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Periodic Dyadic Wavelets and Coding of Fractal Functions

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Abstract—Recently, using the Walsh–Dirichlet type kernels, the first author has defined periodic dyadic wavelets on the positive semiaxis which are similar to the Chui–Mhaskar trigonometric wavelets. In this paper we generalize this construction and give examples of applications of periodic dyadic wavelets for coding the Riemann, Weierstrass, Schwarz, van der Waerden, Hankel, and Takagi fractal functions.

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1. PERIODIC DYADIC WAVELETS

In the paper [1], Chui and Mhaskar use the modified Dirichlet kernels

$$D_m^*(x) := \frac{1}{2}(1+\cos mx) + \sum_{k=1}^{m-1} \cos kx, \quad m \in \mathbb{N}, \quad x \in \mathbb{R},$$

to define the wavelet bases in spaces of trigonometric polynomials. Similar wavelet constructions exist for the classical orthogonal Chebyshev, Legendre, Hermite, Jacobi, and Lagger polynomials [2, 3]. Papers [4–7] and [8] (Chap. 9) are devoted to the construction of multiresolution analysis for periodic wavelets. In this paper we generalize the construction of periodic wavelets on the semiaxis $\mathbb{R}_+ = [0, \infty)$ defined in [9] on the base of the Walsh polynomials and the corresponding Dirichlet type kernels. Some other wavelet constructions in the Walsh analysis are studied in [10–13]. We show that the introduction of the parameter a allows the optimization of the basis for the application of the studied wavelets in signal processing (cf. [14, 15]).

2. PERIODIC DYADIC WAVELETS

Recall that the *system of Walsh functions* $\{w_l \mid l \in \mathbb{Z}_+\}$ on $\mathbb{R}_+$ is defined by the equalities

$$w_0(x) \equiv 1, \quad w_l(x) = \prod_{j=0}^{k} (w_1(2^j x))^{\nu_j}, \quad l \in \mathbb{N}, \quad x \in \mathbb{R}_+,$$

where the numbers k and ν_j are determined from the binary decomposition

$$l = \sum_{j=0}^{k} \nu_j 2^j, \quad \nu_j \in \{0,1\}, \quad \nu_k = 1, \quad k = k(l),$$

and the function $w_1(x)$ is defined on $[0,1)$ as

$$w_1(x) = \begin{cases} 1, & \text{if } x \in [0, 1/2); \\ -1, & \text{if } x \in [1/2, 1), \end{cases}$$

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