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SHEAF OF LOCAL HAMILTONIANS ON SYMPLECTIC MANIFOLD WITH MARTINET SINGULARITIES

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1. Introduction

Let ω_0 be a closed 2-form on $\mathbb{R}^{2n}$. A symplectic manifold with singularities of type ω_0 is a $2n$-dimensional manifold M endowed with a closed form ω such that for any point $p \in M$ there exists a neighborhood $U(p)$ and a diffeomorphism $\phi_p : U(p) \to \mathbb{R}^{2n}$ such that $\omega|_{U(p)} = \phi_p^* \omega_0$ [1].

In [2]–[5] it was proved that on $\mathbb{R}^4$ five generic types of germs of closed 2-forms exist, among them the following four types are stable:

Type 0

$$\omega_0 = dx^1 \wedge dx^2 + dx^3 \wedge dx^4,$$

Type I

$$\omega_0 = x^1 dx^1 \wedge dx^2 + dx^3 \wedge dx^4,$$

Type II-e (the elliptic type)

$$\omega_0 = dx^1 \wedge dx^2 + x^3 dx^1 \wedge dx^4 + x^3 dx^2 \wedge dx^3 + x^4 dx^2 \wedge dx^4 + (x^1 - (x^3)^2) dx^3 \wedge dx^4, \tag{1}$$

Type II-h (the hyperbolic type)

$$\omega_0 = dx^1 \wedge dx^2 + x^3 dx^1 \wedge dx^4 + x^3 dx^2 \wedge dx^3 - x^4 dx^2 \wedge dx^4 + (x^1 - (x^3)^2) dx^3 \wedge dx^4. \tag{2}$$

One can characterize these types in the following way (see [2] for details). Let ω be a closed 2-form on a four-dimensional manifold M. Let us denote by Σ the set of points, where ω is degenerate. In what follows we assume that Σ is a three-dimensional submanifold of M.

If a point p does not lie in Σ, then, by the Darboux theorem, ω has type 0 at p.

Now let $p \in \Sigma$ and $\omega(p) \neq 0$. If the kernel E_p of $\omega(p)$ is transversal to Σ, then ω has type I at p. The set $V = U \cap \Sigma$ is open in Σ and consists of points of type I, and $U \setminus V$ is everywhere dense in U and consists of points of type 0.

Let $\Sigma' \subset \Sigma$ be the set of points p such that $E_p \subset T_p\Sigma$. If Σ' is a submanifold in a neighborhood of p, and E_p is transversal to Σ', then ω has type II-e, or II-h, at p. Let $V = U \cap \Sigma$, $W = U \cap \Sigma'$. Then $U \setminus \Sigma$ is everywhere dense in U and consists of points of type 0, $V \setminus W$ is everywhere dense in V and consists of points of type I, and W consists of points of type II.

Let us consider a closed 2-form ω on a four-dimensional manifold M. If all points of M has type 0, then (M, ω) is a symplectic manifold. If any point of M has type 0 or I, then (M, ω) will be called a *symplectic manifold with Martinet singularities of type* I. If any point of M has type 0, I, or II, then (M, ω) will be called a *symplectic manifold with Martinet singularities of type* II.

Note that a symplectic manifold (M, ω) with Martinet singularities is a symplectic manifold with singularities of type ω_0, where ω_0 is given by (1) or (2). Indeed, let $\phi(p) = (x_0^1, x_0^2, x_0^3, x_0^4)$. Then p has type 0 if and only if $x_0^1 \neq 0$; p has type I if and only if $x_0^1 = 0$ and $(x_0^3)^2 + (x_0^4)^2 \neq 0$;