ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2016, Vol. 60, No. 4, pp. 1–4. © Allerton Press, Inc., 2016.
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On Uniqueness Theorem for a Class of Functions Analytic in a Halfplane

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Received August 28, 2014

Abstract—We obtain a uniqueness theorem for a class of analytic functions of exponential type in a halfplane.

DOI: 10.3103/S1066369X16040010

Keywords: *analytic function, singular limit function.*

1. INTRODUCTION

Let $H^\infty(\mathbb{C}_+)$ be the space of all functions bounded and analytic in $\mathbb{C}_+ = \{z : \operatorname{Re} z > 0\}$. It is well-known (see [1], 6.1, problem 5; [2], Chap. IV, the Phragmén–Lindelöf theorem; [3], theorem 3.5.1)

Theorem 1. *If* $f \in H^\infty(\mathbb{C}_+)$ *and* $\lim\limits_{x\to+\infty} |f(x)|^{1/x} = 0$, *then* $f \equiv 0$.

Some results, connected with Theorem 1, were developed in a series of publications (see, e.g., [1], 26.1; [4]).

Let $H^\infty_\sigma(\mathbb{C}_+)$, $0 \le \sigma < +\infty$, be the space of functions f, analytic in $\mathbb{C}_+$ with

$$\sup\left\{|f(z)|e^{-\sigma|z|} : z \in \mathbb{C}_+\right\} < +\infty.$$

Every function $f \in H^\infty_\sigma(\mathbb{C}_+)$ has angular limits almost everywhere on $\partial\mathbb{C}_+$ with $f(iy)e^{-\sigma|y|} \in L^\infty(\mathbb{R})$ ([5], theorem 2.1). Note that $H^\infty_0(\mathbb{C}_+) = H^\infty(\mathbb{C}_+)$.

The aim of the paper is to find an analog of Theorem 1 for the space $H^\infty_\sigma(\mathbb{C}_+)$. As a supplement to Theorem 1, we obtain

Corollary. If $f \in H^\infty(\mathbb{C}_+)$ has no zeros and $\varliminf\limits_{x\to+\infty} |f(x)|^{1/x} = 0$, then $f \equiv 0$.

The main result of the paper is

Theorem 2. *Let* $0 \le \sigma < +\infty$ *and* $f \in H^\infty_\sigma(\mathbb{C}_+)$.

1) *If*

$$\lim_{x\to+\infty} \left(|f(x)|^{\frac{1}{x}} x^{\frac{2\sigma}{\pi}}\right) = 0, \tag{1}$$

then $f \equiv 0$;

2) *If* f *has no zeros and*

$$\varliminf_{x\to+\infty} \left(|f(x)|^{\frac{1}{x}} x^{\frac{2\sigma}{\pi}}\right) = 0, \tag{2}$$

then $f \equiv 0$.

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