ISSN 1066-369X, Russian Mathematics, 2018, Vol. 62, No. 1, pp. 7–16. © Allerton Press, Inc., 2018.
Original Russian Text © S.S. Volosivets, B.I. Golubov, 2018, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2018, No. 1, pp. 10–20.

Generalized Absolute Convergence of Series of Fourier Coefficients With Respect to Haar Type Systems

S. S. Volosivets[1*] and B. I. Golubov[2]**

[1]*Saratov State University, ul. Astrakhanskya 83, Saratov, 410012 Russia*

[2]*Moscow Institute of Physics and Technology (State University) Institutskii per. 9, Dolgoprudnyi, Moscow Region, 141700 Russia*

Received August 26, 2016

Abstract—For the orthogonal systems of Haar type, introduced by Vilenkin in 1958, we study absolute convergence of series composed from positive powers of Fourier coefficients with multiplicators from the Gogoladze–Meskhia class. The conditions for convergence of the series are given in terms of either best approximations of functions in L^p spaces by polynomials with respect to Haar type systems or fractional modulus of continuity for functions from the Wiener spaces V_p, $p > 1$. We establish the sharpness of the obtained results.

DOI: 10.3103/S1066369X18010024

Keywords: *Haar type system, Fourier coefficients, L^p space, functions of bounded p-variation, best approximation, module of continuity.*

INTRODUCTION

Let $\mathbf{P} = \{p_n\}_{n=1}^{\infty}$ be a sequence of natural numbers such that $2 \le p_n \le N$ for all $n \in \mathbb{N}$. By definition, we put $m_k = p_1 \dots p_k$ for $k \in \mathbb{N}$ and $m_0 = 1$. If A is the set of numbers of the type l/m_k, $l \in [0, m_k] \cap \mathbb{Z}$, $k \in \mathbb{Z}_+ = \mathbb{N} \cup \{0\}$, then every $t \in [0, 1] \setminus A$ can be uniquely represented in the form

$$t = \sum_{k=1}^{\infty} j_k(t)/m_k,$$

where $j_k(t) \in [0, p_k) \cap \mathbb{Z}$. Every integer $n \ge 2$ is uniquely represented in the form $n = m_k + r(p_{k+1} - 1) + s$ where $k \in \mathbb{Z}_+$, $0 \le r < m_k$, $1 \le s < p_{k+1}$, $r, s \in \mathbb{Z}_+$. Let $\psi_1(t) = 1$ on $[0, 1]$; for $n \ge 2$ we put

$$\psi_n(t) = \psi_{k,r}^{(s)}(t) = m_k^{1/2} \exp(2\pi i s j_{k+1}(t)/p_{k+1})$$

for $t \in (r/m_k, (r+1)/m_k) \setminus A$ and $\psi_n(t) = 0$ outside $[r/m_k, (r+1)/m_k]$. At points $t \in A$ lying on $(0, 1)$ we set the value of ψ_n to be equal the half-sum of the left-hand and right-hand limits taken over the set $(0, 1) \setminus A$. We also define ψ_n at the points 0 and 1 by continuity. For the case, when all members of the sequence $\{p_n\}_{n=1}^{\infty}$ are primes, this definition was introduced by Vilenkin ([1], Appendix, § 1) and is a generalization of the definition of the well-known Haar system ([1], Chap. 2, § 2), obtained if we put $p_n = 2$, $n \in \mathbb{N}$. In [1] (Chap. 2, § 3 and Chap. 4, § 4) there was shown completeness of the Haar system and possibility of approximation, in the uniform metric, of every function from $C[0, 1]$ by polynomials with respect to the Haar system with any accuracy. In [2] it was shown that these properties of $\{\psi_n\}_{n=1}^{\infty}$ remain valid for the case when all members of the sequence $\{p_n\}_{n=1}^{\infty}$, $p_n \ge 2$ are arbitrary natural numbers. Some interesting results on the Haar system can be found in [3] (Chap. 10). In [4] for functions from the spaces $C[0, 1]$ and $L^p[0, 1]$ there were obtained estimates of best approximations

*E-mail: VolosivetsSS@mail.ru.
**E-mail: golubov@mail.mipt.ru.