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Projection Methods for the Solution of One Class of Singular Integrodifferential Equations

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Abstract—In this paper we propose a computational scheme of the general projection method for the solution of singular integrodifferential equations in the theory of stream lines and thermal conductivity. We theoretically substantiate this scheme from the standpoint of the theory of approximate methods of functional analysis. We consider particular cases of the general projection method, namely, the method of moments and the collocation method.

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This paper is dedicated to projection methods for the solution of singular integrodifferential equations in the form

$$Kx \equiv x'(t) + a(t)x(t) + \frac{b(t)}{\rho(t)} \int_{-1}^{1} \frac{\rho(\tau)x(\tau)}{\tau - t} d\tau = f(t), \quad -1 < t < 1, \tag{1}$$

subject to the initial condition

$$x(-1) = 0, \tag{2}$$

where $\rho(t)$ is a weight function, $a(t), f(t) \in L_{2\rho}[-1,1]$, $b(t) \in C[-1,1]$, and a singular integral is understood in the sense of the Cauchy principal value [1–3]. Many problems of the theory of stream lines and thermal conductivity result in equations of such a type (e.g., [4–7]).

As a rule, one cannot solve problem (1)–(2) exactly, therefore for several particular cases various approximate solution methods are proposed and substantiated (e.g., [8–11] and references therein). However, in spite of the above said, many questions in this area remain unsolved. This paper fills this gap to some extent.

In this paper we essentially use the following results of monographs [12] and [9]. Particular cases of problem (1)–(2) are considered in papers [10] (with $\rho(t) = \frac{1}{\sqrt{1-t^2}}$) and [13] (with $\rho(t) \equiv 1$).

Let us describe the results and prove them.

Consider the operator K from (1) in a pair of functional spaces (X, Y), where $Y = L_{2\rho}[-1,1]$ is a weight Hilbert space of square summable on $[-1,1]$ functions with the scalar product

$$(f,g) = \int_{-1}^{1} \rho(t) f(t) g(t) dt \quad (f, g \in L_{2\rho}[-1,1])$$

and the norm

$$\|f\|_{L_{2\rho}[-1,1]} = \left\{ \int_{-1}^{1} \rho(t) |f(t)|^2 dt \right\}^{\frac{1}{2}}, \quad f \in L_{2\rho}[-1,1];$$

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