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AN IMPLICIT DIFFERENCE SCHEME FOR SOLVING NONLINEAR EQUATIONS OF HYDRODYNAMICS

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In the present article we propose an implicit method of solution which reduces the dimension of the problem by 1. Here the computations, compared with the methods using explicit schemes, become only slightly more complicated. All the reasoning is carried out for an example of a boundary value problem for the barotropic motion of compressible inviscid gas. In [1], it was shown that the problem can be considered only for a rectangular domain $\overline{\Omega} = [0, a] \times [0, b]$.

Let us write in $\overline{\Omega}$ the system of hydrodynamical equations in the conservative form, using the standard notation, for the Cartesian coordinates x, y

$$\begin{aligned} &(\rho u)_x + (\rho v)_y = 0, \qquad (\rho uv)_x + (p + \rho v^2)_y = 0, \\ &(p + \rho u^2)_x + (\rho uv)_y = 0, \quad p = p(\rho). \end{aligned} \tag{1}$$

On $\partial\Omega$ we prescribe the boundary conditions:

$$\begin{aligned} &\rho u = RU(y), \quad \rho v = RV(y) \text{ for } x = 0; \qquad \rho v/\rho u = \Theta_1(y) \text{ for } x = a; \\ &\rho v/\rho u = \Theta_2(x) \text{ for } y = 0; \qquad \rho v/\rho u = \Theta_3(x) \text{ for } y = b. \end{aligned} \tag{2}$$

Problem (1), (2) is well-posed (see [2], [3]).

We introduce a notation by the following system

$$\begin{aligned} &\rho u = F(1), \qquad \rho v = F(2), \qquad p + \rho u^2 = F(3), \\ &\rho uv = F(4), \qquad \rho vu = F(5), \qquad p + \rho v^2 = F(6) \end{aligned} \tag{3}$$

and rewrite (1) in the form

$$F(n)_x + F(n+1)_y = 0, \quad n = 1, 3, 5. \tag{4}$$

On the grid domain $G = \{(i, j\}$, $i = \overline{1, N_i}$, $j = \overline{1, N_j}$, we replace the system (4) and the conditions (2) up to the second order accuracy by a system of difference equations with respect to a grid function F, corresponding to the grid patterns with interior and boundary nodes enumerated around the central point (i, j),

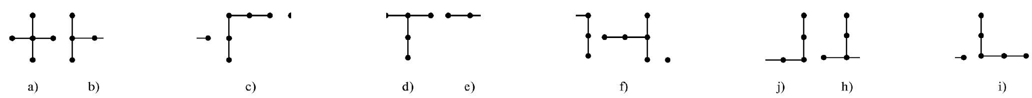

FIGURE 1