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Quasisolutions of One Inverse Boundary-Value Problem of Aerohydromechanics

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Abstract—We study one ill-posed problem, namely, the inverse boundary-value problem of aerohydromechanics, and propose a method for calculating its quasisolution in various normed spaces.

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INTRODUCTION

G. G. Tumashev [1, 2] was first to formulate and solve the inverse boundary-value problem of aerohydromechanics. This problem requires certain solvability conditions which appear only after we find the contour reparametrization and cannot be satisfied initially.

Let us recall the solution procedure.

Let the symbol C stand for an unknown contour. Assume that a velocity distribution $v(s)$ along the contour C is given, s is the curve parameter. We have to find the form of the contour C. Assume that the contour length equals l, the flow bifurcation happens at the point with the coordinate $s_a \in (0, l)$, and the flow vanishes for $s = 0$ and $s = l$. Let $v(s) < 0$ for $s \in [0, s_a)$, and $v(s) > 0$ for $s \in (s_a, l]$.

Comparing the complex flow potential of the contour C in the complex plane $z = x + iy$ with the complex flow potential of the unit circle in the plane $\zeta = re^{i\gamma}$, for parameters of contours we obtain the correlation $s = s(\gamma)$. Then we reconstruct the function $z = z(\zeta)$ which maps the exterior of the unit circle into the flow domain in the plane z.

The first requirement to the function $\log|\widetilde{v}(\gamma)|$ naturally appears when we reconstruct the contour. We write it as a condition imposed on the function $\log|v(\gamma)|$ different from that $\log|\widetilde{v}(\gamma)|$ by a known value.

The closedness condition for the contour C means that the function $\log|v(\gamma)|$ meets the complex equality

$$\int_0^{2\pi} \log|v(\gamma)|e^{i\gamma}d\gamma = A + iB. \tag{1}$$

Thus, the first condition defines the Fourier coefficients of $\cos\gamma$ and $\sin\gamma$ for the function $\log|v(\gamma)|$.

We arrive to the second condition on $\log|v(\gamma)|$ due to purely mechanical reasons. Since the flow velocity is fixed, we obtain the following condition on $\log|v(\gamma)|$:

$$\int_0^{2\pi} \log|v(\gamma)|d\gamma = C. \tag{2}$$

Note that constants A, B, and C subject to (1), (2) depend only on the form of the modified Zhukovskii–Mitchell function [2, 3].

So, in the general case the stated problem has no solution.

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