes. For the low-frequency mode, the Chern-Simons scalar coupling works against the dissipation in the right-handed mode, which works to increase the dissipation for the left-handed mode. On the other hand, for the high frequency mode, the Chern-Simons scalar coupling makes the dissipation of the gravitational wave four times stronger than that in case of the standard Einstein gravity.  $U_0 = 0$  in (46). In addition to the above findings, we demonstrated that there exists a nontrivial mixing between the two different polarization modes, which strongly suggests differences between the standard scalar axion Einstein gravity and the Chern-Simons axion Einstein gravity. Just for comparison, we also investigated the propagation of the gravitational wave in the  $F(R)$  gravity model and found that there is no difference between the right-handed mode and the left-handed mode, which is, of course, because the model does not violate the parity. We should also note that the  $F(R)$  gravity includes a scalar propagating mode as in the Einstein frame action (52). Even in the Chern-Simons axion Einstein gravity, as clear from the action (12), there appears a propagating scalar mode. Both of the scalar modes are massive and there is not an explicit difference. However, regarding the gravitational waves, in the  $f(R)$  gravity case, one has a scalar component of the gravity waves, which is, however, not present in the Chern-Simons axion Einstein case. The only effect of the Chern-Simons term is to discriminate between the two tensor modes of the

gravitational wave, and in fact this is the new feature that the Chern-Simons term induces, and it is a challenge for the gravitational wave astronomy to find any parity violating gravitational modes. A highly nontrivial task is to investigate the  $F(R)$  gravity extension of the axion Chern-Simons gravity. We aim to study this case in a future work.

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## APPENDIX: DETAILED FORM OF EINSTEIN TENSOR COMPONENTS

In this Appendix we present the explicit form of the  $(t, t)$ ,  $(i, j)$ , and  $(t, i)$  components of the Einstein tensor of Eq. (22) in the FRW background (15), which are

$$G_{tt} = -\dot{H}h_{tt} + H^2 \left[ -3h_{tt} + \frac{2}{a^2}(h_{xx} + h_{yy} + h_{zz}) \right] + \frac{H}{a^2} \left[ -\frac{\partial}{\partial t}(h_{xx} + h_{yy} + h_{zz}) + 2 \left( \frac{\partial h_{tx}}{\partial x} + \frac{\partial h_{ty}}{\partial y} + \frac{\partial h_{tz}}{\partial z} \right) \right] + \frac{1}{a^4} \left[ \frac{1}{2} \left\{ \frac{\partial^2}{\partial x^2}(h_{yy} + h_{zz}) + \frac{\partial^2}{\partial y^2}(h_{xx} + h_{zz}) + \frac{\partial^2}{\partial z^2}(h_{xx} + h_{yy}) \right\} - \frac{\partial^2 h_{xy}}{\partial x \partial y} - \frac{\partial^2 h_{xz}}{\partial x \partial z} - \frac{\partial^2 h_{yz}}{\partial y \partial z} \right], \quad (\text{A1})$$

$$G_{xx} = \dot{H} \left[ a^2 h_{tt} - h_{yy} - h_{zz} + \frac{8\dot{U}}{a} \left( \frac{\partial h_{xz}}{\partial y} - \frac{\partial h_{xy}}{\partial z} \right) \right] + H^2 (3h_{tt} - h_{yy} - h_{zz}) + H \left[ a^2 \frac{\partial h_{tt}}{\partial t} - \frac{1}{2} \frac{\partial(h_{yy} + h_{zz})}{\partial t} - \frac{\partial h_{ty}}{\partial y} - \frac{\partial h_{tz}}{\partial z} + \frac{8\ddot{U}}{a} \left( \frac{\partial h_{xz}}{\partial y} - \frac{\partial h_{xy}}{\partial z} \right) + \frac{8\dot{U}}{a} \left( \frac{\partial^2 h_{xz}}{\partial y \partial t} - \frac{\partial^2 h_{xy}}{\partial z \partial t} \right) \right] + \frac{1}{2} \left( \frac{\partial^2(h_{yy} + h_{zz})}{\partial t^2} + \frac{\partial^2 h_{tt}}{\partial y^2} + \frac{\partial^2 h_{tt}}{\partial z^2} \right) - \frac{\partial^2 h_{ty}}{\partial y \partial t} - \frac{\partial^2 h_{tz}}{\partial z \partial t} + \frac{4\ddot{U}}{a} \left( \frac{\partial^2 h_{tz}}{\partial y \partial x} - \frac{\partial^2 h_{ty}}{\partial z \partial x} + \frac{\partial^2 h_{xy}}{\partial z \partial t} - \frac{\partial^2 h_{xz}}{\partial y \partial t} \right) + \frac{4\dot{U}}{a} \left( \frac{\partial^3 h_{tz}}{\partial y \partial x \partial t} - \frac{\partial^3 h_{ty}}{\partial z \partial x \partial t} - \frac{\partial^3 h_{xz}}{\partial y \partial t^2} + \frac{\partial^3 h_{xy}}{\partial z \partial t^2} \right) + \frac{1}{a^2} \left( \frac{\partial^2 h_{yz}}{\partial z \partial y} - \frac{1}{2} \frac{\partial^2 h_{yy}}{\partial z^2} - \frac{1}{2} \frac{\partial^2 h_{zz}}{\partial y^2} \right) + \frac{4\dot{U}}{a^3} \left( \frac{\partial^3 h_{xz}}{\partial y^3} - \frac{\partial^3 h_{xy}}{\partial z^3} - \frac{\partial^3 h_{xy}}{\partial z \partial y^2} + \frac{\partial^3 h_{xz}}{\partial y \partial z^2} - \frac{\partial^3 h_{yz}}{\partial x \partial y^2} + \frac{\partial^3 h_{yz}}{\partial x \partial z^2} + \frac{\partial^3(h_{yy} - h_{zz})}{\partial x \partial y \partial z} \right), \quad (\text{A2})$$

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