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PERTURBATION OF SINGLE-VALUED OPERATOR BY A MULTIVALUED MAPPING OF HAMMERSTEIN TYPE WITH NONCONVEX IMAGES

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In the present article we consider an inclusion whose right side is a multivalued operator consisting of the algebraic sum of the values of a single-valued operator and the values of a mapping of Hammerstein type with nonconvex images. The study of the inclusion of this type is conditioned by the following: The questions of resolvability of boundary value problems for differential inclusions, where linear boundary value conditions (linear vector-functional) "are subject" to some perturbations which can be represented in the form of a nonlinear vector-functional defined on the space of continuous functions, can be reduced to that form of inclusion. Moreover, topological properties of the set of solutions of those boundary value problems can be effectively investigated by means of the inclusion to be considered here.

It turned out that the question on the structure of the set of solutions of the inclusion under investigation can be studied by means of the methods suggested in [1]. Here we shall formulate conditions when the closure (in a space of continuous functions) of the set of solutions of the inclusion coincides with the set of solutions of a "convexized" inclusion and also the "bang-bang" principle. In addition, we shall prove that the intersection in the space of continuous functions of the closures of the sets of δ-solutions of the inclusion coincides with the set of solutions of the "convexized" inclusion. Let us note that the result concerning the δ-solution of the inclusion refines the result in [2], and also complements the results of [3], [4] (p. 62), [5], and [6], in the case, where the multivalued mapping fails to satisfy the Lipschitz condition or a more general condition in which the Hausdorff distance between the values of the multivalued mapping cannot be estimated by the Kamke function.

Notation and some definitions

Let X be a Banach space with the norm $\|\cdot\|$. Let $U \subset X$. Denote by $\overline{U}$ the closure, $\operatorname{co} U$ the convex hull of the set U; $\overline{\operatorname{co}}U = \overline{\operatorname{co} U}$; $\operatorname{ext} U$ is the set of endpoints of the set U; $\overline{\operatorname{ext}}U = \overline{\operatorname{ext} U}$; $\|U\|_X = \sup\{\|u\| : u \in U\}$; $B_X[u;r]$ is an open ball in the space X with the center at the point u and the radius $r > 0$; $U^\varepsilon \equiv \overline{\bigcup_{u\in U} B_X[u;\varepsilon]}$ stands for a closed ε-neighborhood of the set U if $\varepsilon > 0$, and $U^0 \equiv \overline{U}$; $2^X(\Omega(X))$ stands for the set of all nonempty, bounded (nonempty, bounded, convex) subsets of the space X.

Let $\Phi_1, \Phi_2 \subset X$ and $h^+_X[\Phi_1;\Phi_2] = \sup\{\rho_X[y;\Phi_2] : y \in \Phi_1\}$, where $\rho_X[\cdot;\cdot]$ is the distance between a point and a set in the space X, $h_X[\Phi_1;\Phi_2] = \max\{h^+_X[\Phi_1;\Phi_2], h^+_X[\Phi_2;\Phi_1]\}$ is the Hausdorff distance between the sets Φ_1 and Φ_2.

Let $\mathbb{R}^n$ be the space of n-dimensional columns vectors with norm $|\cdot|$; $\operatorname{comp}[\mathbb{R}^n]$ be a set of all nonempty compacts of the space $\mathbb{R}^n$. Let $\mathcal{U} \subset [a,b]$ be a Lebesgue measurable set, moreover, $\mu(\mathcal{U}) > 0$, where μ is the Lebesgue measure. We denote by $L^n(\mathcal{U})$ the space of functions $x : \mathcal{U} \to \mathbb{R}^n$