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Finite Number of Iterations in the Two-Step Extragradient Method

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Abstract—We consider two-step extragradient method. This method is used for solving variational inequalities and related problems. We show the convergence of the method in a finite number of iterations when the condition of sharpness is fulfilled.

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INTRODUCTION

Variational inequalities are a generalization of optimization problems, which include classical problems of mathematical programming, the multicriteria optimization, saddle problems, games, equilibrium problems etc. [1–3]. Extragradient methods [4, 5] present an efficient tool for solving variational inequalities, because for the convergence they require fewer conditions than the gradient ones. For extragradient methods, the monotone convergence with respect to the norm to one of solutions to variational inequality is known, for variational inequalities with a bilinear functional the convergence of extragradient method with the rate of geometric progression is known [5], for variational inequalities with a linear functional under some additional conditions the finiteness of iterations number for the extragradient method [6] has been proved.

A new class of extragradient methods is two-step extragradient methods [7, 8]. We note that two-step and even three-step methods are not new and are investigated by many scientists [9–11], although such methods have obtained their name in view of using several previous iteration points on each iteration, but not due to the use of predictive points, as in [7, 8]. The use of earlier obtained iteration points is applied for the regularization and the solution to problems with inexact initial data. In paper [12] one expresses the idea of multi-step methods, which are structurally similar to the two-step extragradient method [7, 8]. At the same time, in [12] one investigates methods with the one-step extrapolation, which is applied for the construction of strictly separating hyper-plane, with different modes for the choice of the main and extrapolating step.

In the present paper we prove the finiteness of iterations number for two-step extragradient methods in a general nonlinear case.

1. THE PROBLEM DEFINITION

Let us consider the main object under investigation: the variational inequality.

To solve a *variational inequality* means to find a vector $z^* \in \Omega$, which satisfies conditions

$$\langle H(z^*), z - z^* \rangle \geq 0 \ \forall\, z \in \Omega, \tag{1}$$

where $H : \mathbb{R}^n \to \mathbb{R}^n$, Ω is a closed convex set, $\Omega \subset \mathbb{R}^n$, $z^* \in \Omega^*$ is the set of solutions to variational inequality, $\Omega^* \subset \Omega$.

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