ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2008, Vol. 52, No. 8, pp. 58–60. © Allerton Press, Inc., 2008.
Original Russian Text © V.I. Zhegalov, 2008, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2008, No. 8, pp. 70–72.

A Problem with Normal Derivatives in Boundary Conditions for a System of Differential Equations

V. I. Zhegalov[1*]

[1]*Kazan State University, ul. Kremlyovskaya 18, Kazan, 420008 Russia*

Received June 9, 2007

Abstract—We seek for a solution to a system of differential equations, using linear relations connecting normal derivatives of the desired functions at the domain boundary.

DOI: 10.3103/S1066369X08080070

Key words and phrases: *hyperbolic systems, boundary-value problems, solvability conditions.*

Brief communication

The study of problems with normal derivatives in boundary conditions for the equation $u_{xy} + au_x + bu_y + cu = 0$ and its three-dimensional analog was commenced in papers [1, 2] and continued in subsequent publications. In this paper, we study a similar problem for the system of equations

$$u_x = av, \;\; v_y = bu, \;\; a, a_y, b, b_x \in C(\overline{D}) \tag{1}$$

in the domain $D = \{x_0 < x < x_1, \, y_0 < y < y_1\}$. One can extend the reasoning and conclusions (with certain complication of formulas) onto a more general system

$$u_x = av + cu + f_1(x, y), \quad v_y = bu + dv + f_2(x, y).$$

Note that system (1) was studied in [3–6] and other papers from the different points of view. In particular, in [7] (see also [8], pp. 169–174) one obtains conditions for the unique solvability in D of the problem with boundary correlations

$$\begin{aligned} \alpha_{11}(y)u(x_0, y) + \alpha_{12}(y)v(x_0, y) = m_1(y), \quad y \in (y_0, y_1) = h, \\ \alpha_{21}(x)u(x, y_0) + \alpha_{22}(x)v(x, y_0) = m_2(x), \quad x \in (x_0, x_1) = k. \end{aligned} \tag{2}$$

Problem Z. *Find functions* $u, v \in C(\overline{D}) \cap C(D \cup \overline{h} \cup \overline{k})$*, which represent a solution to system* (1) *in* D*, satisfying the conditions*

$$\begin{gathered} \beta_{11}(y)u_x(x_0, y) + \beta_{12}(y)v_x(x_0, y) = n_1(y), \quad y \in \overline{h}, \\ \beta_{21}(x)u_y(x, y_0) + \beta_{22}(x)v_y(x, y_0) = n_2(x), \quad x \in \overline{k}, \\ \beta_{11}, \beta_{12}, n_1 \in C(\overline{h}), \;\; \beta_{21}, \beta_{22}, n_2 \in C(\overline{k}). \end{gathered} \tag{3}$$

Evidently, the correlation between (3) and (2) is similar to that of the conditions of the Neumann and Dirichlet problems in the theory of elliptic equations.

1. Existence of a solution. We seek for a solution, reducing Problem Z to the Goursat problem for the same Eq. (1) with the boundary conditions

$$u(x_0, y) = \varphi(y), \;\; v(x, y_0) = \psi(x). \tag{4}$$

Taking into account (4), we integrate the first equation in (1) with respect to x and we do the second one with respect to y, we substitute the obtained result in (3), and after certain transformations we obtain

*E-mail: valentin.zhegalov@ksu.ru.