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On Scalarization of Vector Optimization Type Problems

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Abstract—We consider scalarization issues for vector problems in the case where the preference relation is represented by a rather arbitrary set. An algorithm for weights choice for a priori unknown preference relations is suggested. Some examples of applications to vector optimization, game equilibrium, and variational inequalities are described.

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Problems with vector criteria often appear in models related to decision making in complex systems. The presence of a vector criterion leads to additional difficulties in definition of a solution concept as well as in investigation of its properties and development of its numerical methods, in comparison with the corresponding scalar problems. Usually, some preference relation in the space of criteria (estimates) is used for the definition of a solution concept, the Pareto relation being most popular; see, e.g., [1, 2]. Such a preference relation is usually incomplete. As a result, the set of solutions (non-dominated variants) appears too broad and it is not sufficient for decision making. The scalarization method based on the (linear) convolution, which is widely applicable for vector problems, meets certain difficulties in weights assignment, especially in the case where the criteria are non homogeneous. At last, the approach to preference definition with the help of a convex cone (see, e.g., [3, 4]) implies a number of additional conditions such as homogeneity and additivity of preferences, which may appear too restrictive in applications.

In this paper, we consider all these drawbacks jointly, reveal new relationships among them and suggest general approaches for their overcoming under rather weak assumptions. In particular, an algorithm for weights choice for a priori unknown preference relations is suggested. Besides, some examples of applications of the results obtained to vector optimization, game equilibrium, and variational inequalities are described.

1. AUXILIARY PROPERTIES

In this Section, we give some results of convex analysis, which will be used in the future considerations. Note that some close results are contained in many works (see e.g. [5–9]) but those are usually proved under somewhat different assumptions. For this reason, we now give full proofs.

A set K is said to be a cone if for any point $x \in K$ it holds that $\lambda x \in K$ for an arbitrary number $\lambda \geq 0$. For a set Q in the space $\mathbb{R}^m$ one can define the polar (or conjugate) cone

$$Q^\star = \{p \in \mathbb{R}^m \mid \langle q, p \rangle \geq 0 \ \ \forall q \in Q\},$$

where $\langle \cdot, \cdot \rangle$ denotes the scalar product. It is clear that $Q^\star$ is a convex and closed cone. If K is a convex and closed cone in $\mathbb{R}^m$ then $K^{\star\star} = \overline{K} = K$. For a cone K we set $S(K) = K \bigcap S(\mathbf{0}, 1)$ where $S(\mathbf{0}, 1) = \{z \mid \|z\| = 1\}$.

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