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The Gellerstedt Problem for a Parabolic-Hyperbolic Equation of the Second Kind

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Abstract—In this paper we consider the Gellerstedt problem for a parabolic-hyperbolic equation of the second kind. We prove the unique solvability of this problem by means of a new representation for a solution to the modified Cauchy problem in a generalized class R.

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It is well-known [1] that the correctness of boundary-value problems for the equation

$$L_\alpha u \equiv u_{xx} + yu_{yy} + \alpha u_y = 0, \quad y < 0,$$

in domains with boundary parabolic degeneration essentially depends on the coefficient α at the lowest terms of the equation. Therefore it is intrinsic to assume that this coefficient also affects the gluing conditions on the degeneration line in boundary value problems for an equations in a mixed domain.

In this paper we consider the Gellerstedt problem for the equation

$$0 = \begin{cases} \frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} + a(x,y)u, & y \geq 0; \\ \frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} + \alpha\frac{\partial u}{\partial y}, & y < 0, \end{cases} \tag{1}$$

in the domain $D = D_1 \cup D_2^* \cup AB$.

Here the domain D_1 is bounded by segments AB, BB_0, A_0B_0, and AA_0 of lines $y = 0$, $x = 1$, $y = 1$, and $x = 0$, correspondingly, and the domain D_2^* (with $y < 0$) is bounded by characteristics of Eq. (1) $AC_1 : x - 2\sqrt{-y} = 0$, $EC_1 : x + 2\sqrt{-y} = x_0$, $EC_2 : x - 2\sqrt{-y} = x_0$, $BC_2 : x + 2\sqrt{-y} = 1$, and $AB : y = 0$, where x_0 is a fixed point in the interval $(0, 1)$.

In Eq. (1) $a(x,y)$ is a given function such that $a(x,y) \in C^{(0,h)}\left(\overline{D}_1\right)$, $0 < h < 1$, $\alpha = -n + \alpha_0$, where $-1/2 < \alpha_0 < 0$ or $0 < \alpha_0 < 1/2$, and $n = 0, 1, 2, 3, \ldots$ In general, this problem with the usual gluing conditions on the degeneration line is ill-posed. Here we replace these conditions with somewhat other ones that depend on α.

Note that with $n = 0, 1$ this problem is studied in paper [2], and with $n = 3$ in [3] one proves the unique solvability of the Tricomi problem for parabolic-hyperbolic equations by means of a new representation of a generalized solution in the class R to the modified Cauchy problem. This paper generalizes these results for the indicated values of n. Unlike the previous papers, here we use two identities (obtained by us) that allow one to find a representation for a generalized solution in the class R to the modified Cauchy problem.

Let D_{21} and D_{22} be parts of the domain D_2^* with boundaries AC_1EA and EC_2BE, correspondingly.

The Gellerstedt problem. *Find in the domain D a function $u(x,y)$ with following properties*:

a) $u(x,y) \in C\left(\overline{D}\right)$;

b) $u(x,y)$ *is a regular solution to Eq.* (1) *in the domain D_1 and its generalized solution from the class R in the domain D_2^** (*see the definition of the class R below*);

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