ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2008, Vol. 52, No. 1, pp. 59–71. © Allerton Press, Inc., 2008.
Original Russian Text © M.I. Sumin, E.V. Trushina, 2008, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2008, No. 1, pp. 63–77.

Regularizing Properties of Pontryagin's Maximum Principle

M. I. Sumin* **and E. V. Trushina****

Nizhni Novgorod State University, pr. Gagarina 23, Nizhni Novgorod, 603950 Russia

Received May 17, 2007

DOI: 10.3103/S1066369X08010076

INTRODUCTION

This work is dedicated to the necessary and sufficient conditions for minimizing sequences in problems with inexact initial data. These conditions are tightly bound with the classical Pontryagin's maximum principle. The paper also covers regularizing properties of these sequences and those of the maximum principle itself, considering a minimizing sequence (rather than the classical optimal control) as the central theoretical notion.

It is well-known that Pontryagin's maximum principle [1] results from real needs, first of all, applied studies ([2], P. 7). However, in most papers which deal with the theory of the necessary conditions in the optimal control, the initial data in problems under consideration are assumed to be known exactly. The papers on optimal control problems, where studying the necessary and sufficient conditions, one takes into account (somehow or other) the possibility of inexact definition of input data, are relatively few [3, 4]. At the same time, it seems natural to develop the theory of the necessary and sufficient conditions in order to tolerate inexact definition of initial data. Consider, for comparison, the development of solution methods for optimization and optimal control problems [5], the theory of ill-posed problems [6]. In favor of this observation, let us adduce the following arguments. First, in numerous applications one inevitably encounters the necessity to use inexact initial data. Second, in the analysis of solution algorithms for optimization and optimal control problems, the necessary and sufficient optimality conditions play the most important role. Finally, third, generally speaking, optimal control problems represent a class of mathematical problems, where the instability of the initial data with respect to a perturbation is anticipated.

In this situation, the difficulties which occur in the study of the necessary and sufficient conditions are connected, first of all, with the following fact. The classical understanding of the optimal control in this case becomes nearly senseless: In a perturbed problem it does not necessarily exist, and even if it does, its relation to the initial optimal control in the initial problem is not quite clear. In this connection, let us adduce two simplest illustrative examples.

Example 0.1. Consider the following minimization problem, whose terminal constraint is specified as an equality:

$$\int_0^1 (x(t)+u(t))dt \to \min, \quad x(1)=1,$$

where $x(t)$ is a solution to the Cauchy problem

$$\dot{x}=u(t),\ \ x(0)=0,\ \ t\in[0,1],\ \ u(t)\in[-1,1].$$

One can easily see that in this problem the optimal control $u_0(t)\equiv 1$. One can identify it with the help of the maximum principle. Since in the problem under consideration the unit belongs to the boundary

*E-mail: msumin@sinn.ru.
**E-mail: lenatr@bk.ru.