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Connection of Some Bilevel and Nonlinear Optimization Problems

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Abstract—In this paper we reduce a quadratic-linear bilevel optimization problem with a guaranteed solution to a family of bilevel problems in the optimistic statement. Then we reduce the obtained bilevel problems to nonconvex one-level optimization problems and solve the latter by nonconvex optimization methods.

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1. INTRODUCTION

Various applied problems [1–4] which arise in modeling hierarchical control systems in energetics, economics, ecology, etc. [2] lead to bilevel optimization problems. Usual solutions to the latter problems are optimistic (cooperative) and guaranteed (non-cooperative, pessimistic) ones [1–4]. In the first case one assumes that the interest of the upper level can be coordinated with the actions of the lower level. In the second case such a coordination of interests is absent.

In the last three decades many papers have been devoted to the search of optimistic solutions to bilevel problems [5], and only few ones have been dedicated to the search of a guaranteed solution [6–8]. Note that in bilevel problems even the search of an optimistic solution (which is simpler) is reduced, as usual, to the search of global solutions to nonconvex optimization problems [4], and therefore from the computational point of view it represents a complex problem [9–12]. Here we reduce the considered bilevel problem with a guaranteed solution to a nonconvex optimization problem.

2. PROBLEMS AND THEIR INTERCONNECTIONS

Consider the following quadratic-linear bilevel optimization problem:

$$\begin{aligned} &W(x,\varepsilon) \triangleq \sup_y\{F(x,y) \mid y \in Y_*(x,\varepsilon)\} \downarrow \min_x, \\ &x \in X,\ \ Y_*(x,\varepsilon) \triangleq \{y \in Y(x) \mid G(y) \le \inf_z[G(z) \mid z \in Y(x)] + \varepsilon\}, \end{aligned} \qquad (\mathcal{BP}(\varepsilon))$$

where $X \triangleq \{x{\in}I\!R^m \mid Ax{\le}a,\ \ x{\ge}0\}$, $Y(x) \triangleq \{y{\in}I\!R^n \mid A_1x{+}B_1y{\le}b,\ \ y{\ge}0\}$, $F(x,y) \triangleq \frac{1}{2}\langle x, Cx\rangle + \langle c,x\rangle - \frac{1}{2}\langle y, C_1y\rangle + \langle c_1,y\rangle$, $G(y) \triangleq \langle d,y\rangle$, $A \in I\!R^{p\times m}$, $A_1 \in I\!R^{q\times m}$, $B_1 \in I\!R^{q\times n}$, $C = C^T \ge 0$, $C_1 = C_1^T \ge 0$, $c \in R^m$, $c_1, d \in R^n$, $a \in R^p$, $b \in R^q$, and $Y_*(x,\varepsilon)$ is the set of ε-solutions to the lower level problem.

Here $F(x,y)$ and $G(y)$ are efficiency criteria of players of the upper and lower levels, therefore $W(x,\varepsilon) = \sup_y\{F(x,y) \mid y \in Y_*(x,\varepsilon)\}$ is an estimate of the efficiency of the strategy of the upper level player [1, 2].

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