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A Method of Algebraic Extension of the Lagrangian of Weak Interactions to Nonassociative Algebra

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Abstract—We propose an algebraic approach to the extension of the Lagrangian of weak interactions to a nonassociative algebra. A special feature of the proposed method is the use of matrix representations (instead of vector ones) for physical fields and their interactions. We construct the Lagrangian of material and interaction fields on the introduced algebra.

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INTRODUCTION

The algebra of octonions, in terminology by Arthur Cayley, or the algebra of octaves, in terminology by John Graves, is the unique generalization of the algebra of quaternions which is a division algebra. It is known [1] that the first publication describing this algebra belongs to A. Cayley and is dated 1845 (two years after the discovering of the quaternions by Hamilton).

Apparently, the first attempt to apply the algebra of octonions to a physical theory was made in [2], where the octonion quantum mechanics was introduced. However, the ideas of that paper had no serious development, possibly for the reason that mathematical and physical concepts of that time were not sufficiently advanced or for the reason that the algebraic octonion apparatus of the paper was too abstract. Not long before, in the paper [3] by M. Zorn, matrix representations of the algebra of octonions were proposed, but elements of Zorn's matrices were special objects: Numbers and vectors. Therefore, this paper specified a way for representation of a nonassociative algebra by matrices, but the proposed form of matrices was very abstract. In the paper [4] by J. Debout and R. Delbago, the octonions were literally "inscribed" into the physical theory: A representation of the octonions by the Dirac matrices with a special multiplication rule was found. But this paper, by author's opinion passed unnoticed by physicists.

At the present time, the algebra of octonions is used rather widely by physicists (for example, in the M-theory), but its application is usually based on the use of the group symmetry G_2. Since the theory is developed on the base of the group approach, the Lagrangian does not contain initially nonassociative terms.

The urgency of author's investigations of the extension of the Weinberg–Salam group theory of weak interactions to a nonassociative algebra is dictated by the necessity to find new approaches for constructing the theory which will combine general coordinate and gauge transformations. In this case, there is a hope to include the group of general coordinate transformations as the manifestation of nonassociativity of interaction. In the present paper, we make only a step in the investigation of the problems which can be met on this way and possible solutions of these problems, and we suggest a Lagrangian of lepton fields on the constructed nonassociative algebra.

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