ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2014, Vol. 58, No. 12, pp. 1–6. © Allerton Press, Inc., 2014.
Original Russian Text © D. Amanov, 2014, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2014, No. 12, pp. 3–8.

Boundary-Value Problem for Degenerate Parabolic Equation of High Order with Varying Time Direction

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Received June 14, 2013

Abstract—In the present paper we investigate a boundary-value problem for a forward-backward parabolic equation in a rectangular domain and prove the existence of unique regular solution to this problem. The proof of the uniqueness of the solution is based on the spectral method, and in the proof of existence of solution we use the method of separation of variables. In the introduction we give a survey of related works.

DOI: 10.3103/S1066369X14120019

Keywords: *degenerate parabolic equation, regular solution, spectral method, separation of variables, Cauchy–Bunyakovskii inequality.*

1. INTRODUCTION

Boundary-value problems for second order parabolic equations with varying time direction

$$u_{xx} - u_t \cdot \operatorname{sgn} x = f(x,t)$$

and more general equations with main part

$$u_{xx} - u_t \cdot \operatorname{sgn} x$$

and non-characteristic degeneration line were studied by a number of authors [1–7].

The boundary-value problem for second order parabolic equation with varying time direction

$$u_{xx} - |x|^p u_t \cdot \operatorname{sgn} x = 0$$

is studied in [8]. The Gevrey problem for second order parabolic equation with varying time direction and with the Caputo derivative is investigated in [9].

The cited works are dealing with second order parabolic equations with varying time direction and with non-characteristic degeneration line. A boundary-value problem for parabolic equation of higher order with varying time direction

$$(-1)^k \frac{\partial^{2k} u}{\partial x^{2k}} + x \frac{\partial u}{\partial t} = f(x,t)$$

is investigated in [10].

In the present paper we study a boundary-value problem for degenerate parabolic equation of higher order with varying time direction

$$|t|^m \frac{\partial^{2k} u}{\partial x^{2k}} + (-1)^k \operatorname{sgn} t \cdot \frac{\partial u}{\partial t} = f(x,t), \tag{1}$$

where $m > 0$, $k \in N$ is a fixed number, $f(x,t)$ and $\Omega = \{(x,t) : 0 < x < p, -T < t < T\}$ are the given function and domain.

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