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A Multiparametric Family of Solutions to a Singular Volterra Integral Equation in a Banach Space

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Abstract—We construct a multiparametric set of solutions to a singular Volterra integral equation of the first kind with a sufficiently smooth kernel in the space of integrable functions whose values belong to a Banach space.

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INTRODUCTION

Despite the great number of publications dedicated to Volterra equations, this theme remains actual and represents certain interest. This research was essentially developed in papers [1–3], where one described the fundamentals of the theory of linear integral equations of the first and second kinds, scalar solutions were found in Banach spaces with weights of a special type. In recent years one studied equations with real finitely smooth coefficients. In [4–9] one considered equations in both finite-dimensional and infinite-dimensional Banach spaces. In paper [10] one studied a singular Volterra integral equation of the Ist kind with a sufficiently smooth kernel in the space of summable on $[0,\delta]$ functions with values in a Banach space E. The present paper is a continuation of [10].

1. THE PROBLEM. THE MAIN RESULT

In a real Banach space E we fix the norm $\|\cdot\|_E$. In the space $L(E)$ of all linear bounded operators on E this norm induces the operator norm

$$\|A\|_{L(E)} = \sup_{\|x\|_E=1} \|Ax\|_E.$$

In the space $C([0,\delta],E)$ of functions continuous in the norm $\|\cdot\|_E$ on $[0,\delta]$ that take on values in E, the norm, as usual, is defined by the formula

$$\|\psi\|_{C([0,\delta],E)} = \max_{0\le x\le\delta} \|\psi(x)\|_E.$$

Finally, in the space C of all functions continuous in the norm $\|\cdot\|_{L(E)}$ on the triangle $0\le t\le x\le\delta$ with values in $L(E)$, we introduce the norm

$$|||Q|||_C = \max_{0\le t\le x\le\delta} \|Q(x,t)\|_{L(E)}.$$

In $L_1([0,\delta],E)$ we consider a Volterra integral equation of the Ist kind in the form

$$\int_0^x K(x,t)u(t)dt = 0 \quad (0\le x\le\delta); \tag{1}$$

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