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CONVERGENCE OF EXPLICIT DIFFERENCE SCHEMES FOR A VARIATIONAL INEQUALITY OF THE THEORY OF NONLINEAR NONSTATIONARY FILTRATION

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1. Introduction

The present article continues investigations started in [1], where, for a variational inequality of the form

$$\int_0^T \left\langle \frac{\partial\varphi(u)}{\partial t}, v-u \right\rangle dt + \int_0^T \langle A(u,\nabla u), v-u\rangle dt \geq \int_0^T \langle f, v-u\rangle dt, \quad \forall\, v \in K, \tag{1.1}$$

a theorem on existence of a generalized solution from the set $K = \{v \in L_p(0,T; \overset{\circ}{W}{}_p^{(1)}(\Omega)) \cap L_\infty(0,T; L_\alpha(\Omega)), v \geq 0$ almost everywhere (a. e.) in $Q_T = \Omega \times (0,T]\}$ was proved, Ω is a bounded domain in the space R^n with the boundary Γ. In the inequality (1.1) $\langle g, v\rangle$ defines the value of the functional g from $L_{p'}(0,T; W_{p'}^{-1}(\Omega))$ on an element from $L_p(0,T; \overset{\circ}{W}{}_p^{(1)}(\Omega)), A(v,\nabla u) = -\sum\limits_{i=1}^n \frac{\partial}{\partial x_i}(a_i(x,v)k_i(x,\nabla u))$. In [1], in the proof of this theorem, there was being established the convergence of the solutions of a semidiscrete problem with a penalty to the solution of the inequality (1.1). In the present article we investigate the convergence of difference schemes (d. s.), which are constructed for a semidiscrete problem with a penalty, to the solution of the inequality (1.1).

The d. s. which are under our investigation are said to be *explicit* since the value of the spatial operator is calculated on the lower layer. These schemes differ by the penalty operators. In the first scheme the penalty operator is chosen as in [1]: $\beta(u) = |u^-|^{p-2} \times (-u^-)$, where $u^- = (|u|-u)/2$. We succeeded in proving this scheme only in a partial case where $k_i(x,\nabla u) = g_i(x,\nabla u)\frac{\partial u}{\partial x_i}$, the functions g_i being non-negative. In the second d. s., the β was chosen in the form of a difference approximation of the operator $Gu = -\sum\limits_{i=1}^n \frac{\partial}{\partial x_i}\left(\left|\frac{\partial(-u^-)}{\partial x_i}\right|^{p-2} \frac{\partial(-u^-)}{\partial x_i}\right)$. The convergence of d. s. in this case was proved under general assumptions on the operator A.

2. Description of the problem. Notation

As in [1], by a *generalized solution* of the inequality (1.1) we shall understand a function $u \in K$ satisfying (1.1) and being such that

$$\frac{\partial\varphi(u)}{\partial t} \in L_{p'}(0,T; W_{p'}^{-1}(\Omega)), \quad u(x,0) = u_0(x) \quad \text{a. e. in } \Omega. \tag{2.1}$$

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