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ON THE COMPLEXES OF TWO-DIMENSIONAL PLANES IN THE PROJECTIVE SPACE WITH A SINGLE CRITICAL LINE

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In the present article we study classes of complexes K_0 of 2-planes in P_n (see [1]) such that every plane has a unique critical line L_1^β (see [2]). The line is invariantly related to the plane of the complex and, therefore, when the parameters of the complex vary, it describes a certain family of lines. By using the family, the structure of complexes K_0 with such lines is clarified.

Let the complex K_0 in the frame $\{A_I\}$, $I = 0, 1, 2, \dots, n$ be defined by the system of equations (see [1]):

$$\Lambda_p^{\alpha i}\omega_i^p = 0, \quad \alpha = 1, 2, \dots, n-2; \quad i = 0, 1, 2; \quad p = 3, 4, \dots, n. \tag{1}$$

In this case the lines L_1^β, $\beta = 1, 2, \dots, n-2$ of the complex K_0 (see [1]) will belong the a curve q of the $(n-2)$-nd class, having the equation

$$\det \|\Lambda_p^{\alpha i} a_i\| = 0, \tag{2}$$

where a_i are the tangential coordinates of the line L_1 in the plane of the complex $L = (A_0A_1A_2)$.

We shall study complexes K_0, in each plane of which there is only one line L_1^β, $2 \le \beta \le n-3$. Exception for the extreme cases $\beta = 1$ and $\beta = n-2$ is made by virtue of the fact that the lines L_1^1 do not exist alone, while the lines L_1^{n-2} appear only in the case where the rank R_i of the matrix $\|\Lambda_p^{\alpha i}\|$ is lowered.

To the line L_1^β we relate the plane of the principal correspondence $l_{2+\beta}$ (see [2]). In view of the uniqueness of the line L_1^β, we fix the frame: $L_1^\beta = (A_0A_1)$, $l_{2+\beta} = (L,\ A_3A_4 \cdots A_{\beta+2})$. Then we have $\Lambda_3^{\alpha 2} = \Lambda_4^{\alpha 2} = \dots = \Lambda_{\beta+2}^{\alpha 2} = 0$, $\alpha = 1, 2, \dots, n-2$, and system (1) takes the form

$$\Lambda_p^{\alpha j}\omega_j^p + \Lambda_q^{\alpha 2}\omega_2^q = 0, \quad j = 0, 1; \quad q = \beta+3, \beta+4, \dots, n, \quad p = 3, 4, \dots, n, \tag{3}$$

furthermore,

$$R\|\Lambda_q^{\alpha 2}\| = n - 2 - \beta. \tag{4}$$

From (4) it follows that we can solve $(n-2-\beta)$ equations of system (3) with respect to the forms ω_2^q and then eliminate the forms from the remaining β equations. By numbering the resulting equations in such a way that the first β equations do not include the forms ω_2^q, we can rewrite system (3) in the form

$$\Lambda_p^{\gamma j}\omega_j^p = 0, \quad \omega_2^q + \Lambda_p^{q-2, j}\omega_j^p = 0, \quad \gamma = 1, 2, \dots, \beta. \tag{5}$$