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Sufficient Conditions for the Local Controllability of Systems with Random Parameters for an Arbitrary Number of System States

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Abstract—We obtain sufficient conditions for the existence of a nonanticipating control for linear systems with stationary random parameters. We consider the case of a bounded control and an arbitrary number of system states. We estimate the probability that the system is nonanticipatingly locally controllable on a fixed time interval. We formulate the main assertions in terms of Lyapunov functions, choosing the latter in the class of piecewise continuously differentiable functions.

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1. INTRODUCTION

We consider the problem on the local controllability for a linear nonstationary system with random parameters. Note that the controllability of dynamic systems with random parameters is studied in many papers (e.g., [1–6]. The data on the "future" behavior of a system of this type may be unknown; therefore a question arises on the existence of a nonanticipating control.

The notion of a "nonanticipating control" was introduced by the Sverdlovsk School of the control theory (see [7], P. 17; [8], P. 45). The problem on the construction of a control of the mentioned type for deterministic systems was also studied in [9, 10]. A control $u(t, x)$ is said to be *nonanticipating,* if its construction at a time moment $t = \tau$ is based only on the data on the system behavior for $t \leqslant \tau$.

In this paper we follow [11–14], where one obtains sufficient conditions for the existence of a nonanticipating control for a linear system with stationary random parameters. In [11, 12] one studies conditions of complete controllability, i.e., no constraints are imposed on the control $u \in \mathbb{R}^m$. In papers [13, 14] one considers the case, when u belongs to a convex compact set $U \subset \mathbb{R}^m$, whose interior contains zero. One also assumes that the system can reside in two states ψ_1 and ψ_2. In this paper we study the case of an arbitrary number of states, what leads to certain additional restrictions on the behavior of system solutions.

Consider the system

$$\dot{x} = A(f^t\omega)x + B(f^t\omega)u, \quad (t, \omega, x, u) \in \mathbb{R} \times \Omega \times \mathbb{R}^n \times \mathbb{R}^m, \tag{1}$$

where the function $t \to \xi(f^t\omega) \doteq \big(A(f^t\omega), B(f^t\omega)\big)$ of the variable t is piecewise constant with each $\omega \in \Omega$ and takes on values in the set $\Psi = \{\psi_j\}_{j=1}^s$; the latter is a finite set of matrix pairs $\psi_j \doteq (A_j, B_j)$. We assume that $u \in U$, where U is a convex compact in $\mathbb{R}^m$ and $0 \in \operatorname{int} U$.

We obtain sufficient conditions for the existence of a nonanticipating control for system (1). In addition, we obtain a lower estimate of the probability that the system is locally controllable on a fixed time interval. We propose an algorithm, constructing a nonanticipating control.

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