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THE RIEMANN METHOD FOR A THREE-DIMENSIONAL HYPERBOLIC EQUATION OF THIRD ORDER

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1. The classical Riemann method for explicit solving the Cauchy problem for the equation

$$u_{xy} + au_x + bu_y + cu = f$$

is well-known (see [1], Chap. 5, § 3, p. 68; [2], Chap. 3, § 3.4, p. 195; [3], Chap. 1, § 4, p. 67, and others). It simultaneously implies the proof of existence, uniqueness, and continuous dependence of the solution on boundary values. In the present article we propose an analogous method for the following equation:

$$L(u) \equiv u_{xyz} + au_{xy} + bu_{yz} + cu_{xz} + du_x + eu_y + fu_z + gu = F. \tag{1}$$

2. We consider a surface S of the class C^3, defined via the equations:

$$\begin{cases} x = x(p,q), \\ y = y(p,q), \\ z = z(p,q), \quad (p,q) \in G^2 \subset R^2, \end{cases} \qquad \text{rang} \begin{pmatrix} x_p & y_p & z_p \\ x_q & y_q & z_q \end{pmatrix} = 2$$

in the space $\mathbb{R}^3$ with the fixed Cartesian coordinate system (x, y, z).

We assume that the tangent plane to S at any of its points is not parallel to any of the coordinate axes, e.g., $z_x > 0$, $z_y > 0$ in G. Let us consider a point $M(x_0, y_0, z_0)$ and planes $x = x_0$, $y = y_0$, $z = z_0$. These planes intersect S along the curves QC, CP, and PQ, respectively. We denote by Ω a finite domain bounded by these planes and by a part QCP of the surface S. We assume that Ω has the positive orientation.

Statement of the problem: *Find a solution of equation* (1) *in the class* $C^3(\overline{\Omega})$, *satisfying the boundary value conditions*

$$\left.\frac{\partial^k u}{\partial l^k}\right|_S = \psi_k, \qquad k = 0, 1, 2, \tag{2}$$

where l is a given field of directions on S, which is non-tangential with respect to this surface.

Thus, we seek for a regular solution of problem (1)–(2) in the whole domain Ω.

3. We so determine a direction field l by terms of the vector $\vec{a}(l_1(p,q), l_2(p,q), l_3(p,q))$, $\vec{a} \in C^3(G^2)$, that $|\vec{a}| \equiv 1$. Let us introduce a coordinate system connected with the surface S

$$\begin{cases} x = x(p,q) + l_1(p,q)l, \\ y = y(p,q) + l_2(p,q)l, \\ z = z(p,q) + l_3(p,q)l, \quad l \in R. \end{cases} \tag{3}$$