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ON THE CONVERGENCE OF GLOBAL SEARCH ALGORITHM IN THE PROBLEM OF CONVEX MAXIMIZATION ON ADMISSIBLE SET

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1. Introduction

In recent years nonconvex problems of optimization obtain an increasing attention, in particular, due to wide field of their applications (see [1]–[6]). An example is the convex maximization problem (see [4]–[9])

$$f(x) \to \max, \quad x \in D, \tag{1}$$

where $f : R^n \to R \cup \{+\infty\}$ and $D \subset R^n$ are convex. In spite of its simple shape, this problem turns to be complex for both investigations and, mainly, numerical solution, because a large quantity of local maxima may exist there not being global (see [1]–[9]). Therefore, for problems of that kind, the application of popular methods of branches and boundaries, cuttings, submersions and so on is assumed to be intrinsic. In contrast to these approaches, in [8]–[11] the methods of global search are based on the necessary and sufficient conditions of global optimality.

A natural question arises concerning the global convergence of such algorithms. The convergence theorems give, seemingly, a certain theoretical support to numerical experiments (see [1], pp. 415–420; [3], pp. 40–42). Therefore the main objective of the present article is the proof of the theorem on convergence of the algorithm of global search suggested above for problem (1). To this end we use the notion of a resolving set introduced here. Earlier (see [9]), a similar theorem was proved only for the quadratic problem. In addition, we succeeded in an essential weakening of some requirements in [9]. For problem (1) we prove also the convergence of the method of local lift.

2. Local lift method

In what follows we assume that in problem (1) the function $f : R^n \to R$ is convex and differentiable in a certain (open) domain Q, containing an admissible set and the Lebesgue set of the function $f(\cdot)$ of the form $L(f) = \{x \in R \mid f(x) \le \sup(f, D)\}$. In this situation, we assume that the function $f(\cdot)$ is bounded from above on D

$$\sup(f, D) \triangleq f_* < +\infty. \tag{2}$$

As known (see [3], theorem 1, p. 179), the condition

$$\langle \nabla f(z), x - z \rangle \le 0 \ \ \forall x \in D \tag{3}$$

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