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The Flattened Infinitesimal Transformations Generated by the Infinitesimal Concircular Transformations

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Abstract—In this work we study the flattening properties of the complete lift of infinitesimal concircular transformation. The tangent bundle is considered as the affinely connected space with the horizontal lift connection. We introduce the concept of the E-lift for a tensor field of arbitrary type which is necessary in the covariant differentiation with respect to the horizontal lift connection.

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INTRODUCTION

There exist at least two branches of generalization of the theory of geodesic mappings of the Riemannian spaces. The first deals with certain specified curve class in the generalized space and with the mappings which transfer the curves of this class into possibly another curves of the same class. For example, the authors specify geodesic circles and consider the concircular mappings in the Riemannian space [1] or analytically planar curves and holomorphically-projective mappings in the Kähler space [2]. The second approach assumes existence of several curve classes in the space and deals with the mappings which transfer curves of one class into curves of another one. This approach was widely exploited by the members of the Odessa Geometrical School under the guidance of N. S. Sinukov and S. G. Leiko. The geodesic curve class can be extended to the class of so-called flattened curves [3]. The flattened curve of order r is a curve whose points are of the flattening order no greater than r-th. In this case an r-geodesic mapping can be defined as the mapping which transforms geodesic curves into the flattened curves of order r. This notion was introduced by S. G. Leiko and is a consistent generalization of an almost geodesic mapping introduced by N. S. Sinukov [4]. Moreover, S. G. Leiko extended the flattened curves theory to infinitesimal transformations with the help of the r-geodesic infinitesimal map (for the sake of brevity, further we will denote it by r-g. i. m.).

The researchers now make an extensive use of the tangent bundles endowed with an affine connection of complete or horizontal lift as generalized spaces. The works of Yano and Ishihara [5, 6] inspired the study of the fibre bundle mappings induced by the base manifolds maps. It seems interesting to find out which geometric properties of the base map are inherited by the induced one and vice versa. There exist some results which reveal the geometrical nature of the induced map from the flattened map viewpoint [7–11]. Also these results help us to construct some flattened maps. Similarly we may extend these constructions to the complete lift of the base infinitesimal mapping.

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