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ON THE LINEAR INVERSE FROM RIGHT OPERATOR FOR THE CONVOLUTION OPERATOR ON THE SPACES OF GERMS OF ANALYTICAL FUNCTIONS ON CONVEX COMPACTS IN $\mathbb{C}$

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Introduction

For a convex compact G in $\mathbb{C}$ we denote by $A(G)$ the space of all germs of the analytical on G functions. Let us provide this space by the natural inductive limit topology. If K is a convex compact in $\mathbb{C}$, then every analytical functional $\mu \in A(\mathbb{C})' \setminus \{0\}$, determined by K, generates a linear continuous operator of convolution

$$T_\mu : A(G+K) \to A(G), \quad T_\mu(g)(z) := \mu(g(\cdot + z)), \quad g \in A(G+K)$$

(here $G+K$ stands for arithmetical sum of the sets G and K). If $K = \{0\}$, then T_μ is a differential operator on $A(G)$ (of a finite or infinite order) with constant coefficients. In this case, T_μ is surjective (see [1], [2]). If $K \neq \{0\}$ and the interior of G is not empty, the characterization of surjective operators T_μ is well-known (see [3]–[6]). In the present article we shall study in what cases a given surjective convolution operator $T_\mu : A(G+K) \to A(G)$ possesses an inverse linear continuous right-side operator (LCRO) R, i. e., when there is a solution $R(f) \in A(G+K)$ of the convolution equation $T_\mu(R(f)) = f$, which depends linearly and continuously on $f \in A(G)$.

In this direction of investigations at the present time there are known the following results. If $K = \{0\}$ and G coincides with a point, then by the result by Meise and Taylor (see [7]) there follows that only a differential operator T_μ of a finite order possesses an LCRO. For a different from a point closed circle G and $K = \{0\}$ it was proved by Yu.F. Korobeĭnik (see [8]) that any operator T_μ possesses an LCRO. Put $\hat{\mu}(z) := \mu(\exp(\cdot z))$, $z \in \mathbb{C}$; we denote by A_μ the set of all limit points of the sequence $(a/|a|)_{\{a \mid \hat{\mu}(a)=0\}}$ (given a finite sequence, we assume $A_\mu = \varnothing$). Langenbruch (see [9]) established in the case where $G = [-1,1] \subseteq \mathbb{R}$, $K = \{0\}$ that T_μ possess an LCRO if and only if $A_\mu \subseteq \{-i, i\}$.

1. Preliminary information

Everywhere in what follows both G and K are convex compacts in $\mathbb{C}$ and $\mu \in A(K)' \setminus \{0\}$. A linear continuous convolution operator $T_\mu : A(G+K) \to A(G)$ is defined via equality

$$T_\mu(g)(z) := \mu(g(\cdot + z)), \quad g \in A(G+K).$$

We put $\hat{\mu}(z) := \mu(\exp(\cdot z))$, $z \in \mathbb{C}$; $\hat{\mu}$ is an entire function of exponential type, whose conjugate diagram is contained in K (see [10], Chap. 1, § 20; [11], theorem 4.5.3). We put $V(\hat{\mu}) := \{z \in \mathbb{C} \mid \hat{\mu}(z) = 0\}$.

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