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Descent Method with Inexact Linesearch for Mixed Variational Inequalities

I. V. Konnov[1*]

[1] *Kazan State University, 18 Kremlyovskaya str., Kazan, 420008 Russia*

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Abstract—We propose a descent method with respect to a merit function for the mixed variational inequality involving a general nonlinear mapping and a convex, but not necessarily differentiable function. The method utilizes an inexact linesearch procedure. Its convergence is proved under the additional assumptions of continuity and strong monotonicity of the cost mapping.

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1. INTRODUCTION

Let U be a convex set in the space R^n, $G : U \to R^n$ a continuous mapping, and $f : U \to R$ a convex, but not necessarily differentiable function. The mixed variational inequality is defined as the problem of finding a point $u^* \in U$ such that

$$\langle G(u^*), u - u^* \rangle + f(u) - f(u^*) \geq 0 \ \ \forall u \in U. \tag{1}$$

Problem (1) was first considered in [1, 2] and afterwards studied by many authors (e.g., [3]). It has a great number of applications in Economics and Mathematical Physics and in the case where $f \equiv 0$ it corresponds to the usual variational inequality. One of the most popular approaches to solve the usual variational inequality consists in its converting into an optimization problem with respect to some artificial merit (or otherwise, gap) function. Such merit functions were first proposed in [4, 5]. They allow one to convert a variational inequality into a constrained differentiable optimization problem.

For more general mixed variational inequalities, various classes of merit functions are contained in [6]–[8]. Then the initial problem reduces to a constrained optimization problem which involves a non-differentiable merit function in general. Usually, substantiation of convergence of solution methods for non-differentiable optimization meets various additional difficulties, which reduce essentially the class of suitable methods for work (e.g., [9, 10]). In particular, convergence of descent methods with respect to a gap function for problem (1) was established only for methods with exact linesearch, whereas various inexact Armijo type linesearch procedures are widely utilized in solution of the usual variational inequalities and nonsmooth optimization problems.

In this paper we propose a descent method with respect to a merit function for the mixed variational inequality (1), which utilizes an inexact linesearch procedure, but its convergence is proved under the same assumptions as those in the methods with exact linesearch.

[*] E-mail: konn-igor@yandex.ru.