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ON THE GEOMETRY OF THE THIRD ORDER EVOLUTIONARY EQUATION

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Introduction

In this article we shall study a differential geometric structure induced by the evolutionary differential equation. We shall construct a fundamental object which determines the equation's geometry, and find a connection enveloped by the prolonged fundamental object. We also find conditions under which the connection can be extended to a connection in a bundle with a greater structure group, as well as conditions under which the extended connection determines the representation of zero curvature for a given third order evolutionary equation.

1. Fundamental object

Let us consider the third order partial differential equation

$$u_t = \varphi(t, x^1, \dots, x^n, u, u_j, u_{jk}, u_{jkl}). \qquad (*)$$

Here $t, x^1, \dots, x^n$ are independent variables, where $t = x^0$ is a distinguished variable ("time") and $x^1, \dots, x^n$ are space variables, u is an unknown function, u_j, u_{jk}, u_{jkl} are the partial derivatives of u with respect to x^i up to the third order. We assume that $t, x^1, \dots, x^n, u$ are adapted local coordinates of a fibered $(n+2)$-dimensional manifold $\mathbf{E}$ with a fibered $(n+1)$-dimensional base $\mathbf{M}$ whose local coordinates are $t, x^1, \dots, x^n$. The admissible transformations of the local coordinates are as follows:

$$\begin{cases} \tilde{t} = \varphi^0(t); \\ \tilde{x}^i = \varphi^i(t, x^1, \dots, x^n), \quad i = 1, \dots, n; \\ \tilde{u} = \varphi^{n+1}(t, x^1, \dots, x^n, u). \end{cases} \qquad (1a)$$

These transformations can be rewritten in the form:

$$\begin{cases} t = \psi^0(\tilde{t}); \\ x^i = \psi^i(\tilde{t}, \tilde{x}^1, \dots, \tilde{x}^n), \quad i = 1, \dots, n; \\ u = \psi^{n+1}(\tilde{t}, \tilde{x}^1, \dots, \tilde{x}^n, u). \end{cases} \qquad (1b)$$

Let us write down equation $(*)$ in a more general form:

$$\lambda_0 - \varphi(t, x^1, \dots, x^n, u, \lambda_j, \lambda_{jk}, \lambda_{jkl}) = 0, \qquad (2)$$