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ON THE SOLVABILITY OF A NONLINEAR EVOLUTIONARY INEQUALITY OF THE THEORY OF JOINT MOTION OF SURFACE AND GROUND WATERS

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In the present article we study an evolutionary variational inequality degenerating on both the solution and its gradient. Such inequalities arise, for instance, in the modeling of the processes of joint motion of ground and surface waters and are characterized by the fact that a process is considered in a domain with a slit on which an extra condition is given in the form of one-dimensional partial differential either equation or inequality.

Under sufficiently general assumptions on the smoothness of the original data, an existence theorem is proved for a non-negative solution in a class of generalized functions. It is proved by a semidiscretization method with a penalty. A sequence of solutions of the penalized semidiscrete problem is shown to converge to the solution of the original inequality.

Equations of that kind and their solution methods were considered in [1], [2].

1. Problem statement

Let Ω be a bounded domain in R_2, Γ being its boundary, Π be a slit lying in Ω and partitioning it into two connected subdomains.

We introduce a notation for the Banach spaces to be used in the sequel. Let $\overset{\circ}{V}$, $\overset{\circ}{V}(0,T)$, $W(0,T)$ be functional Banach spaces with the norms

$$\|u\|_{\overset{\circ}{V}} = \|u\|_{\overset{\circ}{W}{}^1_{p_1}(\Omega)} + \|u\|_{\overset{\circ}{W}{}^1_{p_2}(\Pi)} < \infty,$$

$$\|u\|_{\overset{\circ}{V}(0,T)} = \|u\|_{L_{p_1}(0,T;\overset{\circ}{W}{}^1_{p_1}(\Omega))} + \|u\|_{L_{p_2}(0,T;\overset{\circ}{W}{}^1_{p_2}(\Pi))} < \infty,$$

$$\|u\|_{W(0,T)} = \|u\|_{\overset{\circ}{V}(0,T)} + \|u\|_{L_\infty(0,T;L_{\alpha_1}(\Omega))} + \|u\|_{L_\infty(0,T;L_{\alpha_2}(\Pi))} < \infty.$$

Further, let z be an element from $\overset{\circ}{V}(0,T)$. We denote by $\Phi(z(t),v)$ a functional, whose value on $v \in \overset{\circ}{V}$ for $t \in [0,T]$ is determined by

$$\Phi(z(t),v) = \frac{d}{dt}\Big(\int_\Omega \omega_1(z(t))v(x)dx + \int_\Pi \omega_2(z(t))v(s)ds\Big).$$

Consider the following problem: Find a function

$$u \in K = \{v \in W(0,T), v(x,t) \geq 0$$
$$\text{almost everywhere (a.e.) in } Q_T = \Omega_T \times [0,T] \text{ and on } \Pi_T = \Pi \times [0,T]\},$$

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