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On Sobolev Problems Associated with Actions of Lie Groups

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Abstract—We consider Sobolev problems with nonlocal boundary conditions associated with an action of a compact Lie group. We find a natural conditions of ellipticity of such problems, obtain the corresponding finiteness theorem, and give the index formula.

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1. STATEMENT OF THE PROBLEM

Let M be a smooth closed manifold of dimension n, and X be its submanifold of codimension $\nu = n/2$. We also assume that there is a compact Lie group G acting on M.

The action of G induces a representation of the group in function spaces on M via the shift operators T_g, acting on functions u by the formula

$$(T_g u)(x) = u(g^{-1}x).$$

In the paper we investigate the following Sobolev problem with nonlocal boundary condition:

$$\begin{cases} Du \equiv f \pmod{X}, & u \in H^s(M),\ f \in H^{s-m}(M); \\ i^* Bu = \varphi, & \varphi \in H^{s-b-\nu/2}(X), \end{cases} \tag{1}$$

where D is a pseudodifferential operator (for short, further we write PDO) on M of order m, $i^* : H^{s-b}(M) \longrightarrow H^{s-b-\frac{\nu}{2}}(X)$ is the restriction operator on the manifold, induced by the embedding $i : X \subset M$, and the equivalence "$\equiv$" means that Du and f coincide outside X. Finally, the boundary condition in (1) is defined by the nonlocal operator

$$Bu = B_0 u + \int_G B_g T_g u\, dg, \tag{2}$$

associated with G (cf. [1]). Here B_0 is a PDO on M of order b, and B_g ($g \in G$) is a family of PDO on M of the same order b, smoothly depending on g.

We will assume that the order of operators is connected with indices of Sobolev spaces by the inequalities $s - b - \nu/2 > 0$ (it offers boundedness of the boundary operator) and $0 < m - s - \nu/2 \leq 1$ (both the inequalities guarantee that we need only one boundary condition in the Sobolev problem).

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