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CORRECTION OF THE HAAR POLYNOMIALS APPLIED FOR COMPRESSION OF GRAPHIC INFORMATION

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The article is dedicated to the theoretical substantiation of a possibility to correct a polynomial in a two-dimensional Haar system representing an image after a nonlinear approximation. This approximation consists of nulling the small by their absolute value Fourier–Haar coefficients independently on their position in the initial polynomial, which represents the image before the information compression. In addition, the aim is a compression of graphic information without essential distortions (see [1], p. 103).

In this article a two-dimensional Haar system is represented in two forms: by both one-scale splashes and a simple product of one-dimensional Haar systems. The problem is arisen by a comparison with the following fact known for the Riesz projectors (see [2], p. 38): If for the initial function $f(x)$ the inequalities are valid $m \le f(x) \le M$, then any partial sum $S_N(f,x)$ of the Fourier–Haar series satisfies the same inequalities, i. e., $m \le S_N(f,x) \le M \; \forall N$. The rejection of the small coefficients results in a polynomial which, in general case, does not coincide with the partial sum $S_N(f,x)$. On a set, where for the resulting lacunar polynomial analogous inequalities are violated, one must apply a correction related to a cutting (truncation) at some given levels m and M. In what follows we shall estimate the measure of this set.

1. In this item a two-dimensional Haar system is considered to be a one-scale system of splashes. Denote by

$$\varphi(t) = \begin{cases} 1, & t \in [0,1]; \\ 0, & t \notin [0,1], \end{cases}$$

the scaling Haar function, while by

$$\psi(t) = \begin{cases} +1, & t \in [0,1/2); \\ -1, & t \in [1/2,1]; \\ 0, & t \notin [0,1], \end{cases}$$

a Haar splash. Set

$$\begin{aligned} \Phi^{(-1)}(x_1,x_2) &= \varphi(x_1)\psi(x_2), \\ \Phi^{(1)}(x_1,x_2) &= \psi(x_1)\varphi(x_2), \\ \Phi^{(0)}(x_1,x_2) &= \psi(x_1)\psi(x_2). \end{aligned}$$

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