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An Algebra Generated by Multiplicative Discrete Convolution Operators

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Abstract—We consider a Banach algebra generated by multiplicative discrete convolution operators. We construct a symbolic calculus for this algebra and in terms of this calculus we describe criteria for the Noetherian property of operators and obtain a formula for their index.

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Introduction. Presently we know a number of published works dealing with integral operators with homogeneous kernels and with the corresponding Banach algebras (see, e.g, [1–5] and references therein). The discrete operators with homogeneous kernels (the so called operators of multiplicative discrete convolution) are studied less. Unlike the one-dimensional continual case, such operators are not reducible to the convolution operators, and one needs other approaches to studying them. These operators were first systematically investigated in [6, 7]. In particular, in [6] one obtained a criterion for the Noetherian property and an index formula for multiplicative discrete convolutions.

In this paper we continue the research commenced in [6]. Our main goal is the study of the Banach algebra $\mathfrak{A}$ generated by one-dimensional operators of multiplicative discrete convolution. In what follows we construct symbolic calculus for this algebra. It enables us to obtain a criterion for the Noetherian property and a formula for the index. Our technique is based on studying the factor-algebra over the ideal of compact operators. In addition, we describe the space of maximal ideals. Evidently, this description is of independent interest.

Below we use the following denotations: $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{C}$ stand for the sets of positive integer, real, and complex numbers, correspondingly; $\mathbb{R}_+ = (0, \infty)$; $\dot{\mathbb{R}}$ is a compactification of $\mathbb{R}$ by a single point at infinity; for a space X the symbol $\mathcal{L}(X)$ stands for the Banach algebra of all linear bounded operators acting in X; for an element b of a Banach algebra $\mathfrak{B}$ symbols $\mathrm{Sp}_{\mathfrak{B}}(b)$ and $\mathrm{Spr}_{\mathfrak{B}}(b)$ stand for the spectrum and the spectral radius of $\mathfrak{B}$, correspondingly; $\ell_p = \ell_p(\mathbb{N})$, where $1 < p < \infty$, is the Banach space of complex sequences $\{\varphi_n\}_{n=1}^{\infty}$ such that $\sum\limits_{n=1}^{\infty} |\varphi_n|^p < \infty$.

1. We define an operator K in the space ℓ_p, $1 < p < \infty$, by the formula

$$(K\varphi)_m = \sum_{n=1}^{\infty} k(m,n)\varphi_n, \quad m \in \mathbb{N}, \tag{1}$$

where $\varphi = \{\varphi_n\}_{n=1}^{\infty}$, and the function $k(x, y)$ is defined on $\mathbb{R}_+ \times \mathbb{R}_+$ and satisfies the following conditions:

1) the homogeneity of degree (-1), i.e.,

$$k(\alpha x, \alpha y) = \alpha^{-1} k(x, y) \ \ \forall\, \alpha > 0;$$

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