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The Application of the Wavelet Transform in the Numeric Averaging of Differential Equations with Quickly Oscillating Coefficients and in Calculation of Effective Characteristics

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INTRODUCTION

We consider the problem of the numerical averaging of elliptic differential equations in the form

$$-\nabla^T(K(x,y)\nabla u) + Q = 0, \quad x, y \in [0,1], \tag{1}$$

where $K(x,y)$ is a quickly oscillating coefficient. The direct numeric solution of equations in form (1) (for example, by the method of finite differences or the method of finite elements) requires significant computational costs, because it implies the use of a computational mesh with a very small step.

The most known among the methods of numerical averaging is the asymptotic averaging method for periodic media proposed by N. S. Bakhvalov [1]. In this method one deduces the correlations which connect two scales, micro and macro ones, for the boundary-value problem $L_\varepsilon u_\varepsilon = f_\varepsilon$; here ε is a small parameter such that $u_\varepsilon \to \overline{u}$ with $\varepsilon \to 0$, $\overline{u}$ is the averaged solution. The averaging problem implies that one has to find $\overline{L}$ and $\overline{f}$ such that $\overline{u}$ satisfies the differential equation $\overline{L}\overline{u} = \overline{f}$.

In this paper, we propose an averaging method based on the multiscale analysis with the wavelet projection and approximation of the discrete operator.

1. THE WAVELET AVERAGING ON THE BASE OF THE METHOD OF FINITE ELEMENTS

In the one-dimensional case we understand the wavelets and the scaling functions as the functions which form a basis of the space $L_2(\mathbb{R})$ and are obtained by a shift and a compression of one function $\phi_{j,k}(x) = 2^{j/2}\phi(2^j x - k)$, $\psi_{j,k}(x) = 2^{j/2}\psi(2^j x - k)$, $j, k \in \mathbb{Z}$; here $\mathbb{Z}$ is the space of integer numbers, ψ and ϕ are, respectively, the wavelet and the scaling function of the one-dimensional Haar basis:

$$\psi(x) = \begin{cases} 1, & x \in [0, 1/2); \\ -1, & x \in [1/2, 1], \end{cases} \qquad \phi(x) = \begin{cases} 1, & x \in [0,1]; \\ 0, & x \notin [0,1]. \end{cases}$$

A two-dimensional wavelet basis is formed by functions of the one-dimensional basis. Consider the space $L_2([0,1] \times [0,1])$. Introduce the sequence of nested spaces

$$\mathcal{V}_j = \operatorname{span}\{\phi_{j,k_1} \otimes \phi_{j,k_2},\ k_i \in \mathbb{Z}\},$$
$$\mathcal{W}_j = \operatorname{span}\{\psi_{j,k_3} \otimes \phi_{j,k_4}, \phi_{j,k_5} \otimes \psi_{j,k_6}, \psi_{j,k_7} \otimes \psi_{j,k_8},\ k_i \in \mathbb{Z}\},$$

where the symbol $\otimes$ means the operation of the tensor product of functions of the one-dimensional Haar basis $\psi_{j,k_i} \otimes \psi_{j,k_l}(x,y) = \psi_{j,k_i}(x)\psi_{j,k_l}(y)$.

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