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p-SATURATED FORMATIONS WITH COMPLEMENTED p-SATURATED SUBFORMATIONS

N. G. Zhevnova and A. N. Skiba

All groups under consideration in the present article are supposed to be finite.

A formation of groups $\mathfrak{F}$ is said to be *p-saturated* if from $G/L \in \mathfrak{F}$, where $L \subseteq O_p(G) \cap \Phi(G)$, it always follows $G \in \mathfrak{F}$. The meaning of the notion of a p-saturated formation is related first of all to the following observation by L.A. Shemetkov (see [2]): If the product $\mathfrak{M}\mathfrak{H}$ of formations $\mathfrak{M}$ and $\mathfrak{H}$ is saturated, then the formation $\mathfrak{H}$ is p-saturated for all $p \in \pi(\mathfrak{H}) \setminus \pi(\mathfrak{M})$.

In [3], there were found various characterizations of the classes of p-saturated formations. In particular, there was proved that a formation $\mathfrak{F}$ is p-saturated if and only if $\mathfrak{N}_p\mathfrak{F}(p) \subseteq \mathfrak{F}$, where

$$\mathfrak{F}(p) = \begin{cases} \mathrm{form}(G/F_p(G) \mid G \in \mathfrak{F}) & \text{for } p \in \pi(\mathfrak{F}); \\ \varnothing & \text{for } p \in \pi'(\mathfrak{F}). \end{cases}$$

Basing on this result, we shall describe in the present article p-saturated formations whose lattice of p-saturated subformations is Boolean.

Let us recall that a subformation $\mathfrak{M}$ of a formation $\mathfrak{F}$ is said to be *complemented* in $\mathfrak{F}$ [4] if $\mathfrak{M}$ is complemented in the lattice of subformations of the formation $\mathfrak{F}$, i. e., if in $\mathfrak{F}$ there is a subformation $\mathfrak{H}$ such that

$$\mathfrak{M} \cap \mathfrak{H} = (1), \quad \mathfrak{M} \vee \mathfrak{H} = \mathrm{form}(\mathfrak{M} \cup \mathfrak{H}) = \mathfrak{F}.$$

We should note that study of formations with systems of complemented saturated subformations can be found in [5]–[7].

We shall base on terminology adopted in [8], [9], and [11]. We denote by $\mathrm{lform}_p\,\mathfrak{X}$ the meet of all those p-saturated formations which contain the class of groups $\mathfrak{X}$. For arbitrary class of groups $\mathfrak{X}$ we denote by $\mathfrak{X}/O_p(\mathfrak{X})$ the following: $\{A/O_p(A) \mid A \in \mathfrak{X}\}$.

Lemma 1 ([10]). *For any class of groups $\mathfrak{X}$ there takes place* $\mathrm{lform}_p\,\mathfrak{X} = \mathrm{form}(\mathfrak{X} \cup \mathfrak{N}_p\mathfrak{X}(p))$.

Lemma 2. *Let $\mathfrak{H}$ and $\mathfrak{M}$ be subformations of a formation $\mathfrak{F}$ and let, for arbitrary $p \in \pi(\mathfrak{M})$, the formation $\mathfrak{F}$ be p-saturated. In this situation, if the formation $\mathfrak{M}$ is a hereditary one and $\mathfrak{H}$ is the complement to $\mathfrak{M}$ in $\mathfrak{F}$, then $\pi(\mathfrak{M}) \cap \pi(\mathfrak{H}) = \varnothing$.*

Proof. Let us suppose that $p \in \pi(\mathfrak{M}) \cap \pi(\mathfrak{H})$. Then in $\mathfrak{H}$ there can be found a non-unit group G, whose certain main factor H/K has an order dividable by p. Let $T = G/K$ and $L_1 \times \cdots \times L_{i-1} \times L_i \times L_{i+1} \times \cdots \times L_t$ be the socle of the group T, where L_j is a minimal normal in T subgroup ($1 \le j \le t$) and $L_i = H/K$. We denote by M the normal subgroup in group T of the most order among all its normal subgroups which contain a subgroup $L_1 \times \cdots \times L_{i-1} \times L_{i+1} \times \cdots \times L_t$, but do not contain the subgroup L_i. Then the group $A = T/M$ is evidently monolithic and its monolith coincides with L_iM/M. Since $L_iM/M \simeq H/K$, we have $O_{p'}(A) = 1$.