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ON INFINITE BOUNDED POLYHEDRA WITH ISOGONAL FACES

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Introduction

One of the lines of the "anschauliche geometrie", constructing polyhedra with specific metric properties, originates in Plato's and Archimedes' studies. The necessary condition in the definition of the Plato polyhedra and of the Archimedes polyhedra is that the faces are required to be regular. It has taken almost two thousand years to find all the polytopes whose faces are regular polygons (see [1], [2]). In the present article the requirement for faces to be regular is replaced with a more general condition that the faces are to be isogonal.

1. Bounded polyhedra with isogonal faces

The geometry of bounded polytopes with regular faces in three-dimensional Euclidean space has been studied in [1], [2], where a complete list of these polyhedra was given. This class was expanded by B.A. Ivanov (see [3]) who considered faces partitioned into regular polygons via "conventional" edges (without taking new vertices). B.A. Ivanov and Yu.A. Pryakhin (see [4]) listed all the so-called "simple" polyhedra with conventional edges.

Yu.A. Pryahin proposed a further generalization and demonstrated in [5] that the polyhedra with isogonal faces, and also with "parqueted" faces (partitioned via conventional edges into isogonal faces) can be studied in a way similar to the polyhedra with regular faces, because the number of these polyhedra is finite up to the combinatorial equivalence, except for four simple infinite sequences.

Theorem 1 (see [6]). *Up to the combinatorial equivalence, to each bounded polytope with isogonal faces one and only one bounded polytope with regular faces corresponds, and the number of different polytopes in one set is equal to that in the other one.*

The Theorem follows from two facts. First, in the construction of the edge net for polytopes with regular faces (see [2]) the equality of edges of all faces is used only to ensure the possibility of gluing faces together over the entire edges. This possibility occurs for polyhedra with isogonal faces. Second, in verifying that the edge net may be metrically realized as a polytope with regular faces, only the sizes of flat angles of faces and the dihedral angles between faces were used. This is also sufficient to verify that a polyhedron with isogonal faces exists.

Theorem 1 evidently holds for polyhedra with conventional edges.

Theorem 2. *In the three-dimensional Euclidean space the set of combinatorially different bounded polytopes with isogonal faces consists of one hundred and ten polytopes and two infinite series of prisms and antiprisms.*

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