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SINGULAR INTEGRAL EQUATIONS ON A COUNTABLE SET OF CLOSED NON-RECTIFIABLE AND FRACTAL CURVES

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In the present article we consider the following characteristical singular integral equation

$$\mathcal{K}^0\varphi \equiv a(t)\varphi(t) + b(t)S_\Gamma\varphi(t) = f(t), \tag{1}$$

where S_Γ is a singular integral operator with the Cauchy kernel on the system of curves Γ. The contour Γ (considered as a system of curves) is assumed to consist of a countable set of closed non-rectifiable curves Γ_k which are situated outside of each other and have a single condensation point $z_0 \neq \infty$. Our assumptions concerning the given functions $a(t)$, $b(t)$, $f(t)$ and the sought-for function $\varphi(t)$ will be formulated below in terms of fractal dimension of the set Γ. Since the contour Γ consists of non-rectifiable curves, the integral operator S_Γ is understood in the generalized sense.

1. *Introduction.* Integral equation (1) is well-studied in the case where the contour Γ consists of a finite number of closed and open simple smooth and piecewise-smooth curves (see, e. g., [1]–[5]).

In the classical monographs by F.D. Gakhov (see [1]) and N.I. Muskhelishvili (see [2]), the singular integral is defined by the equality

$$S^*_\Gamma\varphi(t) = \frac{1}{\pi i}\,\text{v. p.}\int_\Gamma \frac{\varphi(\tau)d\tau}{\tau - t}.$$

In the works by D.A. Kveselava (see [3]), V.D. Kupradze (see [4]), and T.G. Gegeliya (see [5]) it is defined by the formula

$$\widetilde{S}_\Gamma\varphi(t) = \frac{1}{\pi i}\,\text{v. p.}\int_\Gamma \frac{\varphi(\tau) - \varphi(t)}{\tau - t}d\tau + \varphi(t).$$

Provided that Γ is a smooth curve, we have $S^*_\Gamma\varphi = \widetilde{S}_\Gamma\varphi$.

Recently a number of works appeared in which an integral along a non-rectifiable curve is defined and studied (see, e. g., [6]–[10]). We shall take advantage of this generalized curvilinear integral in order to define the integral operator $\widetilde{S}_\Gamma$ on the contour Γ consisting of non-rectifiable curves. We shall obtain Sokhotskiĭ's formulas for this operator and study the questions of equivalence between the singular equation (1) and the Riemann boundary value problem for a countable set of non-rectifiable curves, investigated by B.A. Kats in [11].

2. *An integral along a non-rectifiable curve or along a countable system of non-rectifiable curves.* All proposed in [7]–[10] definitions of an integral along a non-rectifiable (fractal) curve can be reduced, in the case of a closed plane curve, to the following one. Let Γ be a simple closed curve

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