ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2013, Vol. 57, No. 3, pp. 46–50. © Allerton Press, Inc., 2013.
Original Russian Text © F.M. Ablayev and K.R. Khadiev, 2013, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2013, No. 3, pp. 56–61.

Extension of the Hierarchy for k-OBDDs of Small Width

F. M. Ablayev* **and K. R. Khadiev****

Kazan (Volga Region) Federal University, ul. Kremlyovskaya 18, Kazan, 420008 Russia

Received July 05, 2012

Abstract—In this paper we explore the well-known k-OBDD model of branching programs. We develop a method of representation of the k-OBDD computation process as an "automata-communication protocol" computation process. Our method allows us to extend the hierarchy proved by Bolling–Sauerhoff–Sieling–Wegener in 1996 for k-OBDDs. Moreover, using the PJM function (a modification of well-known PJ and ISA functions), we prove a new hierarchy.

DOI: 10.3103/S1066369X13030067

Keywords and phrases: *branching programs, binary decision diagrams, OBDD, k-OBDD, communication protocol, communication complexity.*

1. PRELIMINARIES AND RESULTS

The k-IBDD, k-OBDD, and OBDD models are well-known models of branching programs. Various models of branching programs are described in the book [1].

A branching program P over a set X of n Boolean variables is a directed acyclic graph with a source node and sink nodes. Sink nodes are labeled with 1 (Accept) or 0 (Reject). Each inner node v is associated with a variable $x \in X$ and has two outgoing edges labeled as $x = 0$ and $x = 1$, respectively. An input $\nu \in \{0,1\}^n$ determines a computation (consistent) path from the source node of P to one of the sink nodes of P. We denote by $P(\nu)$ the label of the sink finally reached by P on the input ν. The input ν is accepted or rejected if $P(\nu) = 1$ or $P(\nu) = 0$, respectively.

Program P computes (presents) Boolean a function $f(X)$ ($f : \{0,1\}^n \to \{0,1\}$) if $f(\nu) = P(\nu)$ for all $\nu \in \{0,1\}^n$.

A branching program is *leveled* if the nodes can be partitioned into levels $V_1, \ldots, V_\ell$ and a level $V_{\ell+1}$ such that nodes in $V_{\ell+1}$ are the sink nodes, nodes in each level V_j with $j \leq \ell$ have outgoing edges only to nodes in the next level V_{j+1}.

The *width* $w(P)$ of a leveled branching program P is the maximum of the number of nodes in levels of P

$$w(P) = \max_{1 \leq j \leq \ell} |V_j|.$$

A leveled branching program is called *oblivious* if all inner nodes of one level are labeled with the same variable. A branching program is called *read once* if each variable is tested on each path only once.

An oblivious leveled read once branching program also called an Ordinary Binary Decision Diagram (OBDD).

An OBDD P reads variables in a particular for P order $\theta(P) = (j_1, \ldots, j_n)$.

A branching program P is called k-OBDD with order $\theta(P)$ if it consist of k layers and each ith layer is an OBDD with the same order $\theta(P)$.

*E-mail: fablayev@gmail.com.
**E-mail: kamilhadi@gmail.com.