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Multimodular Spaces and Their Properties

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Abstract—We introduce the so-called multimodular spaces and their particular cases. These investigations generalize some definitions and theorems obtained by S. Mazur, W. Orlicz, and J. Muzielak.

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INTRODUCTION

In this paper we present generalizations of modular spaces and generalized Orlicz spaces, which were introduced in the book by J. Musielak [1]. The author has papers [2–4] about functional systems in spaces E_φ, where final results were obtained. Therefore, we consider the introduction of multimodular spaces which describe physical process more precisely and in more natural way. At that, one mathematical model takes into account a lot of physical parameters. With all the abundance of mathematical models, there was no such space connection before. In this paper we do not consider questions on convergence of functional series in any spaces (see, e.g., [2, 5, 6]), and study topological properties (metricity, completeness, separability) and the embedding of new spaces. Definitions 1–4, 5(A), 9, 11, 12 we take from [1].

Definition 1. Functional $\|\cdot\| : E \to [0,\infty)$ in vector space E is called F-pseudonorm, if following conditions are satisfied: 1) $\|0\| = 0$; 2) $\|ax\| = \|x\|$ for $a \in L$ with $|a| = 1$, where L is a set of real or complex numbers; 3) $\|x + y\| \le \|x\| + \|y\|$; 4) if $\gamma_n \to \gamma$ and $\|x_n - x\| \to 0$, then $\|\gamma_n x_n - \gamma x\| \to 0$.

Definition 2. If conditions 1)–4) from Definition 1 and condition 5) $\|x\| = 0$ implies $x = 0$ are satisfied, then the functional $\|\cdot\| : E \to [0,\infty)$ is called F-norm.

Definition 3. If the functional $\|\cdot\| : E \to [0,\infty)$ satisfies conditions 1)–3) from Definition 1 and condition 4′) $\|\alpha x\| = |\alpha|^s\|x\|$, $\alpha \in L$, $0 < s \le 1$, then it is called s-homogeneous pseudonorm. If, besides, condition 5) from Definition 2 is satisfied, then it is called s-norm in E and is denoted by $\|\cdot\|^s$.

A space E with F-norm (norm) we will call F-space (B-space) if it is complete.

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