

RIEMANN PROBLEM ON ULTRAHYPERELLIPTIC SURFACE

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In [1], [2], the solution in quadratures of the boundary value Riemann problem was obtained on a compact Riemann surface. In [3] and [4], on a noncompact Riemann surface, its Noether theory was constructed and the defect numbers were evaluated. On an arbitrary open Riemann surface this problem was solved explicitly only in cases where the coefficient of the problem equals either 1 (see [3]) or -1 (see [5]). In the present article we obtain conditions for the resolvability of the boundary value Riemann problem on an ultrahyperelliptic surface R in case of an arbitrary piecewise smooth contour Γ and give its explicit solution. In addition, this solution (in general case) is expressed via integrals whose kernels are analogs of the Cauchy kernel on a certain auxiliary hyperelliptic surface. In case the contour Γ fails to partition the surface R , it turns out that one can apply more elementary means than in solving the Riemann problem on a compact surface (in particular, hyperelliptic one). Namely, the Jacobi inversion problem (or any its analog) is not used, instead of analogs of the Cauchy kernel on a Riemann surface we use the ordinary Cauchy kernel on the complex plane. Note that the requirement of Λ_0 -behavior of the desired function (which was used in [3]–[5]) means its boundedness in a neighborhood of the ideal boundary.

1. Preliminary information and notation

Let (R, z) be an unbounded two-sheeted covering of the complex plane \mathbb{C} , whose branching points are condensed to a unique point of the ideal boundary, lying over $z = \infty$. The covering (R, z) can be realized as the Riemann surface of a function $u(z)$, which is determined via the equation $u^2 = P(z)$, where $P(z)$ is an entire function for an infinite quantity of simple zeros. The Riemann surface R , admitting such a realization, is said to be ultrahyperelliptic. We denote by j_R a certain, different from identical, covering transformation of the covering (R, z) , i.e., a conformal automorphism of the surface R , satisfying the relation $z(j_R(q)) = z(q)$ for all $q \in R$. Let Γ be a piecewise smooth line on R and $\zeta(q)$ a holomorphic function in a neighborhood of Γ , such that $d\zeta$ has no zeros on Γ . Then ζ is a local uniformization in a neighborhood of any point $\tau \in \Gamma$. If Γ contains no branching points of the covering (R, z) , then one can take $\zeta = z$. If $\tau \in \Gamma$ is a branching point, then in its certain neighborhood both z and ζ are connected via relation of the form $z - z(\tau) = (\zeta - \zeta(\tau))^2 a(\zeta)$, where $a(\zeta)$ is a function holomorphic at the point $\zeta = \zeta(\tau)$ and $a(\zeta(\tau)) \neq 0$.

Let $\alpha : [0, 1] \rightarrow R$ be a path on R . By its *length* we shall call the length of the flat path $z \circ \alpha : [0, 1] \rightarrow \mathbb{C}$, i.e., $\int_0^1 |d(z \circ \alpha)|$. By a *distance* $\rho(p, q)$ between the points p and q on R we shall call the greatest lower bound of lengths of all paths connecting p and q . Using the local univalence of the function $\zeta(q)$, one can easily prove that a number $\epsilon > 0$ can be found such that from the inequality $\rho(t_1, t_2) \leq \epsilon$, $t_1 \neq t_2$ the inequality $\zeta(t_1) \neq \zeta(t_2)$, $t_1, t_2 \in \Gamma$, follows.

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